# ASTRONOMY

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## A TEXTBOOK

FOURTH EDITION

By

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ASTRONOMY



F-W

SIR,

We are told, and, if I remember right, it is also your Opinion, that three of the finest sights in Nature, are a rising Sun at Sea, a verdant Landskip with a Rainbow, and a clear Star-light Evening.

THOMAS WRIGHT, *1750*

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# PREFACE TO THE FOURTH EDITION

This book is intended to present a general view of the science of the stars, and to be suitable for the use of beginning classes in Astronomy in colleges and universities. While assuming the degree of mental maturity commonly expected of college students, it avoids difficult mathematical discussions and requires no previous study of any science.

The original manuscript was begun in 1924, and the first edition appeared in 1926. The second edition followed in 1930, and the third was published in 1935. About half of my work on the four editions was done at Wellesley College and the other half at the offices of the Mount Wilson Observatory.

To the Directors of the Mount Wilson Observatory I am grateful for the privilege which I have enjoyed during many years, as a member of their staff and as a visitor, of using at times the unrivaled equipment of their Observatory in making many of the photographs which are here reproduced as illustrations. Many other illustrations were contributed by various observatories and individuals, to whom acknowledgment is made in the captions. For advice, assistance, and suggestions I am indebted to many friends, among whom are numerous astronomers, my assistants at Wellesley College, and a number of students and other readers of the book.

JOHN C. DUNCAN

*Wellesley, Massachusetts*
*September, 1945*

# ASTRONOMY

# INTRODUCTION

**Astronomy** is the science of the heavenly bodies. Literally, the word (ἄστρον, star, νέμειν, arrange) means mapping, classifying, or describing the stars.

From analogy with such words as geology, psychology, and zoölogy, it might be expected that the science of the stars would be called astrology (ἄστρον, λέγειν, speaking about stars), and in fact this word was so used centuries ago; but astrologers wandered from the paths of reality into the realms of fancy and superstition, and the name astrology is now applied not to any science, but to fortune-telling by the stars.

**The Heavenly Bodies** are:

1. **The Stars,** which appear to us as tiny glittering points in the night sky, and which are really vast globes of intensely heated gas shining by their own light. They are the most numerous of visible heavenly bodies. They are so far away that their relative motions do not become appreciable to the eye in centuries, and so they are sometimes referred to as "the fixed stars."

2. **The Sun,** one of the stars but much nearer than any other; the source of light, heat, and life upon the Earth.

3. **The Planets,** which look like stars to the unaided eye, but may be distinguished by the fact that their position among the stars may be seen to change in the course of a few nights. In reality, the planets are opaque spheres that shine by reflected sunlight and revolve in nearly circular orbits around the Sun. The Earth itself is a planet, and as such forms a part of the domain of astronomy. The names of the principal planets, in the order of distance from the Sun, are: Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune, and Pluto.

4. **Satellites,** bodies resembling small planets, but revolving around planets instead of around the Sun.

5. **The Moon,** the satellite of the Earth and the nearest of the heavenly bodies.

6. **Comets,** flimsy bodies, some of great size, which revolve around the Sun in elliptic or parabolic orbits. The brightest comets are spectacular objects with long trains or tails that extend over a large part of the sky.

7. **Meteors,** tiny solid objects which fly through space around the Sun like the comets, but which are so small that they cannot be seen until they encounter the Earth and become entangled in its atmosphere, when the heat generated by the stoppage of their swift motion makes them luminous.

8. **Nebulae,** vast aggregations of matter at distances comparable to those of the stars, but so large as to have perceptible size and shape instead of appearing, like the stars, as mere points. The nebulae are all faint, only a few of the many thousand known being visible to the unaided eye.

The Sun, planets, satellites, comets, and meteors comprise the **solar system,** which, vast though it is, forms but a speck in the universe of stars and nebulae.

Astronomy is concerned not only with the heavenly bodies themselves—their nature, distribution, motions, physical conditions, and effects on each other—but also with their **light,** the sole means by which we are made aware of their existence. Light is energy which, emitted by the stars and nebulae, flows through the vast abysses of space with a speed that exceeds all other speeds. The study of its nature, structure, production, and effects is shared by the astronomer and the physicist.

**The Constellations.** In contemplating the stars, which at first glance seem sprinkled at random over the sky, it is natural to group them into geometric figures—triangles, squares, and winding rows; and the imagination can easily people the sky with more elaborate forms. Star-groups to which definite names have been given are called **constellations.** Forty-eight were named by the ancients, mostly in prehistoric times, and mostly for objects or heroes of mythology. Since A.D. 1600 the spaces between these ancient groups, spaces which for the most part contain only inconspicuous stars, have also been named, bringing the whole number of constellations up to eighty-five;[1] and their boundaries have been arbitrarily fixed by an international committee of astronomers so that no portion of the sky is omitted.

Most of the groups bear little resemblance to the objects for which they are named: for example, Pegasus, the winged horse, is most easily recog-

[1] Or eighty-eight, if Argo is divided into four (page 6).

nized by a square of four stars; Andromeda, the chained maiden, is represented chiefly by a nearly straight row of three; and the most conspicuous part of Ursa Major, the bear into which the nymph Callisto was transformed, is called in America the Great Dipper and in Europe the Wain or the Plow. The names of all the constellations are given in Table 1 (page 6); the mythical figures are pictured in Figures 1 and 2; and the principal star groups are represented in Maps 1 to 8. For the fainter stars and the constellation boundaries, the reader may consult the large-scale maps in a modern star atlas.

**Designations of Stars.** The most common method of designating the brighter stars is by a small letter of the Greek alphabet followed by the genitive form of the Latin name of the star's constellation. This method was introduced in A.D. 1601 by Bayer, who generally applied the letters, beginning with $\alpha$, in the order of brightness (but there are notable exceptions). If the number of stars in a constellation exceeds the number of Greek letters, Roman letters also are used.

The Greek alphabet is as follows:

| | | | | | |
|---|---|---|---|---|---|
| Α, $\alpha$ | Alpha | Ι, $\iota$ | Iota | Ρ, $\rho$ | Rho |
| Β, $\beta$ | Beta | Κ, $\kappa$ | Kappa | Σ, $\sigma$, ς | Sigma |
| Γ, $\gamma$ | Gamma | Λ, $\lambda$ | Lambda | Τ, $\tau$ | Tau |
| Δ, $\delta$ | Delta | Μ, $\mu$ | Mu | Υ, $\upsilon$ | Upsilon |
| Ε, $\epsilon$ | Epsilon | Ν, $\nu$ | Nu | Φ, $\phi$ | Phi |
| Ζ, $\zeta$ | Zeta | Ξ, $\xi$ | Xi | Χ, $\chi$ | Chi |
| Η, $\eta$ | Eta | Ο, ο | Omicron | Ψ, $\psi$ | Psi |
| Θ, $\theta$ | Theta | Π, $\pi$, $\varpi$ | Pi | Ω, $\omega$ | Omega |

A few stars are well known by individual names, mostly Latin words such as Polaris ($\alpha$ Ursae Minoris), Pollux ($\beta$ Geminorum), Spica ($\alpha$ Virginis), and Bellatrix ($\gamma$ Orionis); or words taken from the Arabic, as Algenib ($\gamma$ Pegasi), Deneb Kaitos ($\beta$ Ceti), and Mizar ($\zeta$ Ursae Majoris).

For a few faint naked-eye stars which were not given letters in the Bayer system, numbers are used. These were assigned by Flamsteed in the seventeenth century in the order of the right ascension (page 22) of the star within its constellation.

For the vast numbers of telescopic stars, to which no designation has been assigned in any of the above systems, it is customary to give the star's number in some well-known star catalogue (page 319), or, if it is not listed in a catalogue, to give its right ascension and declination at a designated epoch.

**Star "Magnitudes."** The brightness of a star is denoted by a number known as its **magnitude.** This system has come down from Ptolemy, who, about A.D. 150, classified the brightest twenty stars as of the *first* magnitude, those just visible to the naked eye as of the *sixth*, and those of intermediate brightness by intermediate numbers that increase as the

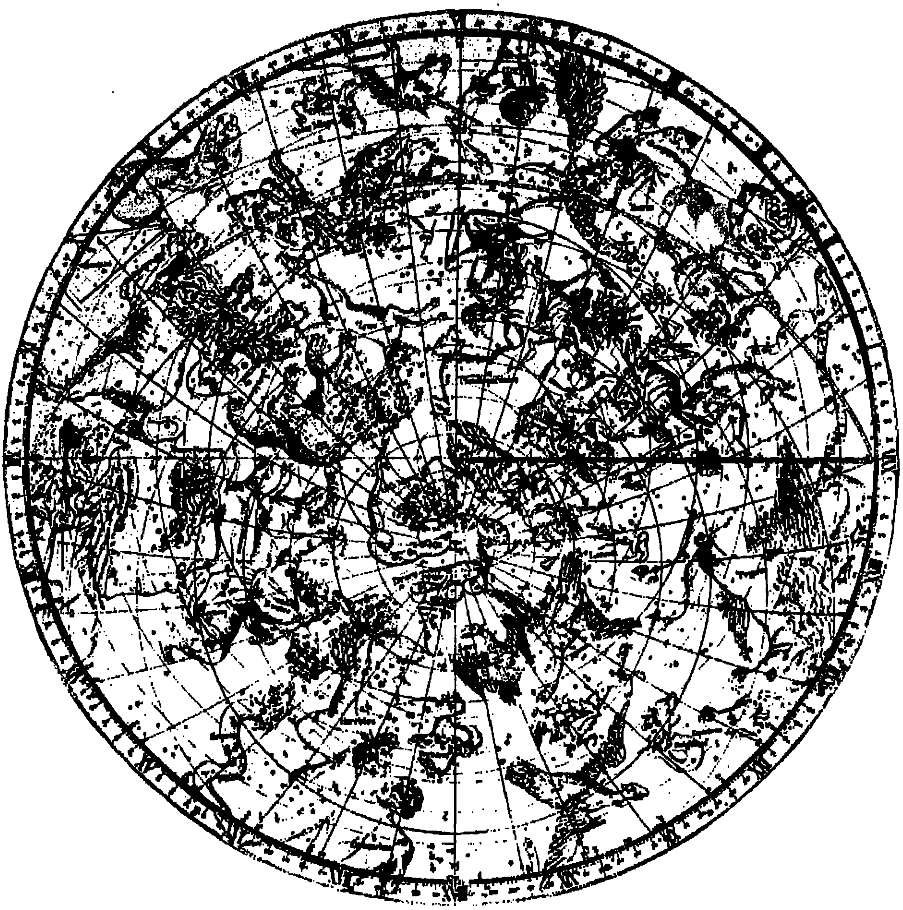

Fig. 1. *Mythical Figures of the Constellations of the Northern Hemisphere.* (*From the Flamsteed* Atlas Coelestis, *1729.*)

brightness grows less. Modern astronomers have refined and extended the scale, using numbers greater than 6 for telescopic stars and numbers less than 1 for the few stars that are brighter than the average of Ptolemy's "first." On this scale the magnitude of Sirius, the brightest of all the stars, and that of Canopus, second-brightest, are negative.

In beginning the study of astronomy, the student should acquaint himself as soon as possible with the more important constellations then visible, and should add others to his acquaintance as they are brought to view in the progress of the year. For this purpose the star maps on the following eight pages should prove useful, especially if supplemented by

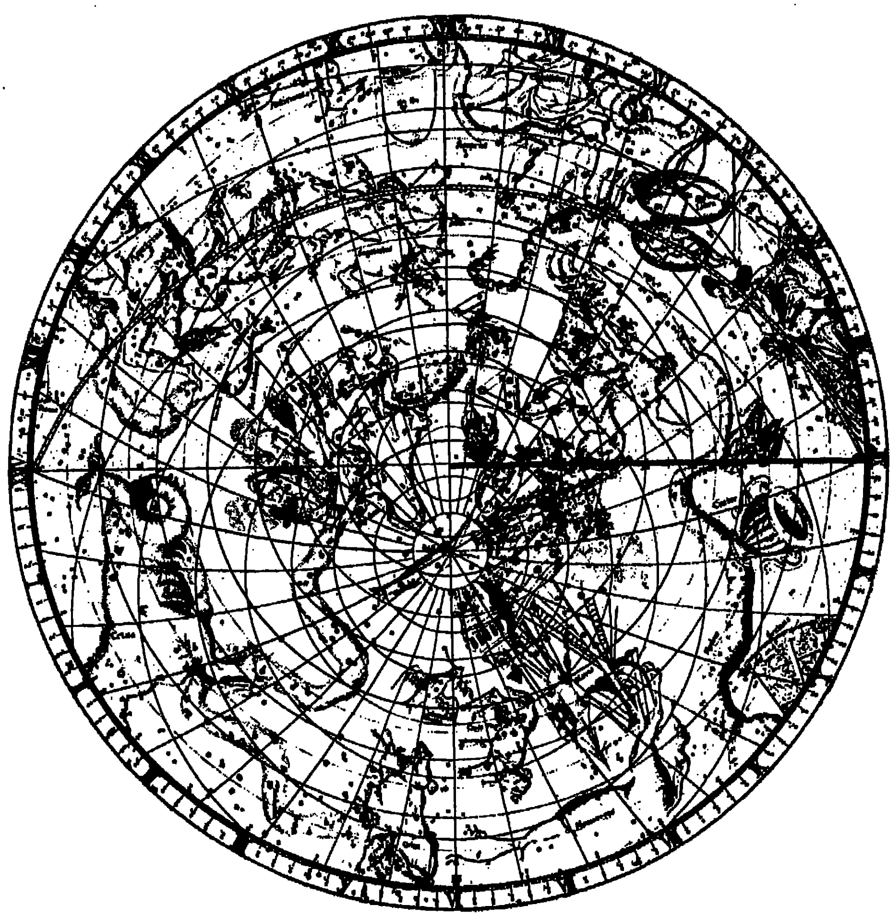

Fig. 2. *Mythical Figures of the Constellations of the Southern Hemisphere.* (*From the Flamsteed* Atlas Coelestis, *1729.*)

the help of a friend who has already made a start. In these maps, only those stars are shown which are rendered important by their brightness or by their salient positions in the constellation figures, and many of the less important constellations are omitted. The more familiar constellation figures are traced by dark lines which connect the stars. The magnitude

Table 1

THE CONSTELLATIONS

| Name | Genitive | Abbreviation[a] | Name | Genitive | Abbreviation[a] |
|---|---|---|---|---|---|
| Andromeda | Andromedae | And | Lacerta[b] | Lacertae | Lac |
| Antlia[b] | Antliae | Ant | Leo | Leonis | Leo |
| Apus[b] | Apodis | Aps | Leo Minor[b] | Leonis Minoris | LMi |
| Aquarius | Aquarii | Aqr | Lepus | Leporis | Lep |
| Aquila | Aquilae | Aql | Libra | Librae | Lib |
| Ara | Arae | Ara | Lupus | Lupi | Lup |
| Argo (Navis)[c] | Argus | | Lynx[b] | Lyncis | Lyn |
| Aries | Arietis | Ari | Lyra | Lyrae | Lyr |
| Auriga | Aurigae | Aur | Mensa[b] | Mensae | Men |
| Bootes | Bootis | Boo | Microscopium[b] | Microscopii | Mic |
| Caelum[b] | Caeli | Cae | Monoceros[b] | Monocerotis | Mon |
| Camelopardalis[b] | Camelopardalis | Cam | Musca[b] | Muscae | Mus |
| Cancer | Cancri | Cnc | Norma[b] | Normae | Nor |
| Canes Venatici[b] | Canum Venaticorum | CVn | Octans[b] | Octantis | Oct |
| Canis Major | Canis Majoris | CMa | Ophiuchus | Ophiuchi | Oph |
| Canis Minor | Canis Minoris | CMi | Orion | Orionis | Ori |
| Capricornus | Capricorni | Cap | Pavo[b] | Pavonis | Pav |
| Cassiopeia | Cassiopeiae | Cas | Pegasus | Pegasi | Peg |
| Centaurus | Centauri | Cen | Perseus | Persei | Per |
| Cepheus | Cephei | Cep | Phoenix[b] | Phoenicis | Phe |
| Cetus | Ceti | Cet | Pictor[b] | Pictoris | Pic |
| Chamaeleon[b] | Chamaeleontis | Cha | Pisces | Piscium | Psc |
| Circinus[b] | Circini | Cir | Piscis Australis | Piscis Australis | PsA |
| Columba[b] | Columbae | Col | Reticulum[b] | Reticuli | Ret |
| Coma Berenices[b] | Comae Berenices | Com | Sagitta | Sagittae | Sge |
| Corona Australis | Coronae Australis | CrA | Sagittarius | Sagittarii | Sgr |
| Corona Borealis | Coronae Borealis | CrB | Scorpius | Scorpii | Sco |
| Corvus | Corvi | Crv | Sculptor[b] | Sculptoris | Scl |
| Crater | Crateris | Crt | Scutum (Sobieskii)[b] | Scuti | Sct |
| Crux[b] | Crucis | Cru | Serpens | Serpentis | Ser |
| Cygnus | Cygni | Cyg | Sextans[b] | Sextantis | Sex |
| Delphinus | Delphini | Del | Taurus | Tauri | Tau |
| Dorado[b] | Doradus | Dor | Telescopium[b] | Telescopii | Tel |
| Draco | Draconis | Dra | Triangulum | Trianguli | Tri |
| Equuleus | Equulei | Equ | Triangulum Australe[b] | Trianguli Australis | TrA |
| Eridanus | Eridani | Eri | Tucana[b] | Tucanae | Tuc |
| Fornax[b] | Fornacis | For | Ursa Major | Ursae Majoris | UMa |
| Gemini | Geminorum | Gem | Ursa Minor | Ursae Minoris | UMi |
| Grus[b] | Gruis | Gru | Virgo | Virginis | Vir |
| Hercules | Herculis | Her | (Piscis) Volans[b] | Volantis | Vol |
| Horologium[b] | Horologii | Hor | Vulpecula[b] | Vulpeculae | Vul |
| Hydra | Hydrae | Hya | | | |
| Hydrus[b] | Hydri | Hyi | | | |
| Indus[b] | Indi | Ind | | | |

[a] Adopted in 1922 by the International Astronomical Union.
[b] Of modern origin.
[c] This constellation, which is very large, is often divided into four: Carina (Car), Puppis (Pup), Pyxis (Pyx), and Vela (Vel).

scale is such that the areas of the white dots are proportional to the amount of light which we receive from the respective stars.

For one who lives within the latitudes of the United States it is probably easiest to begin with the Great Dipper, since it is conspicuous, always above the horizon, and almost universally known. A line drawn through the two end stars of the Dipper bowl, which are called the Pointers, and prolonged five times the distance between them, passes very near Polaris, the North Star. On the opposite side of Polaris from the Dipper is Cassiopeia, which has the form of a W. After these groups are located in the sky, it is easy to find from the north polar map (Map 8) the remaining stars of Ursa Major, also Ursa Minor, Draco, and Cepheus; and these groups will then serve, with the help of the other maps, to identify constellations farther from the Pole.

# CHAPTER 1

## THE CELESTIAL SPHERE

**The Celestial Sphere.** When we look upward from some point where the view is unobstructed by roofs or trees, we seem to see what we call the **sky**, which is like a great hollow sphere with the observer at the center. The space between the heavenly bodies is really empty and the light of the sky is due to the air within less than a hundred miles of the Earth, which reflects to our eyes a little of the light of the Sun and stars, the blue more copiously than the other colors; and yet this **celestial sphere** seems so real and of so firm a construction that it is sometimes called the *firmament.* If we attempt to reach the sky or to find out how far away it is, its distance turns out to be greater than that of any of the heavenly bodies, prodigious as these distances are. For convenience in describing the apparent positions of the stars, astronomers speak of the celestial sphere as if it really existed, and conceive of it as having mathematically infinite dimensions, in comparison with which the observer and the whole Earth shrink to a mere point located at the center of the sphere.

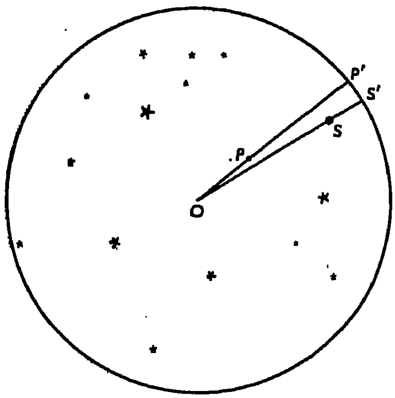


Fig. 3. *Apparent Position of a Heavenly Body.*

**Apparent Position of a Heavenly Body.** The distances of the stars, the planets, and even the Moon are so great that they cannot be determined except by delicate and indirect measurements, and so, to all ordinary appearances, these distances are also infinite and the heavenly bodies appear as if set in the surface of the celestial sphere itself. Two bodies that are in reality separated by a vast space, as the star $S$ and the planet $P$ (Figure 3), will,

if nearly in the same direction from the observer $O$, appear close together, since $S$ seems attached to the celestial sphere at $S'$ and $P$ at $P'$. The apparent position of a heavenly body is described by locating its projection upon the celestial sphere, and so only its *direction* from the observer is taken into account, its distance from him being disregarded.

**Apparent, or Angular, Distance.** Distances laid out upon the surface of the infinite celestial sphere—for example, the apparent separation $P'S'$ of two heavenly bodies—can evidently not be expressed in such units as feet, miles, or kilometers, and such statements as that two stars "look to be about a foot apart" are without meaning unless the distance of the foot rule from the observer is specified. A foot rule would cover the space between the two Pointers of the Big Dipper if held about twelve feet from the eye; but if brought within four feet, it would appear larger and could cover a side of the Square of Pegasus. The apparent distance of $P$ from $S$ can in fact be definitely stated only as the difference of direction between the two objects as seen from $O$—that is, the angle $POS$ or its equivalent arc $P'S'$.

The arc $P'S'$ is a portion of a circle whose center is $O$, the center of the sphere. Such a circle is called a **great circle**, since no larger circle can be drawn upon the surface of the sphere, although any number of smaller circles can be drawn parallel to it. A great circle divides the sphere into halves and lies in a plane that passes through the center. Apparent distances on the celestial sphere are always specified as arcs of great circles.

**Units of Angular Measurement.** The most familiar unit of arc or of angular distance is the **degree**, the 360th part of a circumference or 1/90 of a right angle. It was chosen many centuries ago because it represents very nearly the apparent daily motion of the Sun among the stars. It is divided into sixty equal parts called **minutes**, and these into sixty **seconds** each, so that a degree contains 3600 seconds. A side of the Square of Pegasus is about fifteen degrees long, the Pointers are about five degrees apart, and the diameters of the Sun and Moon are each about thirty minutes. The smallest angular distance that can easily be perceived by the unaided eye is about $3\frac{1}{2}'$, which is the distance between the principal components of the multiple star $\epsilon$ Lyrae. Two stars that are only a second apart require a fairly good telescope to show them separately, and yet the results of modern astronomical measurements are reliable to the hundredth of a second—the angle subtended by the diameter of a ten-cent piece at a distance of about two hundred miles.

Certain angular distances in the sky are conveniently measured by the angle through which the Earth rotates in a given time. These are often expressed in hours, minutes, and seconds, the hour being one-twenty-fourth of a circumference, or fifteen degrees. The subdivisions of the hour are called minutes and seconds of time and must be carefully distinguished from those of the degree, which are only one-fifteenth as long and are called minutes and seconds of arc. The units of the "time" system are abbreviated by $^h$, $^m$, $^s$, while those of the "arc" system are written $^\circ$, $'$, $''$. Thus, $22^\circ 18' 44.''25$ designates the same angular distance as $1^h 29^m 14^s.95$.

A third unit of angular measurement, very useful in computations, is called a **radian**, and is defined as the angle subtended at the center of any circle by an arc equal in length to the radius. In Figure 4 the arc $AB$ is laid off so that, measured along the curve, its length is equal to the radius, $R$, of the circle; hence, the angle $AOB$, with vertex at the center of the circle and sides extending to the extremities $A$ and $B$ of the arc, is a radian. Any other angle $BOC$ is the same fraction of a radian that its arc $BC$ is of $AB$, or of the radius $R$, because angles are measured by their subtending arcs; hence, in general, if we represent by $R$ the radius of any circle and by $S$ the length of an arc subtending an angle at its center, *the value of that angle in radians is* $S/R$.

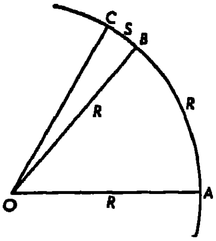

Fig. 4. *The Radian.*

There are $2\pi$ arcs of length $R$ in the whole circumference (where $\pi = 3.14159 \cdots$), and so $360^\circ = 2\pi$ radians. One radian, therefore, is equal to $360^\circ \div 2\pi$, or

$$1 \text{ radian} = 57^\circ.3 = 3438'; \text{ or, more precisely, } 1 \text{ radian} = 206265''.$$

**Relations Between Angles and Lengths.** With the vertex of any angle $\theta$ as center and with any radius $r$, let an arc of a circle be drawn so as to intersect both sides of the angle; and from the point of its intersection with one side let a perpendicular be dropped upon the other side. Call the length of this perpendicular $a$, the distance of its foot from the vertex $b$, and the length of the intercepted arc $s$. The ratios of these lengths are very useful in computations, and are named as follows:[1]

$s/r = \theta$, expressed in radians

$206265\ s/r = \theta$, expressed in seconds of arc

| | | |
|---|---|---|
| $a/r =$ | the sine of $\theta$, | written $\sin\theta$ |
| $b/r =$ | the cosine of $\theta$, | written $\cos\theta$ |
| $a/b =$ | the tangent of $\theta$, | written $\tan\theta$ |
| $b/a =$ | the cotangent of $\theta$, | written $\cot\theta$ |
| $r/b =$ | the secant of $\theta$, | written $\sec\theta$ |
| $r/a =$ | the cosecant of $\theta$, | written $\csc\theta$ |

The study of these ratios is the province of trigonometry, and their values, corresponding to all values of the angle $\theta$, are contained in conveniently printed trigonometric tables. By their aid, when the three sides, or an angle and two sides, or a side and two angles, of any plane triangle are known, the other parts, whether sides or angles, may be calculated—a process called solving the triangle.

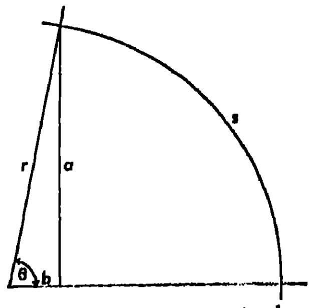


Fig. 5. *A Large Angle.*

If $\theta$ is nearly a right angle (that is, nearly 90 degrees or $\pi/2$ radians as in Figure 5), then sin $\theta$ is nearly 1, cos $\theta$ is small, and tan $\theta$ is very large, becoming infinite when $\theta = \pi/2$; but if $\theta$ is small (Figure 6), then cos $\theta$ is nearly 1 while both sin $\theta$ and tan $\theta$ are small and nearly equal to the value of $\theta$ in radians. If, for example, the angle $\theta$ is less than 6′ (as it is in many astronomical problems), the error of putting $\theta = \sin\theta = \tan\theta$ is no greater than the decimals neglected in using six-place tables.

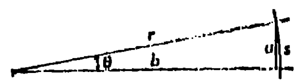


Fig. 6. *A Small Angle.*

**Linear and Angular Dimensions of a Heavenly Body.** The real size of a spherical body, such as the Moon or a planet, may be expressed by stating its diameter in miles. This is called the **linear diameter,** being the length of a straight line. The **apparent diameter** or **angular diameter** is the angle subtended by the linear diameter at the observer's eye; or the angle made by the two light-rays that reach his eye from points at opposite ends of the linear diameter. The angular diameter depends on distance: the more remote the body, the smaller it looks.

The heavenly bodies generally have small apparent diameters, and the relations of these to the linear diameters and distances may be computed without the use of trigonometric tables. Let the line $AB$ or $d$ (Figure 7) be the linear diameter of a

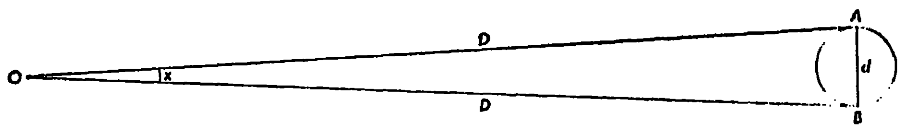


Fig. 7. *Solution of a Slender Triangle.*

planet situated at distance $D$ from an observer $O$; then the apparent diameter is the small angle $AOB$ or $x$. An arc of a circle drawn with $O$ as center and $D$ as radius would so nearly coincide with $AB$ that we may, without appreciable error, take this arc as the linear diameter $d$. Then the value of $x$ in radians is $d/D$, or, if $x''$ represent the value of $x$ in seconds of arc,

$$x'' = \frac{d}{D} \cdot 206265, \quad d = \frac{x''D}{206265}, \quad \text{and} \quad D = \frac{206265\, d}{x''}.$$

For example, it has been found by measurement that the apparent diameter of Mars is 24″ when its distance from the Earth is 36,354,000 miles; its linear diameter is therefore

$$d = \frac{24 \times 36354000}{\cdots} = 4250 \text{ miles}$$

**Practical Measurement of Angular Distance.** The actual measurement of apparent distances in the sky depends upon two principles—the physical principle that a ray of light in free space is straight and the geometric principle that the arcs of concentric circles included between two

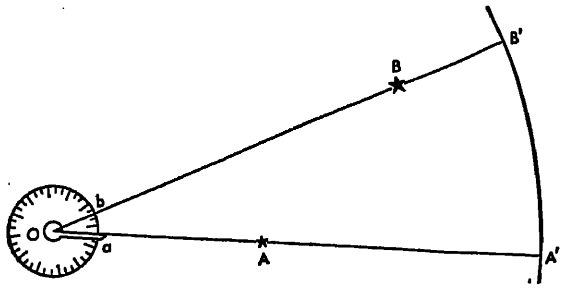

Fig. 8. *Principle of the Astrolabe.*

straight lines that meet at their common center have each the same value in degrees or other angular units as the angle between the lines. One of the earliest of astronomical instruments, the **astrolabe,** which was used by the ancient Greek astronomers and for many centuries after them, consists in its simplest form of a circular plate, graduated to degrees around its circumference and having a movable pointer, called the **alidade,** pivoted at its center. To measure the angular distance between the stars $A$ and $B$ (Figure 8) the circular plate is held in the plane of the observer and the two stars, and the position of the alidade is read upon the graduated circle, first when the alidade is pointed to $A$ and again when it is pointed to $B$. The difference of the readings is the desired angular distance; for the center of the astrolabe, being on the Earth which is a mere point in comparison with the celestial sphere, is also the center of the sphere; and so the angle $AOB$, between the straight rays of light from the two stars, is measured by the arc $ab$ between the two settings of the alidade, and this is equal to the arc $A'B'$ of the great circle of the celestial sphere included between the apparent places of the two stars.

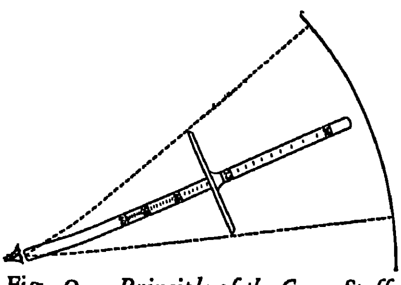
Fig. 9. *Principle of the Cross-Staff.*

A second type of instrument, called the **cross-staff,** was also used by early astronomers and navigators. It consists of a straight staff of wood about a meter long, with a crosspiece that slides upon it and is used as

illustrated in Figure 9. The end of the staff is placed near the eye and the crosspiece is adjusted so that it covers the arc in the sky which it is desired to measure. The number of degrees in this arc is the same as that in the angle formed at the eye by the rays of light that just pass the ends of the crosspiece; and this angle increases as the crosspiece is brought nearer the eye. Its value in degrees is read from a scale engraved on the staff.

**Systems of Coördinates.** The position of a point in a plane may be completely described by two numbers which give the distances of the point from two lines or axes that intersect at right angles—a device familiar to every student of algebra. The two numbers are called **coördinates,** the horizontal distance being an **abscissa** and the vertical one an **ordinate.** A practical example is found in the method often used for designating an address in a city, the streets of which intersect at right angles. Thus, for a residence at 234 East 116th Street, 234 may be regarded as the abscissa and 116 as the ordinate.

For designating the positions of stars, astronomers conceive of the celestial sphere as covered with imaginary circles which intersect at right angles like the city streets. They are precisely analogous to the circles used for indicating the latitude and longitude of places on the Earth, which appear on all geographic maps. Four such **systems of coördinates** are in common use in astronomy; they are known as the **horizon, equator, ecliptic,** and **galactic** systems.

Each system consists of a **fundamental circle,** which is a great circle of the sphere and for which the system is named: the **poles** of this great circle, which are the points of the sphere 90° from it; a set of **secondary great circles,** indefinite in number, which pass through the poles and cut the fundamental circle at right angles; and a set of **parallels,** small circles parallel to the fundamental circle. A point on the fundamental circle is chosen as **origin** and one coördinate of a star is counted from this point along the fundamental circle, while the other coördinate is measured along a secondary from the fundamental circle to the star. In the familiar geographic system the poles are the points where the Earth's axis intersects the surface of the Earth; the fundamental circle is the Earth's equator; the secondaries are the meridians,[2] which are cut at right angles by the parallels of latitude; the origin is the point where the meridian of Greenwich intersects the equator; and the coördinates of a place are its latitude and longitude.

[2] The geographic meridians, however, are not circles. but nearly ellipses

**Practical Measurement of Angular Distance.** The actual measurement of apparent distances in the sky depends upon two principles—the physical principle that a ray of light in free space is straight and the geometric principle that the arcs of concentric circles included between two

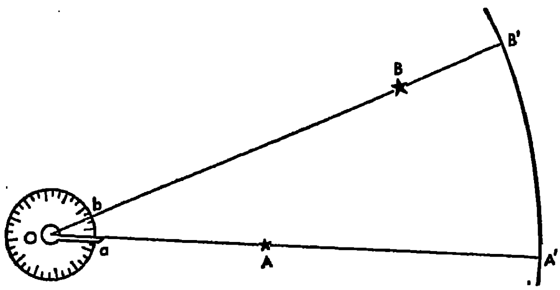


Fig. 8. *Principle of the Astrolabe.*

straight lines that meet at their common center have each the same value in degrees or other angular units as the angle between the lines. One of the earliest of astronomical instruments, the **astrolabe,** which was used by the ancient Greek astronomers and for many centuries after them, consists in its simplest form of a circular plate, graduated to degrees around its circumference and having a movable pointer, called the **alidade,** pivoted at its center. To measure the angular distance between the stars $A$ and $B$ (Figure 8) the circular plate is held in the plane of the observer and the two stars, and the position of the alidade is read upon the graduated circle, first when the alidade is pointed to $A$ and again when it is pointed to $B$. The difference of the readings is the desired angular distance; for the center of the astrolabe, being on the Earth which is a mere point in comparison with the celestial sphere, is also the center of the sphere; and so the angle $AOB$, between the straight rays of light from the two stars, is measured by the arc $ab$ between the two settings of the alidade, and this is equal to the arc $A'B'$ of the great circle of the celestial sphere included between the apparent places of the two stars.

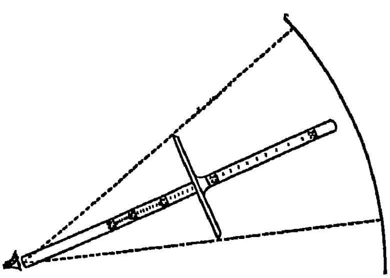

Fig. 9. *Principle of the Cross-Staff.*

A second type of instrument, called the **cross-staff,** was also used by early astronomers and navigators. It consists of a straight staff of wood about a meter long, with a crosspiece that slides upon it, and is used as

illustrated in Figure 9. The end of the staff is placed near the eye and the crosspiece is adjusted so that it covers the arc in the sky which it is desired to measure. The number of degrees in this arc is the same as that in the angle formed at the eye by the rays of light that just pass the ends of the crosspiece; and this angle increases as the crosspiece is brought nearer the eye. Its value in degrees is read from a scale engraved on the staff.

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**The Horizon System of Coördinates.** The fundamental circle in the horizon system of coördinates is the horizon and its poles are the zenith and nadir. Their position is determined by the direction of gravity.

Suppose a weight is suspended by a string (plumb line) and allowed to come to rest. The string will take the direction of gravity, and if prolonged, in imagination, indefinitely upward will meet the celestial sphere at the zenith. The **zenith** is therefore defined as the point where the direction of gravity produced upward meets the celestial sphere. The **nadir** is the (invisible) point of the celestial sphere exactly opposite the zenith.

The **true horizon** is the great circle midway between the zenith and nadir, 90° from each. It may also be defined as the great circle along which a level plane meets the celestial sphere.

The term horizon is loosely applied to the line where the sky seems to meet the surface of the Earth; this may properly be distinguished as the **visible** or **apparent** horizon. It is irregular in shape on land, and at sea it is a small circle lying below the true horizon by an angular distance called the **dip**.

**Vertical circles** are the secondaries of the system and are defined as great circles passing through the zenith or at right angles to the horizon. The vertical circle that extends north and south is called the **meridian**, since it is the trace on the celestial sphere of the plane of the observer's geographic meridian; a more precise definition will be given later (page 21). The vertical circle that extends east and west is called the **prime vertical**. **Almucantars** are small circles parallel to the horizon.

The coördinates of a star in the horizon system are its altitude and azimuth. Both are measured in degrees. The **altitude** is the arc of the star's vertical circle that is included between the star and the horizon. Its complement is the star's **zenith distance**. The **azimuth** is the arc of the horizon included between a point chosen as origin and the foot of the star's vertical circle. Astronomers usually choose the south point of the horizon as the zero of azimuth and reckon clockwise all the way around to 360°. Surveyors and navigators more often take the north point as origin and reckon azimuth

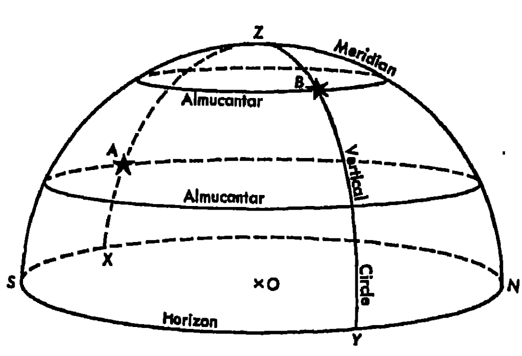

Fig. 10. *The Horizon System of Coördinates.*

(or **true bearing,** as they sometimes call it) in either direction to 180°. (The exact definition of the south and north points is postponed to page 21.)

The base of Figure 10 represents the horizon, *O* the observer, and *Z* the zenith. The azimuth of the star *A* is the arc *SX*—about 35°—and its altitude is *AX*, or about 25°. The azimuth of *B* is reckoned through the west and north around to *Y* and is about 250°, while its altitude *BY* is about 70°.

**Practical Measurement of Altitude and Azimuth.** In the most common form of the ancient astrolabe, the instrument was supported by a ring held in the observer's hand so that the weight of the instrument caused it to hang vertically. The graduations of the circle being so arranged that the alidade read zero when pointing horizontally, its reading when pointed to a star gave the star's altitude—i.e., the angle which its ray made with a horizontal line.

A more accurate measurement of altitude could be made with the **quadrant,** which was highly developed in the sixteenth century by the German Hevelius and the Dane, Tycho Brahe. This instrument had the form of a ninety-degree circular sector sometimes several feet in radius. It was arranged to turn in a vertical plane about a pivot at its center, from which point was suspended a fine plumb line. Sights resembling modern rifle sights were attached to one edge and the arc was graduated to degrees and minutes, the zero being 90° from the edge that bore the sights. The altitude to which the sights were directed was read by using the plumb line as an index on the graduated arc.


Fig. 11. *Using a Small Quadrant.*

Figure 11, from Regiomontanus's *De Trianguli,* 1532, shows the use of a small quadrant in measuring the altitude of the Sun for the purpose of calculating the height of a tower. The latter can be obtained from the Sun's altitude together with the length of the tower's shadow, since the Sun is so far away that its rays are sensibly parallel, and its altitude is thus equal to the angle at the base of the right triangle formed by the tower and its shadow. The

height of the tower equals the length of the shadow multiplied by the tangent of this angle, and the value of the tangent may be found in trigonometric tables when the value of the angle is known.

At sea, the altitude of a heavenly body is found by measuring with a sextant the shortest distance of the body from the visible horizon and correcting for the dip, which depends on the observer's height above the water.

Altitude and azimuth may be determined from a single pointing by means of the **altazimuth instrument**, of which the modern **surveyor's transit** or **theodolite** (Figure 12) is an example. It consists of a small telescope mounted on an axis which is in turn supported upon a second axis at right angles to the first. Each axis bears a graduated circle which is read by an index attached to the frame of the instrument. The base is provided with spirit levels by means of which the axes may be made respectively horizontal and vertical. If the indexes are adjusted to read zero when the telescope is level and pointed due south, their readings when it is directed to a star give the star's altitude and azimuth.

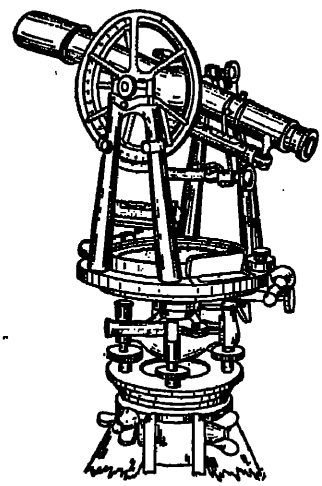

Fig. 12. *A Surveyor's Transit.*

**The Diurnal Motion.** If the Earth were flat and stationary, no other system of coördinates than the horizon system would be needed for describing the positions of the stars; but since the plane of the horizon is tangent to the Earth's surface, the motions of the Earth and also any change in the observer's geographic position must move his horizon and all its associated circles and so change the altitudes and azimuths of the heavenly bodies. The rotation of the Earth on its axis, which is completed each day, results in an apparent revolution of all the heavenly bodies around the Earth in the same time. This apparent revolution [3] is called the **diurnal motion.**

One result of the diurnal motion is the most obvious of all astronomical phenomena—the rising and setting of the Sun; but the manner of this rising and setting is not so obvious. At no place except the equator does

[3] Throughout this book the word rotation will be used to designate a turning or spinning of a body around an axis that passes through the body itself, while revolution will mean motion around an external point.

ıe Sun ever rise or set vertically, and for no observer outside the torrid ɔne does it ever reach the zenith. As seen within the latitudes of the

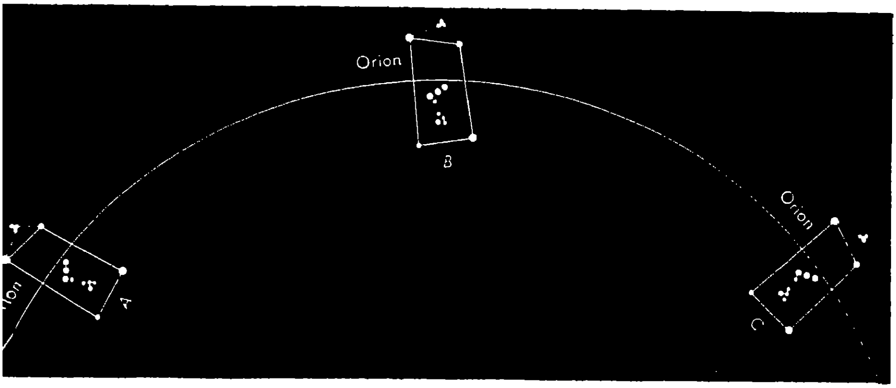


Fig. 13. *Three Positions of Orion.* (*After McKready.*)

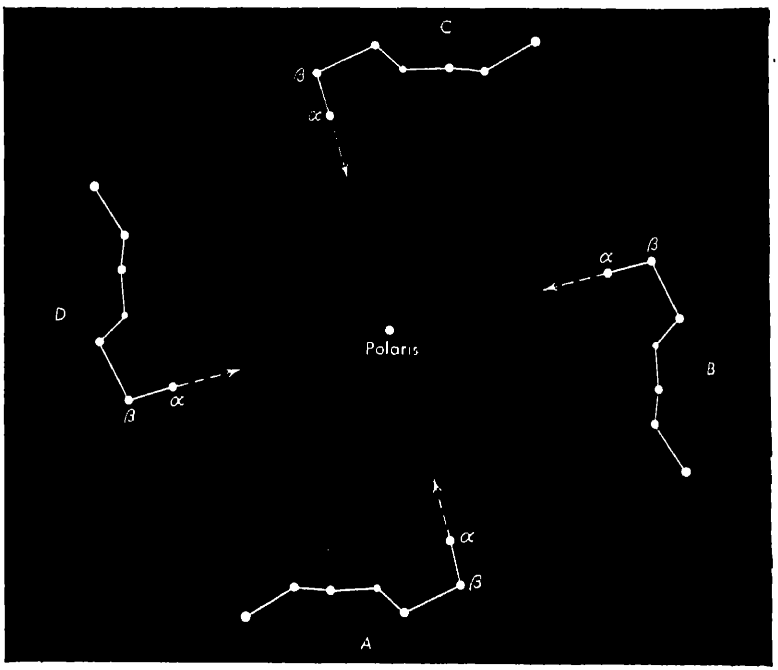


Fig. 14. *Four Positions of the Dipper.* (*After McKready.*)

nited States, the Sun follows in the forenoon a path that slants upward ›ward the right, crosses the meridian south of the zenith at midday, and ͻscends toward the right in the afternoon—moving like the hands of a atch that lies on a sloping roof facing the north. Such stars as rise

approximately in the east follow similar paths; the constellation Orion, for example, may be seen on a December night successively in the positions shown in Figure 13.

If one faces the north, he sees stars that do not descend below the horizon, but follow diurnal circles centered upon a point which, in the northern part of the United States, is about halfway between the horizon and the zenith. Four successive positions occupied by the Great Dipper at intervals of six hours are shown in Figure 14. The stars far in the south, on the other hand, are visible only a short time, following diurnal arcs that extend but little above the horizon.

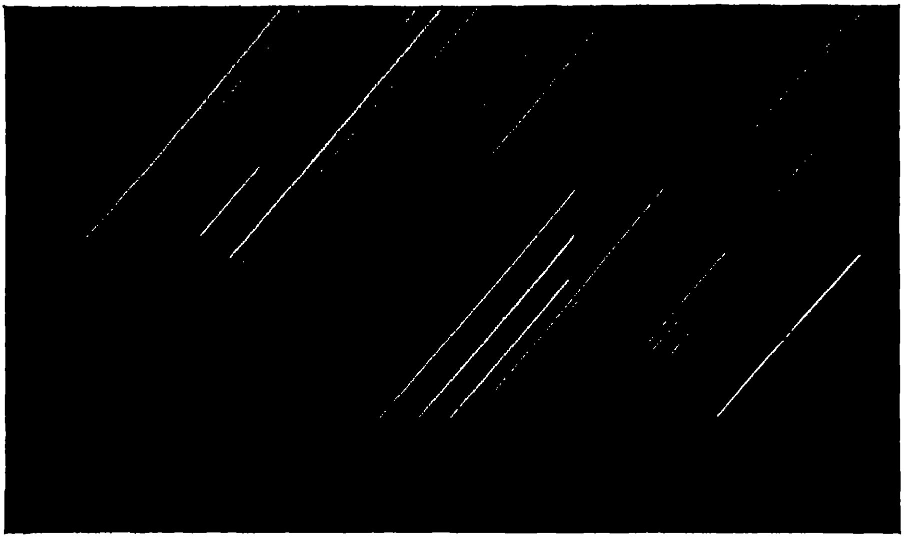

Fig. 15. *Photographic Star Trails: Orion Rising (Whitin Observatory).*

Fig. 16. *Photographic Trail of Venus Setting (Halls of Wellesley College).*

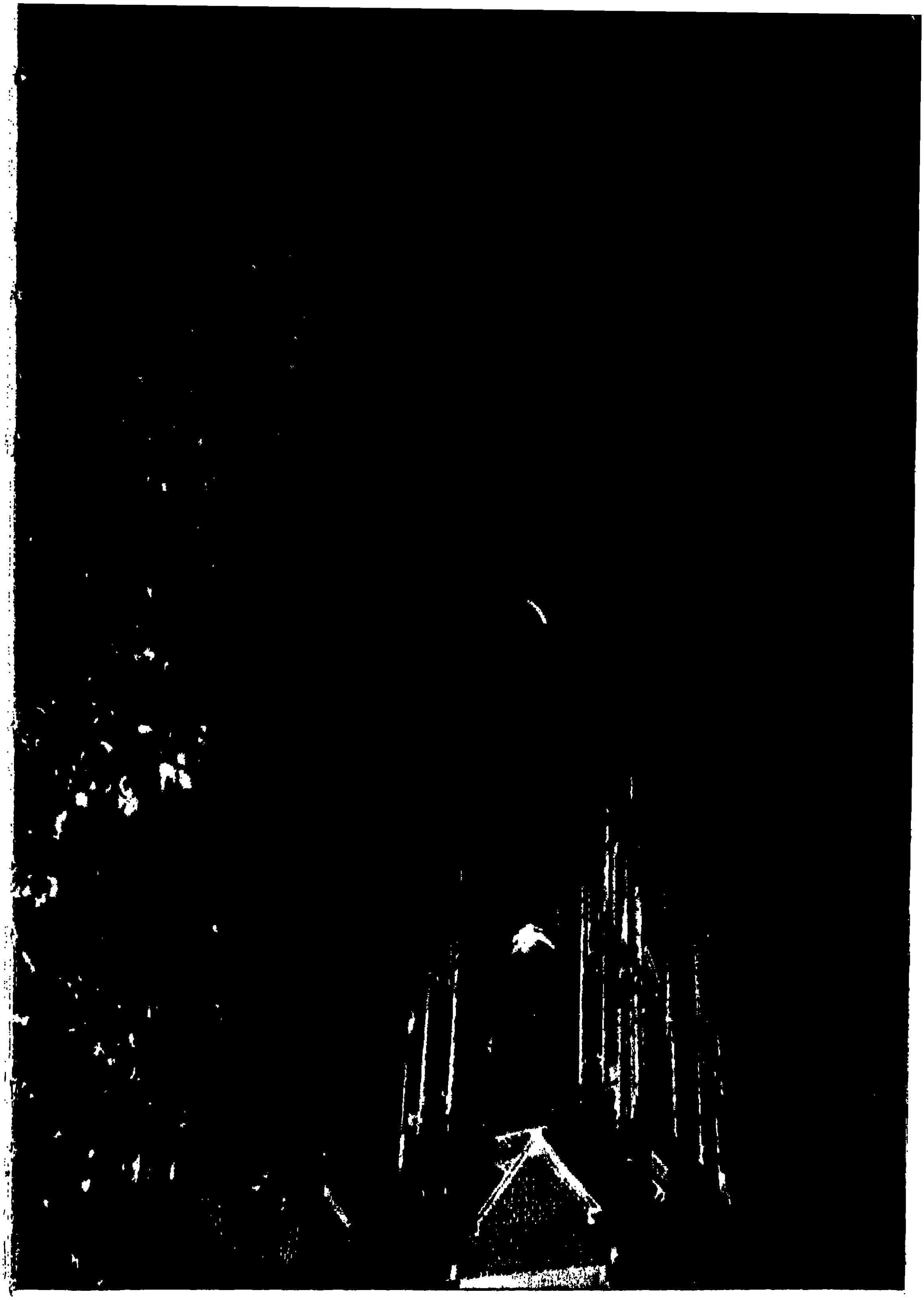

Fig. 17. *Photographic Polar Star Trails (Carillon Tower of Wellesley College).*

The nature of the diurnal motion is readily made evident by photographing the stars, using a fixed camera and exposing the plate an hour or more, so that the star images trail upon the plate. The three photographs reproduced in Figures 15–17 were obtained in this manner, the camera being pointed east, west, and north, respectively, in latitude 42°.3.

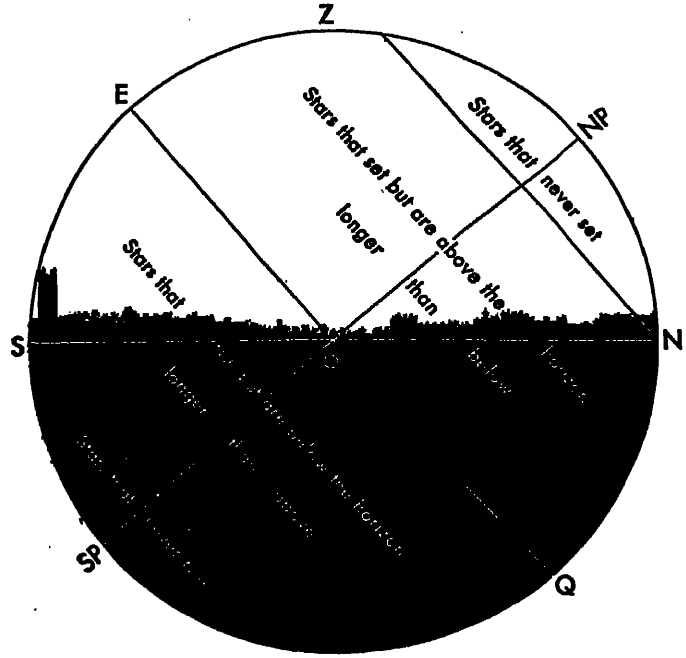


Fig. 18. *The Diurnal Motion.*

The diurnal motion is shown diagrammatically in Figure 18. It is as if the whole celestial sphere, with the heavenly bodies attached to its inner surface, rotated each day around an axis passing north and south and inclined to the plane of the horizon—just what the Earth is really doing, but in the opposite direction. Evidently, the celestial sphere must contain stars that are south of us and that never rise; and this inference is confirmed by the observations of people living farther south, to whom these stars become visible. The boundary of the portion of the celestial sphere containing stars that never rise is called the **circle of perpetual occultation**; that of the region of stars that never set, the **circle of perpetual apparition**.

**The Equator System of Coördinates.** The diurnal motion both limits the usefulness of the horizon system of coördinates and affords a

basis of another system, the **equator system.** The fundamental circle of this system is the **celestial equator** and its poles are known as the **north and south poles of rotation,** or often simply the poles.

The **poles of rotation** are those points in the sky about which the diurnal motion takes place, or, more accurately, those points which have no diurnal motion. They may also be defined as the points where the Earth's axis, produced, intersects the celestial sphere. The **north** pole is the one in the constellation of Ursa Minor around which the diurnal motion appears counterclockwise.

The **celestial equator** is the great circle of the celestial sphere that lies midway between the poles—90° from each.

**Hour circles** are great circles that pass through the poles. Each hour circle in turn, as the diurnal motion proceeds, passes through the zenith, when it coincides with the **meridian,** which therefore is common to the two systems and may be defined as the hour circle that passes through the zenith or as the vertical circle that passes through the poles.

The **cardinal points** are the north, south, east, and west points of the horizon. The north point is that one of the intersections of the meridian with the horizon which is the nearer to the north pole. The cardinal points are 90° apart, the south being opposite the north, and the east being at the observer's right when he faces the north.

**Parallels of declination** are circles parallel to the celestial equator.

The **declination** of a star is the arc of an hour circle included between the star and the celestial equator. It is reckoned in degrees and is considered + if the star is north of the equator and − if south. Declination is often abbreviated by the Greek letter $\delta$. Since the diurnal motion takes place parallel to the celestial equator, it does not change the declination of the star as it does the altitude and azimuth.

The **hour angle** of a star is the arc of the celestial equator included between the meridian and the star's hour circle. It is usually measured westward so that it continually increases with the time. The rate of increase of hour angle is uniform, and in fact can be measured by a clock, and so the hour angle is usually expressed in hours.

It is very desirable to use coördinates which, unlike the altitude, azimuth, and hour angle, are not changed by the diurnal motion. We have seen that the declination is not changed, and we may obtain an equally permanent coördinate to accompany it if, instead of measuring the arc of the equator from the meridian, we measure it from a point that is carried

along by the diurnal motion. The point chosen by astronomers for this purpose is called the **vernal equinox**, and is located on the celestial equator in the comparatively starless region just south of the Square of Pegasus, as shown in Figure 19. Its exact definition will be given on page 26.

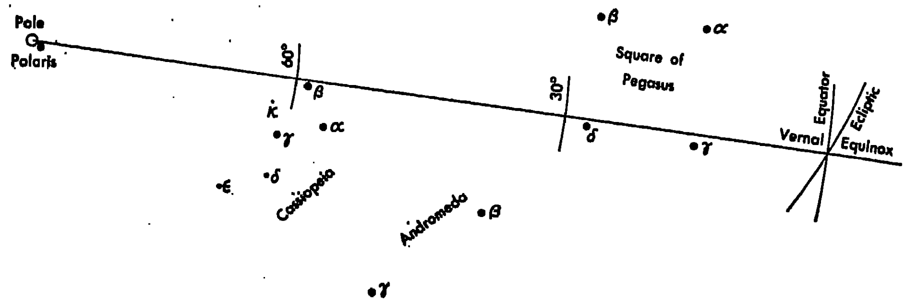


Fig. 19. *How to Locate the Vernal Equinox.*

The **right ascension** of a star is the arc of the celestial equator included between the vernal equinox and the star's hour circle. It is reckoned eastward from the vernal equinox and is usually expressed in hours. Its common abbreviation is the Greek letter $\alpha$.

**The Equatorial Telescope.** Most large telescopes are supported on an **equatorial mounting**, the principle of which may be seen in Figure 20, a photograph of the great refractor of the Lick Observatory. A massive pier supports an axis of steel which is placed parallel to the Earth's axis and therefore points to the pole of rotation and is called the **polar axis** of the instrument. Attached to this is a steel sleeve that holds the **declination axis** in a direction at right angles to that of the polar axis; and the tube of the telescope is fixed at right angles to this. Motion of the telescope around the declination axis causes it to sweep along an hour circle, and the angle turned through from the equator is the declination of the star to which the telescope is directed, which may be read upon a graduated circle, called the **declination circle**, attached to the declination axis. Motion around the polar axis is parallel to the equator, and the hour angle may be read upon the graduated circle, called the **hour circle** (although not corresponding to the hour circles in the sky), attached to the polar axis. To follow a star in its diurnal motion, the instrument need only be turned about the polar axis, the declination setting being kept fixed; and, since the motion in hour angle is uniform, the telescope is ordinarily provided with a clock, electrical or otherwise, which keeps a wheel on the polar axis turning westward at a uniform rate of one rotation

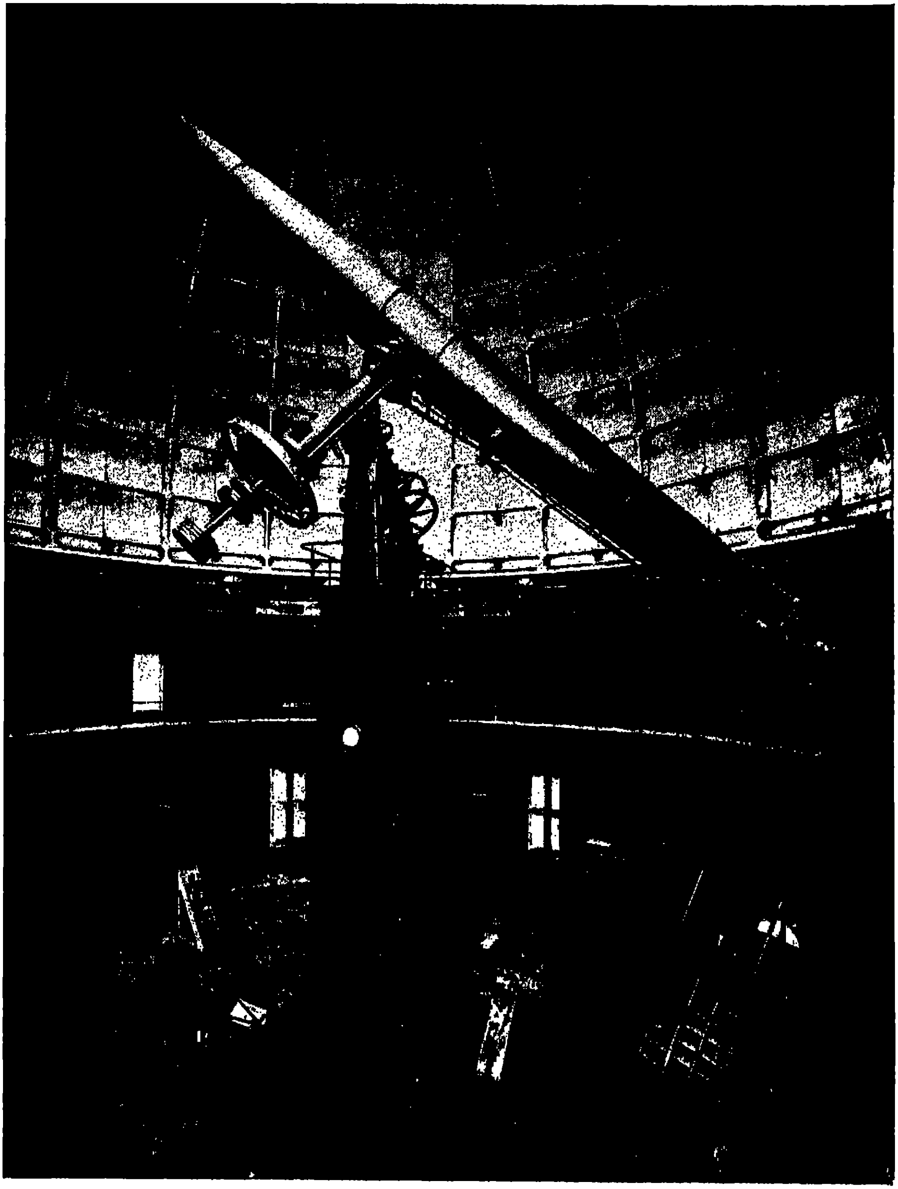

Fig. 20. *The 36-Inch Equatorial Refracting Telescope of the Lick Observatory.*

per day. When the telescope is set on an object the polar axis is clamped to the driving wheel, which then keeps the object in the field of view.

To set the equatorial telescope on a star, whether the star is visible to the unaided eye or not, it is only necessary to know the star's declination and its hour angle at the moment, and to set the circles accordingly. On some modern equatorials, a second circle is mounted on the polar axis in such a way that its zero may be made to correspond with the vernal equinox instead of the meridian. So long as the clock is kept running correctly, this circle reads the right ascension.

**Sidereal Time.** The period of one rotation of the Earth relative to the stars, which is the time interval occupied by one complete apparent rotation of the celestial sphere, is called the **sidereal day.** It is about four minutes shorter than the ordinary solar day, and is divided into twenty-four sidereal hours, each a little shorter than an ordinary solar hour. **Sidereal noon** occurs at the moment when the vernal equinox is on the meridian, which is at different times of the day or night at different times of the year. The **sidereal time** at any moment is the hour angle of the vernal equinox, or, what is exactly the same thing, the right ascension of the meridian. The sidereal time is therefore the interval in sidereal hours, minutes, and seconds since sidereal noon.

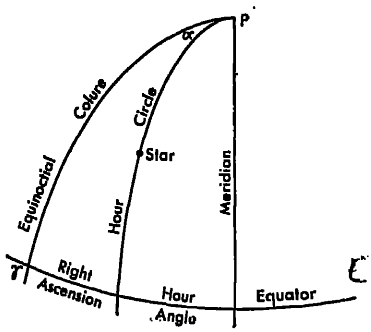


Fig. 21. *Right Ascension, Hour Angle, and Sidereal Time.*

The hour angle and right ascension of a star are connected very simply with the sidereal time; for, the hour angle being measured westward from the meridian and the right ascension eastward from the vernal equinox, each to the star's hour circle, their sum is the angular distance between the meridian and equinox (Figure 21)—the hour angle of the vernal equinox, or the right ascension of the meridian, which we have already defined as the sidereal time. That is,

$$\text{Sidereal time} = \text{right ascension} + \text{hour angle}.$$

This simple relation is very useful, for example, when it is desirable to set an equatorial telescope on an unseen planet whose position in the sky is known. Subtracting the right ascension from the time taken from a sidereal clock gives the hour angle, and the hour angle and declination are the necessary circle settings.

**Solar Time.** Sidereal time would be inconvenient for regulating the affairs of life, since its noon occurs at all times of the day and night. **Apparent solar time** is the hour angle of the Sun, which differs from sidereal time by the Sun's right ascension, and is the time indicated by a properly erected **sundial.** Many forms of this instrument are known, now valued chiefly for ornamental purposes. In each, a gnomon, or straight-edge, placed parallel to the Earth's axis, casts its shadow upon a dial which is graduated to hours and subdivisions and which is placed in such a way as to show the hour angle of the Sun. The interval between two successive meridian passages of the Sun is called the apparent solar day. Its length is not quite the same throughout the year, and so our clocks are regulated to **mean solar time,** which is the hour angle that the Sun would have if the solar days were all of their average length. The subject of Time is discussed more completely on pages 73 *et seq.*

**The Annual Apparent Motion of the Sun.** In the temperate zones, a striking feature of the change of the seasons is the difference of the length of day and night in summer and winter. In midsummer in the

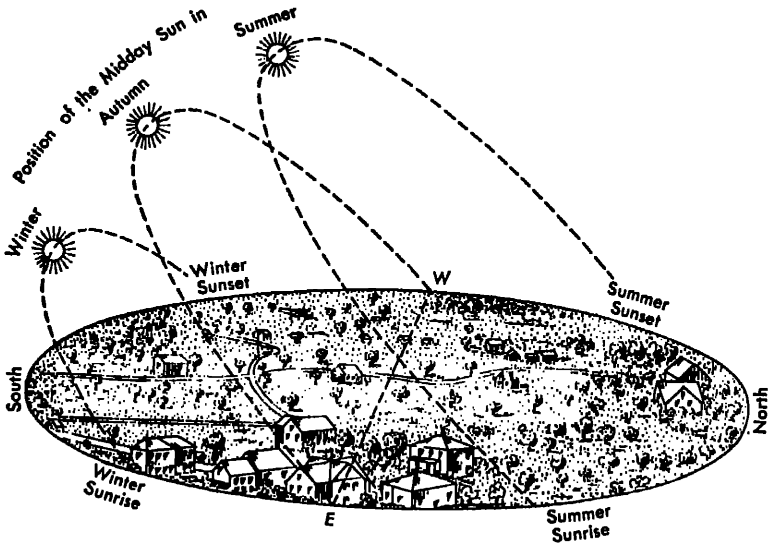


Fig. 22. *Diurnal Paths of the Sun in Different Seasons.* (*After Todd.*)

northern hemisphere the Sun rises far to the north of the east point, crosses the meridian high in the sky, and sets far north of the west point, the part of its diurnal path that is above the horizon being much greater than that which is below, so that the day is longer than the night. In midwinter it rises and sets far south of the prime vertical, reaching but a moderate

altitude even at noon, so that the day is shorter than the night (Figure 22). Evidently, the Sun crosses the celestial equator twice each year. A study of the constellations throughout the year shows that the apparent motion of the Sun is not directly northward or southward. In late September, for example, the constellation Scorpius is low in the southwest just after sunset, but by November this group cannot be seen and its place in the evening sky is taken by Sagittarius, which adjoins it on the east. By January, Sagittarius has disappeared and an early observer may see Scorpius rising in the southeast just before sunrise. During the four months, in fact, the Sun seems to have passed eastward through these constellations, and it may be traced in similar fashion until in the following September it will be found again at the starting point west of Scorpius. Measurements of the Sun's right ascension and declination show accurately the nature of its path on the celestial sphere, and it is found to be a great circle that is inclined to the equator at an angle of 23½°, in which the Sun moves always toward the east, completing the circuit in one year. Of course it is now known that this apparent motion is due to the revolution of the Earth, which, like all the other planets, moves in an orbit around the Sun.

We might represent the celestial sphere by a hollow globe with dots on it for the stars. The diurnal motion would be a spinning of the globe upon its axis, and the Sun might be represented by a trained firefly if it would accommodatingly creep around the globe in a direction making an angle of 23½° with the equator and opposite the diurnal motion, and at such a rate as to complete the circuit in 366¼ turns of the globe. The firefly would then be carried around by the globe, but more slowly than the dots of the star map, which it would pass on its way.

**The Ecliptic System of Coördinates.** The apparent path of the Sun among the stars is called the **ecliptic**, the angle at which it intersects the equator (23½°) the **obliquity**, and the points of intersection the **equinoxes**. The equinox where the Sun crosses from south to north of the equator, since it is passed by the Sun in the spring, is called the **vernal** equinox, and the other is the **autumnal** equinox. Ninety degrees from the equinoxes, as measured on the ecliptic, are the summer and winter **solstices**. The hour circles that pass through the equinoxes and solstices are known as the **equinoctial** and **solstitial colures**, respectively.

The belt of sky 18° wide which has the ecliptic as its central line is called the **zodiac** (circle of animals) because, from the earliest recorded times, its twelve constellations with the exception of Libra, the Balance (which was probably formed at a later epoch from the claws of the Scorpion), were given the names of living creatures. Within the zodiac are always to be found not only the Sun, but also the Moon

and all the principal planets. It was divided into twelve parts of 30° each, which were called the **signs** of the zodiac and were named for the twelve zodiacal constellations; and the position of a planet was indicated by the sign in which it could be seen—a practice still followed by some makers of almanacs.

Ninety degrees from the ecliptic are the north and south **poles of the ecliptic,** and passing through them are the great circles called **secondaries to the ecliptic;** small circles parallel to the ecliptic are called **parallels of latitude.**

The **celestial longitude** of a star is the arc of the ecliptic measured eastward from the vernal equinox to the secondary that passes through the star. The star's **celestial latitude** is the arc of its secondary between the star and the ecliptic, + if the star lies north of the ecliptic and − if south.

**The Galactic System of Coördinates.** Every watcher of the sky is familiar with the beautiful band of misty light that is known as the Milky Way or Galaxy. It is composed of a vast number of faint stars and a few nebulae and is of great importance in studies of the distribution of the stars. Its central line follows closely a great circle of the celestial sphere which makes an angle of 62° with the celestial equator. This circle is called the **galactic circle** and is the fundamental circle of the galactic system of coördinates. The north pole of the Galaxy is in the constellation Coma Berenices, in right ascension $12^h\ 44^m$ and declination +27°, and the opposite pole is in the constellation Sculptor, about 10° south of the bright star $\beta$ Ceti. **Galactic latitude** and **longitude** are related to the galactic circle exactly as celestial latitude and longitude are related to the ecliptic. The origin from which galactic longitude is reckoned is the intersection of the galactic circle with the celestial equator, in $\alpha = 18^h\ 44^m$.

**General Remarks on Astronomical Coördinates.** The coördinates of the horizon system, altitude and azimuth, may be said to depend on the personal outlook of the observer; they are based on the direction of gravity, which is very nearly the line joining the observer with the center of the Earth. Wherever the observer goes, he takes his zenith and his entire horizon system with him. Right ascension and declination depend on the direction of the Earth's rotational axis, and are the same for every observer on the Earth, while an observer on another planet would doubtless have a different set of equatorial coördinates depending on the plane of that planet's rotation. Celestial latitude and longitude are also on a terrestrial basis, but in this case it is the plane of the Earth's revolution that is the determining factor instead of the plane of its rotation. The

position of the circles of the galactic system of coördinates depends on the structure of the visible universe of stars and would be the same for an observer on any planet of the Solar System or among its neighboring stars.

Table 2

COMPARISON OF SYSTEMS OF COÖRDINATES

| System | Geographic | Horizon | Equator | Ecliptic | Galactic |
|---|---|---|---|---|---|
| Basis | Rotation of Earth | Direction of gravity | Diurnal motion of cel. sphere | Earth's orbital motion | Structure of visible universe |
| Fundamental Circle | Terrestrial equator | True horizon | Celestial equator | Ecliptic | Central line of Milky Way |
| Poles | Terrestrial poles | Zenith and nadir | Poles of rotation | Poles of ecliptic | Poles of galaxy |
| Secondary Great Circles | Meridians | Vertical circles | Hour circles | Secondaries to ecliptic | Secondaries to galactic circle |
| Parallels | Parallels of latitude | Almucantars | Parallels of declination | Parallels of latitude | Parallels of gal. latitude |
| Coördinates | Latitude, $\phi$ Longitude, $\lambda$ | Altitude Azimuth | Declination, $\delta$ Right ascension, $\alpha$ Hour angle | Latitude, $\beta$ Longitude, $\lambda$ | Galactic lat. Galactic longitude |
| Origin for second coördinate | Meridian of Greenwich | South point of horizon | Vernal equinox, meridian | Vernal equinox | Intersection of galactic circle and equator |
| Circles fixed with respect to | Surface of Earth | The observer | The stars | The stars | The stars |

Probably the altitude and azimuth of stars are more easily estimated by the eye than are any of the other coördinates, since they are referred to the horizon, the position of which can be easily noted; but the coördinates of the equator system can be measured with greatest precision, and it is usual to obtain the others from them by a trigonometric transformation.

A better understanding of the different systems of coördinates may be obtained by studying Table 2.

**Star Maps and Celestial Globes.** A star map is a chart that shows the relative apparent positions of stars in the sky as an ordinary map shows the relative positions of places on the Earth. Like the terrestrial map, it represents on a flat surface bodies that lie upon a sphere; and as a result, if a large area is shown on a single map, there is considerable distortion. For this reason, in the best star atlases only a limited portion of the sky is shown in each map.

As in the terrestrial map, it is customary to represent north at the top; but since the star map represents objects seen overhead, the right-hand side of the equatorial maps is west instead of east. It must be understood that "west" means the direction of the diurnal motion of the heavens, or contrary to that of the Earth's rotation; hence it is really a circular direction, and on the north polar map "west" follows the parallels of declination counterclockwise around the pole. Astronomers often use the words **preceding** and **following** instead of west and east in describing directions in the sky.

The star maps in this book (Maps 1 to 8, between pages 6 and 7) are drawn on the **globular projection**; each map representing a portion of the sky approximately as if the stars had been plotted on the inside of a hemispherical shell and photographed from a point on the opposite side of its center, at a distance from the center of 1.7 times the radius of the hemisphere. The use of this projection and the fact that each map shows only a small section of the sky render the distortion of the constellation figures almost negligible. Hour circles, parallels of declination, the ecliptic, and the galactic circle are all drawn as dark lines. Right ascension, declination, celestial longitude, and galactic longitude are indicated by dark figures. To follow the stars in the order of their diurnal or seasonal appearance, the pages containing the equatorial maps (Maps 2 to 7) should be used in order from right to left.

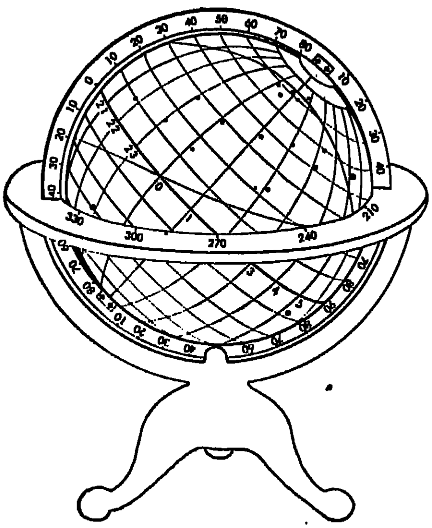

Fig. 23. *A Celestial Globe.*

A **celestial globe** (Figure 23) is a small model of the celestial sphere, on which the positions of stars are represented without distortion. It is only at great expense that a globe can be made large enough for the student to go inside of it, and the usual type is only a few inches in diameter. The map on the surface of the globe must therefore show the stars as they would appear if we could go outside the celestial sphere. If the globe be viewed from the north, "west" is the direction clockwise around the globe—the reverse of the case of maps in star atlases.

The globe is mounted on an axis passing through the poles of rotation. On its surface, in addition to the map of the stars, are printed the equator and ecliptic, and sometimes the galactic circle, together with either the secondaries to the ecliptic and parallels of latitude or the hour circles and parallels of declination. In either case, the two points where each of the twelve principal hour circles intersects the equator are indicated by numerals along the equator, reading from the vernal equinox toward the east from 0 to 24 and designating the hours of right ascension. The ecliptic is divided into degrees, and on most globes there are printed along the ecliptic dates showing the position of the Sun throughout the year. The globe is supported at the ends of its axis by a vertical ring, usually made of brass and called the brass meridian.

The center of the globe lies in the plane of one side of this ring, and this side is graduated to degrees. The graduations on the upper half read from 0 at the equator to 90 at the poles to designate declination, and those on the lower half read from 0 at the poles to 90 at the equator and are used in setting the globe to correspond to the observer's latitude. The brass meridian is supported in a frame, the upper part of which is a ring that represents the horizon, which is also graduated to degrees to designate azimuth. A thin brass or paper quadrant, when attached to the zenith, or highest point of the brass meridian, represents a vertical circle. It is graduated to designate altitude. When set so that the altitude of the pole equals the observer's latitude (page 70) and the right ascension of the meridian equals the sidereal time, the globe exhibits correctly the aspect of the sky and is said to be "rectified."

**The Zeiss Planetarium.** By far the most effective device yet produced for picturing the celestial sphere and illustrating the apparent motions of the heavenly bodies is the *optical planetarium*, designed by the engineers of the firm of Carl Zeiss in Jena, Germany, at the suggestion of the astronomer Max Wolf. A hemispherical dome forms the ceiling of a large room in which an audience may be seated, and represents the sky. At its center is an instrument consisting of many stereopticons or optical lanterns which project upon the smooth, white, inner surface of the dome images representing the naked-eye stars, the Sun, the Moon, and the brighter planets, and, at the will of the operator, the reference lines of the coördinate systems and the names of the constellations. The projectors are mounted on a polar axis so that, by a simple rotation, the diurnal motion of the celestial sphere is faithfully reproduced. As they rotate, automatic shutters close to obscure the setting stars and open to reveal those which are rising. The projectors of Sun, Moon, and planets may be operated independently, each in its appropriate plane, to imitate accurately the motions of those bodies. The whole projecting apparatus may be rotated in the plane of the meridian so as to be rectified for any latitude, and may also be adjusted to correspond to the changes due to precession (page 115). The planetarium may thus be made to exhibit, with close fidelity, the appearance of the sky at any place on the Earth, and at any time of any day or night of any year in many thousands. The motions are produced by electric motors controlled by switches at the hands of the demonstrator, who may operate them at various speeds so as to represent in a few minutes, for example, changes which actually require days, years, or centuries. Even for an observer of the picture presented by the planetarium, however, the problem of distortion is not overcome unless he is very near the center of the hemispherical chamber.

Zeiss planetaria have been in use for a number of years as instruments of public education and entertainment. In America there are the following:

1. The Adler Planetarium and Astronomical Museum at Chicago. Opened in 1930, under the direction of Dr. Philip Fox, it admitted more than three million visitors during its first three years.

2. The Fels Planetarium of the Franklin Institute at Philadelphia, opened in 1934. A public observatory is operated on the roof of the Institute, which houses a large scientific museum.

3. The planetarium of the Griffith Observatory at Los Angeles. Completed in 1935, it is operated in connection with a magnificent public observatory which is situated on the slope of Mount Hollywood, a thousand feet above the city.

4. The Hayden Planetarium of the American Museum of Natural History in New

York City, opened in 1935. In addition to the optical planetarium, there is an electrically operated Copernican planetarium, a mechanical model which shows the Solar System as it might appear from a distant point in space.

5. The Buhl Planetarium and People's Observatory at Pittsburgh, opened in 1941.

At the Museum of Natural History in Springfield, Massachusetts, is a device that projects a star map upon the inner surface of a dome, as in the Zeiss planetarium, but without representing the planets. It was constructed locally and opened in 1937.

**Practical Determination of Sidereal Time, Right Ascension, and Declination.** The declination and hour angle of a star may be found with a considerable degree of accuracy from the circle readings of an equatorially mounted telescope; and if the sidereal time is known, the right ascension is obtained by subtracting from it the hour angle (page 24). Similarly, the sidereal time is obtained by setting the equatorial on a star of known right ascension, reading the hour angle, and adding hour angle and right ascension together. The tube of the equatorial, however, when turned into different positions, bends by a small amount that is difficult to determine, and the observed positions are affected by this and by the even more troublesome effects of atmospheric refraction (page 66).

The most accurate determinations of star positions and of sidereal time are made by observing the stars as they cross the meridian. This method was used extensively by Tycho Brahe before the invention of the telescope, and is practiced with the modern **meridian circle** and **transit instrument**, which are often referred to as the fundamental instruments of practical astronomy. The meridian circle is a small or moderate-sized telescope attached firmly at right angles to a rigid axis which is supported in a level east-west direction on massive piers, and on which is mounted concentrically a finely graduated circle. The telescope and circle are thus free to swing in the plane of the meridian, but in no other direction. The transit instrument is like the meridian circle except that its circle is not so finely graduated and is used only for setting. In meridian instruments of the nineteenth-century type, which are still much used, there is placed in the tube of the telescope, in such a way as to be seen with the star (page 46), a **reticle** consisting of fine wires or filaments of spider web which are illuminated by a miniature lamp. An

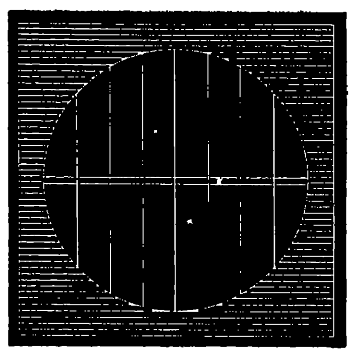

Fig. 24. *The Field of View of a Transit Instrument.*

odd number of lines—five, seven, sometimes as many as fifteen—are placed at equal intervals in the north-south direction, and when the instrument is in perfect adjustment the middle line of the group lies exactly on the meridian. At right angles to these parallel lines are one or more similar lines to mark the center of the field (Figure 24).

Since no instrument can be made perfect, it is never assumed that the middle wire follows the meridian with absolute precision. Its departure from the meridian can be expressed in terms of the **errors of the instrument**, of which there are three: (1) the collimation error, the angle which the line joining the middle wire of the reticle with the center of the object glass makes with a plane perpendicular to the axis; (2) the level error, the angle made by the axis with the plane of the horizon; and (3) the azimuth error, the angle made by the axis with the plane of the prime vertical. These errors can all be determined and allowed for, as explained in works on practical astronomy, but their discussion is outside the scope of this book.

For determining sidereal time or the right ascension of a star, the meridian circle or transit instrument is used by noting the reading of a clock at the instant the star is on the meridian. Instead of noting the time when the star crosses the middle line of the reticle only, a more accurate determination of the clock time of meridian passage is obtained by taking the average of the times of crossing the different lines. Since the star's hour angle at the moment of meridian passage is zero, its right ascension equals the sidereal time at that moment; and so, if the clock gives the correct sidereal time, the right ascension of the star is determined, or, vice versa, if the right ascension is known, the error of the sidereal clock is found by subtracting from its reading the star's right ascension. No effort is ordinarily made to set a sidereal clock "right"; it is better to leave it undisturbed and to keep a record of its error.

The observations may be made with a reticle either by the **eye-and-ear method**—watching the star and listening to the beats of the clock (Figure 25)—or by means of a **chronograph** which, being connected with the clock and with a key at the observer's finger tips, makes a record electrically. In either case, they are affected by **personal equation,** a small error by which even the best observer will judge the arrival of the star at the wire a little too early or too late. Personal equation is avoided in the best modern observations by replacing the reticle with a moving wire which, kept continually bisecting the star's image, makes electric contacts automatically; or by using photography.

Declination cannot be determined with the transit instrument, but is measured with the meridian circle. For this, the reading of the circle when

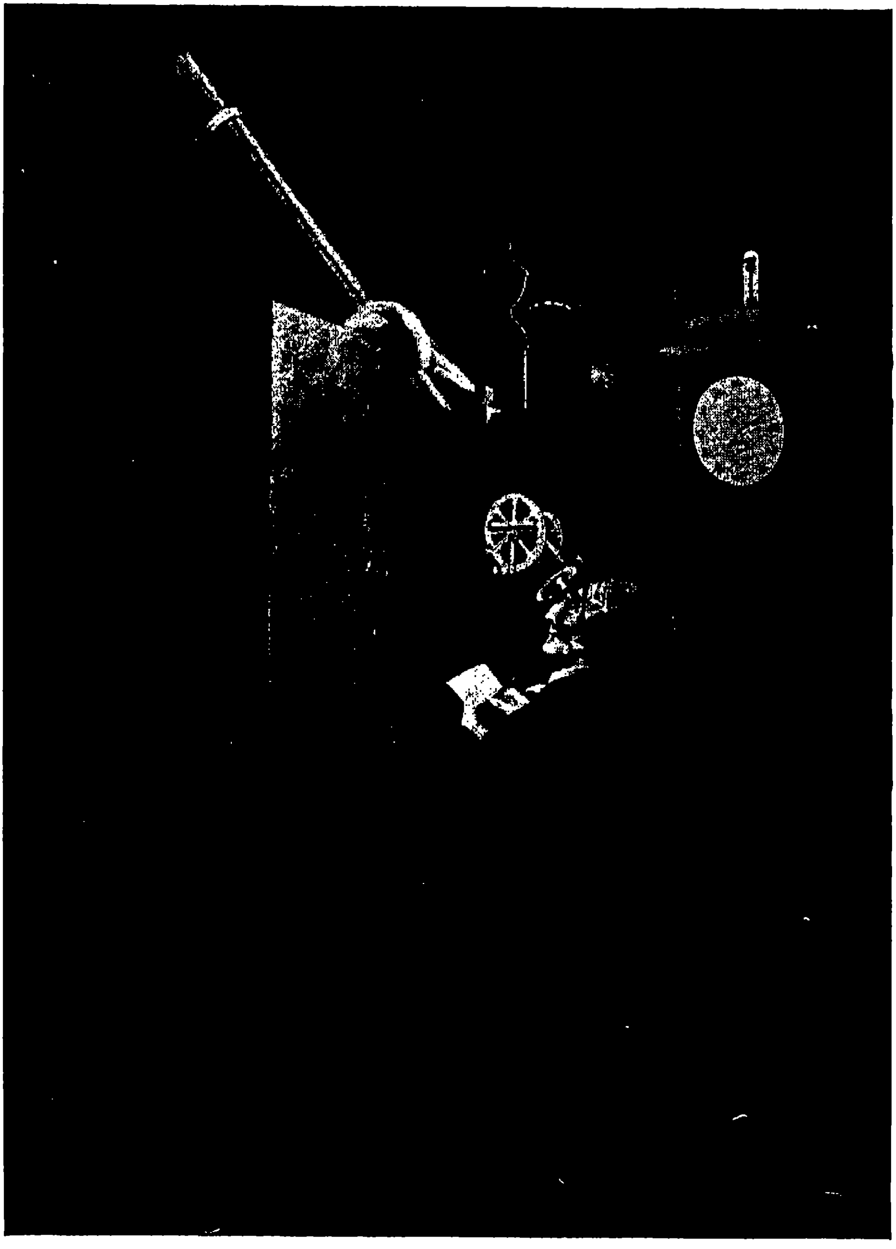

Fig. 25. *Using a Transit Instrument by the Eye-and-Ear Method.* (*From* Speculum Hartwellianum *by W. H. Smyth, 1860.*)

the telescope is directed to the pole must first be found by observing a circumpolar star as it crosses the meridian above the pole (upper transit) and again at lower transit, twelve hours later. When corrected for atmospheric refraction (page 66), the mean of these two readings gives the polar reading, which, so long as the meridian circle is undisturbed, remains unchanged. The angular distance of any star from the pole may then be determined by observing it at meridian passage and subtracting the reading of the circle (corrected for refraction) from the polar reading. This polar distance is the complement of the star's declination.

The best meridian circles of the present day are graduated with exquisite accuracy and are read by means of microscopes to the tenth of a second of arc. With modern astronomical clocks and transit instruments, sidereal time and the right ascension of stars may be determined to the hundredth of a second of time.

The positions of objects that cannot be conveniently observed with the meridian circle, such as faint comets that cross the meridian in daylight, are determined by measuring, with a micrometer (page 48) or upon a photograph, the *difference* of right ascension and of declination between the object and a star whose position is already known.

**Locating the Equinox.** The meridian circle method of determining sidereal time requires a knowledge of the right ascension of the stars observed, and the determination of right ascension requires a knowledge of sidereal time. The reader may naturally inquire how the astronomer makes a start. The fact is that the right ascensions of hundreds of standard stars are now known with great accuracy and that the methods we have described are the ones actually used in practice; but since right ascension is measured from the vernal equinox, the system must rest ultimately upon a knowledge of the position of that point among the stars. The vernal equinox being defined as the point where the Sun crosses from the south to the north side of the equator, the determination of its position must be made by observations of the Sun. Observations of the Sun's declination throughout the year (but best near the times of the equinoxes) give an accurate value of the obliquity of the ecliptic, which, with the observed declination, gives by a trigonometric calculation the Sun's right ascension independently of the sidereal time or observations of the stars; hence, observations of the clock time of the meridian passage of the Sun made with the meridian circle simultaneously with those of the declination give the error of the sidereal clock, and the possession of this error

provides for the determination of the right ascensions of the standard stars.

## EXERCISES

1. At a distance of 39,080,000 miles, the planet Venus subtends an angle of 40″. What is the approximate diameter of Venus?

*Ans.* 7580 miles

2. At what distance must a toy balloon one foot in diameter be placed in order to appear the same size as the Moon (apparent diameter ½ degree)?

*Ans.* 115 feet

3. A cylindrical water tower one mile distant just hides the setting Sun. What is the diameter of the tower?

*Ans.* About 47 feet

4. A man six feet tall is seen to cover the vertical diameter of the rising Moon. How far is he from the observer?

*Ans.* About 690 feet

5. If two stars are seen at the same altitude below the pole by an observer in the United States, which is "east" of the other—the right or the left? Which, if the two stars are between the pole and the zenith?

6. What are the altitude, azimuth, and hour angle of the west point of the horizon? Of the north point?

7. What is the sidereal time when $\beta$ Cassiopeiae is directly above Polaris? Below Polaris?

8. What is the approximate solar time when $\beta$ Cassiopeiae is directly above Polaris on December 21?

9. What are the right ascension and declination of the north pole of the ecliptic?

# CHAPTER 2

## THE OPTICS OF THE TELESCOPE

**Invention of the Telescope; Its Importance to Astronomy.** The first telescope of which we have any authoritative account was made in 1608 by Jan Lippershey, a Dutch spectacle-maker. He made no astronomical application of his invention, but in 1609 news of this wonderful instrument which could extend the power of the human eye reached Galileo Galilei, the great Florentine physicist, who at once discovered the principle for himself, made a number of telescopes, and within a year had used them to make discoveries that revolutionized the astronomy of his time. With improvements in the making of telescopes their usefulness has increased, and at the present time almost all observations that contribute to the advancement of astronomy are made with the help of this instrument.

**Essential Parts and Principal Properties of the Telescope.** The most essential part of a telescope is the **objective**, a lens or mirror which, by refraction or reflection, concentrates a beam of light from a luminous point such as a star [1] into a tiny dot, an image of the star. Star images are formed in a nearly flat geometrical surface called the **focal plane**, and the distance of the center of the objective from this surface is its **focal length.** If a photographically sensitized plate or film is placed in the focal plane, the dots of light are recorded as a photograph of the stars. The distance between their centers is proportional to the focal length of the objective, which thus determines the scale of the picture. In photography lies one of the principal uses of the telescope in modern astronomical research.

Because of the wave nature of light, the star image formed by a large objective is smaller than one formed by a small objective, so that the images of a close pair of stars that overlap in a small telescope may be clearly resolved in a large one. The

[1] Every star is so far away as to have no sensible diameter, and the rays forming any beam of its light that reaches the Earth are sensibly parallel.

theoretical **resolving power**, in fact, is directly proportional to the diameter of the opening, or **aperture**, of the objective.

For visual use (that is, for seeing instead of photographing), there is added a small lens called the **eyepiece**, through which the contracted beam passes to the observer's eye. Objective and eyepiece (or objective and plate holder) are held in their proper relative position by the **tube** of the telescope, which is usually mounted equatorially. As a funnel restricts a large stream of water into a small one, so the visual telescope concentrates a beam of starlight that is as large as the aperture of the objective into a much brighter beam that is small enough to enter the eye. One of the most important properties of the telescope, whether photographic or visual, is thus its **light-gathering power**, which is proportional to the area of the aperture of the objective and therefore to the square of its diameter. Through the eyepiece the observer sees the imaged stars at a greater angular distance apart than they appear to the unaided eye, and this **magnifying power** is independent of the aperture and equals the ratio of the focal lengths of objective and eyepiece. Finally, in the visual astronomical telescope a small object such as a filament of spider web, if placed in the focal plane and artificially illuminated, becomes visible as if it were among the stars and makes the telescope an exquisitely sensitive pointer or instrument for measuring angular distances.

In summary, the principal properties of a telescope (which will be explained in some detail in the remainder of this chapter) are:

1. Usefulness for both photographic and visual observation.
2. Resolving power, proportional to the diameter of the objective, determining the sharpness and clearness of the picture. An advantage of large telescopes.
3. Light-gathering power, proportional to the square of the diameter of the objective, permitting the study of faint objects. The most important advantage of large telescopes.
4. Scale of the picture in the focal plane, proportional to the focal length of the objective. The advantage of long telescopes, especially important in photography.
5. Magnifying power (of a visual telescope), equal to the ratio of the focal lengths of objective and eyepiece.
6. Usefulness for accurate measurement of angular distances.

Galileo's telescopes were refractors and were so small that they could be held in the hand. The first reflecting telescope was made by Sir Isaac Newton about 1670. Reflectors having apertures up to 48 inches were made by Sir William Herschel late in the eighteenth century; a 72-inch reflector was built by Lord Rosse in 1845; and the construction of giant reflectors in the twentieth century has culminated in the 200-inch reflector of Palomar Mountain. Refractors reached their maximum size for the present in the 40-inch of the Yerkes Observatory, the objective of which was made by Clark in 1895. Astronomical photography has been practiced since about 1850, and has now largely superseded visual observation.

**Reflection and Refraction of Light.** When light travels through empty space, as between the Sun and the Earth, or through a uniform transparent substance, it travels in straight lines. If a ray of light, traveling in such a medium, encounters the surface of a second medium, say a piece of glass, the ray will be divided (Figure 26); one part will be turned back into the first medium and the remainder will pass into the second. If this second medium is homogeneous like the first, the portion of the original ray that enters it will also travel in a straight line, but not, in general, the same straight line as that of the original ray. In other words, the light is bent abruptly at the common surface of the two media. The turning back into the first medium is called **reflection**, and the bending into the second medium is called **refraction**.

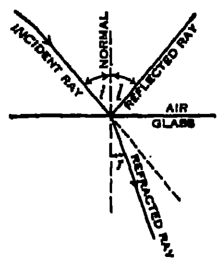


Fig. 26. *Incident, Reflected, and Refracted Rays.*

The angle $i$, made with the normal to the common surface of the two media by the original (incident) ray, is called the **angle of incidence**; that made by the reflected ray, the **angle of reflection** ($l$); and that made by the refracted ray, the **angle of refraction** ($r$). The **law of reflection** states that the angles of incidence and reflection are equal, or

$$l = i.$$

The **law of refraction**, known as Snell's law, states that the ratio of the sines of the angles of incidence and refraction is a constant which depends on the color of the light and the nature of the two media; that is,

$$\frac{\sin i}{\sin r} = \mu.$$

This constant, $\mu$, is called the relative **index of refraction** of the two media. Its value for air and water is about 1.33. For air and glass of different kinds, it ranges from about 1.4 to about 2.0.

**Mirrors.** The mirrors of telescopes are no ordinary looking-glasses. Their front surface must be made the reflecting surface to avoid multiple

reflections; they must be thick and, if large, must be supported effectually from the back to avoid flexure when they are moved; the material should be such as to change but little through wide changes of temperature; and their surface must be shaped to a mathematical curve with a minimum tolerance of a millionth of an inch.

Until about 1860, telescope mirrors were usually made of speculum metal, a hard, bright, brittle alloy of tin and copper; but since that time they have mostly been made of glass, coated on the front with an exceedingly thin film of metal. The glass need not be transparent, since the light does not enter it; but it should have a low coefficient of expansion. "Pyrex," the glass of which many cooking utensils are made, fulfills this condition and is now being much used for astronomical mirrors. Until recently, the metal used for the reflecting surface was chemically deposited silver; but this has been lately superseded by aluminum, distilled upon the glass in an electrically heated vacuum chamber. For visible light, fresh silver is slightly the better reflector; but it tarnishes readily and, moreover, it is transparent to much of the invisible light known as ultra-violet (page 166). Distilled aluminum is highly reflective throughout the range of wave-lengths used by astronomers, and it long preserves an untarnished surface. All the work of grinding, polishing, and testing the glass to achieve the correct figure is of course completed before the metal is applied. The production of a good mirror (or any other fine optical surface) requires great patience and long-continued effort as well as skill and intelligence.

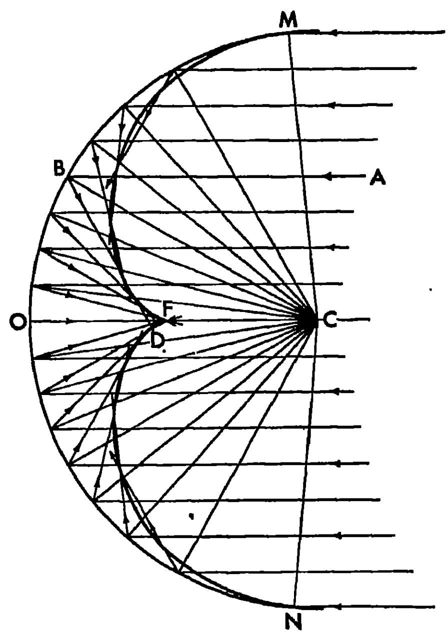


Fig. 27. *Action of a Spherical Mirror (Caustic Curve).*

The optical surface that is easiest to produce is a spherical surface. Since all the normals to the surface of a sphere pass through the center, it is easy to determine the effect of such a mirror by constructing a diagram like that in Figure 27, which represents rays of starlight reflected from the inner surface of a hemispherical shell *MON*, whose center is *C*. Let *AB* be any one of the parallel rays coming from the star. Draw the radius (normal) *BC* and, in harmony with the law of reflection, draw the reflected ray *BD* so that the angles *ABC* and *CBD* are equal. When this is done for many rays, it is found that each reflected ray

meets its neighbor on a curved surface that is sharply pointed at *F*, midway between *C* and the vertex of the mirror, *O*. This surface is called a **caustic** surface, and its cross section, *MFN*, is called a caustic curve.

The caustic curve may be seen beautifully within a finger ring laid in sunshine on white paper. When formed in parallel light as in Figure 27, it is mathematically identical with the **epicycloid** that might be traced by a point on the circumference of a circle of radius one-quarter that of the mirror, rolling around a fixed circle with center at *C* and radius one-half that of the mirror.

Evidently, a spherical mirror fails to bring all the light falling upon it to a common focus as required in a telescope objective. This failure constitutes a defect that is called **spherical aberration.** By using only a small saucer-like part of the hemisphere near *O* (or, what is the same thing, making a mirror with a focal length several times its aperture), the most undesirable part of the caustic is eliminated; but even so, the spherical aberration is too great to be tolerated in an astronomical telescope. It is therefore necessary to make a mirror with a surface that is other than spherical, whose normals are so directed that the reflected rays will meet at a point. It is shown in elementary works on analytic geometry that the required surface is a portion of a **paraboloid of revolution.** A cross-section is represented in Figure 28, with reflected rays of starlight meeting at the focus of the paraboloid, *F*. The usual procedure in making a paraboloidal mirror is first to grind and polish the glass to a spherical surface with a radius about twice the desired focal length and then, by a long process of local polishing and testing, to transform it to the parabolic form. The layer of glass that is removed in this transformation is exceedingly thin, but its removal makes a great difference in the value of the mirror.

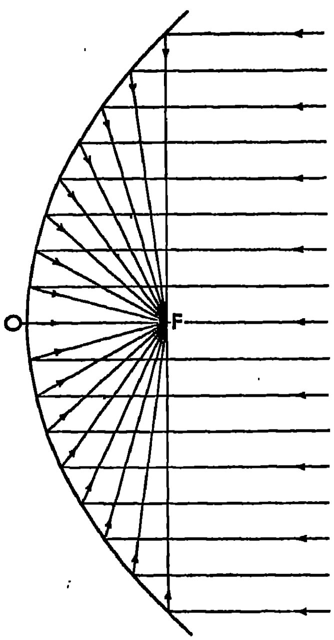

Fig. 28. *Action of a Parabolic Mirror.*

The path of the light is reversible: light from a small source placed at *F* would follow the lines of Figure 28 backward and so be reflected in a parallel beam. The

reflectors of searchlights operate on this principle and are deep "saucers" like the one represented in Figure 28. In the objectives of reflecting telescopes, the focal length is ordinarily from three to ten times the aperture, and the "saucer" is shallow, having an aperture from 50 to 150 times its depth.

**Reflecting Telescopes.** In the reflecting telescope, the paraboloidal mirror is mounted in a cell at the lower end of the tube (which in large reflectors is a skeleton tube composed of steel members braced for rigidity), and its focus is at the upper end where it is somewhat difficult of access. Different methods of overcoming this difficulty produce different forms of the telescope.

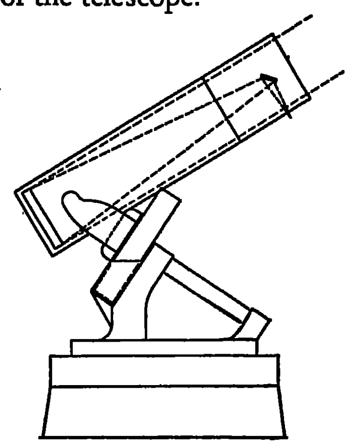

Fig. 29. *Newtonian Reflecting Telescope.*

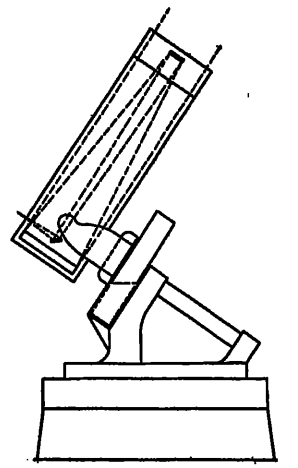

Fig. 30. *Cassegrainian Reflecting Telescope.*

In the **Newtonian** type of reflector (Figure 29) a plane mirror, set at an angle of 45° to the axis of the paraboloid, intercepts the reflected light just before it reaches the focus and sends it to the side of the tube near the top. In the **Cassegrainian** form (Figure 30) a convex hyperboloidal mirror at the upper end of the tube increases the equivalent focal length and sends the rays back toward the objective, where they either pass through a central hole, as in most Cassegrains, or are reflected to the side by a plane mirror just above the objective, as in the reflectors at the Mount Wilson Observatory. In still another form of reflector, the American **coudé**, the third reflection sends the light down the polar axis into an underground chamber. Large modern reflectors are equipped with various auxiliary mirrors mounted in removable sections of the tube or otherwise, so that the type and equivalent focal length of the telescope may be changed. In one form of the 200-inch telescope on Palomar Mountain (Figure 31), all auxiliary

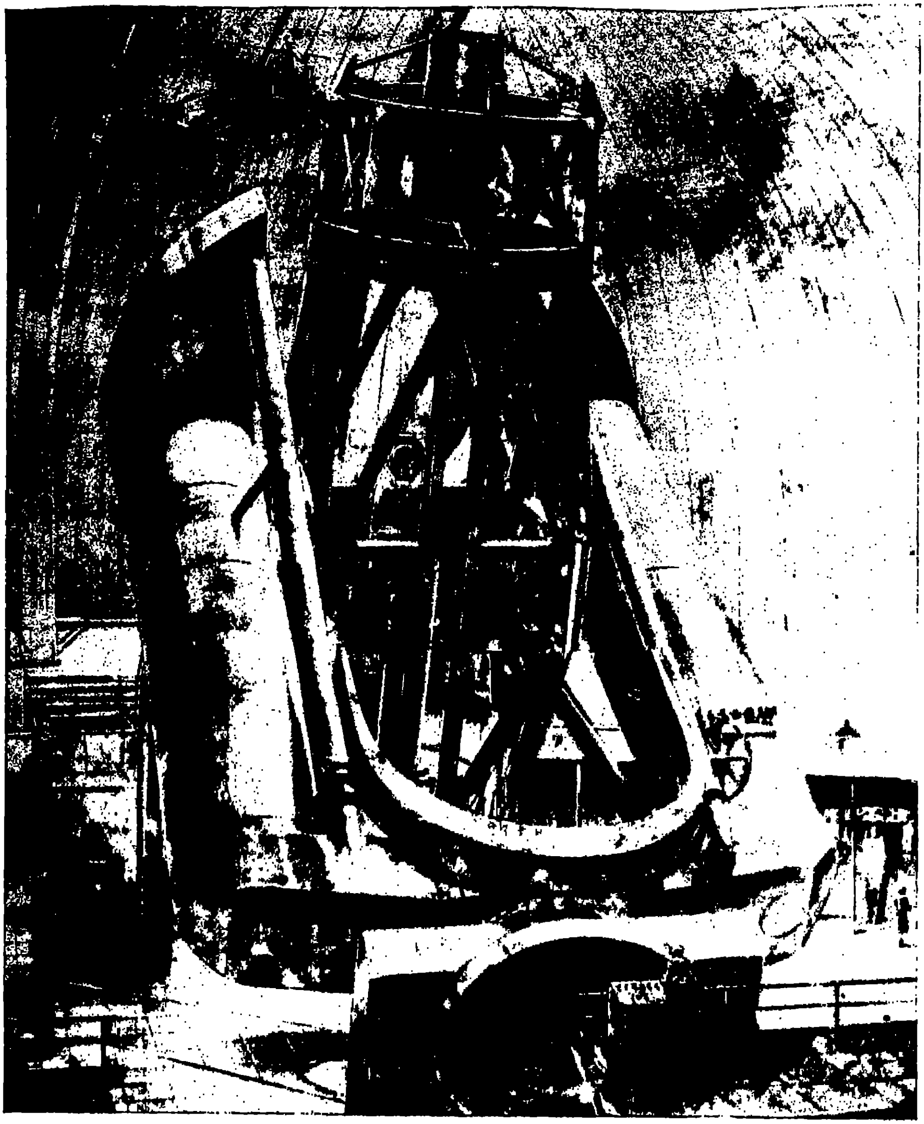

Fig. 31. *The 200-Inch Telescope, Viewed from the South. (Drawing by Russell Porter.)*

mirrors are dispensed with and the observer has direct access to the prime focus while riding in an air-conditioned cage in the middle of the upper end of the tube. The cage, about five feet in diameter, intercepts only about a tenth of the 200-inch beam of incident light.

**Path of a Light Ray Through Glasses of Different Forms.** When light enters glass from air or empty space, it is refracted toward the normal,

and when it passes from glass to air it is refracted away from the normal. In passing through a plate of glass with plane parallel sides (Figure 32), the two refractions are equal and the emergent ray is parallel to the incident ray. The total effect is thus to shift the ray sidewise without changing its direction.

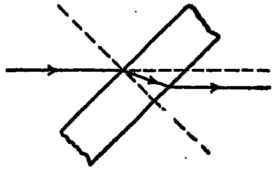

Fig. 32. *Action of a Plate with Parallel Sides.*

The case is different if the two glass faces through which the light passes are not parallel. This is true of the optical **prism**, a block of glass with three plane faces that meet in parallel edges (Figure 33). Here the normal to the second face is not parallel to that of the first, and the ray suffers a deviation, which is always in a direction away from the edge in which the two refracting faces meet. Light of different colors is deviated by different amounts, for the index of refraction of glass is greatest for violet light and least for red, the indices for the different colors being in the order violet, blue, green, yellow, orange, red. White light is in reality a mixture of all the different colors, and so when white light is passed through a prism, it is spread out in a colored **spectrum**, red at the end toward the refracting edge and violet at the other. This property of transparent substances is called **dispersion**, and is exceedingly useful to astronomers, although somewhat troublesome to makers of telescopes.

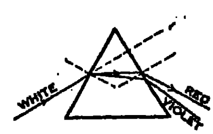


Fig. 33. *Action of a Prism.*

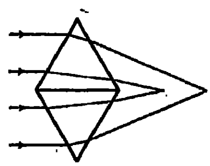

Fig. 34. *Action of a Double Prism.*

Suppose that two prisms are placed with their bases together (Figure 34) and illuminated by a beam of parallel rays—say the light of a star, which is always so far away that its rays that fall on the Earth are sensibly parallel. If the rays through the extreme parts of the combination could be made to deviate more than those through the middle part, all the rays might be made to meet. This is in fact accomplished by the **convex lens** (Figure 35). The point where rays that were originally parallel are made to meet is called the principal **focus**, and its distance from the center of the lens, the latter's **focal length**.

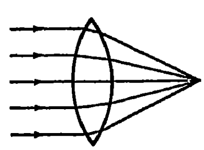

Fig. 35. *Action of a Convex Lens.*

If the rays are not parallel, but diverge from a point within a finite distance from the lens, they are converged by the lens to a point more distant than the principal focus. Thus light diverging from the point *A*, Figure 36, is focused to the point *B*.

The points $A$ and $B$ are called **conjugate foci.** In the use of a projecting lantern, as in a motion-picture theater, one of a pair of conjugate foci of the projecting lens

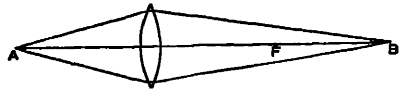


Fig. 36. *Conjugate Foci.*

is occupied by the slide or film carrying the photograph, and the other by the screen on which the audience sees the picture.

Figure 37 shows a **concave lens,** the effect of which upon parallel rays is to make them diverge from each other.

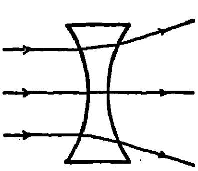

Fig. 37. *Action of a Concave Lens.*

**The Photographic Camera.** A photographic camera is, in its simplest form, a light-tight box with a convex lens in an opening in one wall and means for holding a sensitive plate or film in the opposite wall, at a distance from the lens equal to its focal length. If the camera is directed at night to the sky, the parallel rays from each star within its field are brought to a focus upon the plate, which registers a dot in a position corresponding to that of the star. Each point of an extended object may be similarly recorded, the aggregate of the point-images making up a picture of the object. For taking "close-ups" of terrestrial objects, where each point of the object sends a divergent pencil of light to the lens, it is necessary to "focus" the camera by changing the distance of the plate from the lens, so that the plate and the object are at conjugate foci; and all except the simplest box cameras are provided with means for doing this.

**The Human Eye.** Physiologically considered, the eyeball is a camera, in which light is focused by the **crystalline lens** and **cornea** upon the **retina,** a network of sensitive nerves which, instead of photographing the light, transmit to the brain the sensation produced by its presence. In the normal eye, when it is at rest, the retina lies in the principal focus of the lens system. The nearsighted, or myopic, eye is too long, and in it the retina is behind the principal focus; in the farsighted, or hypermetropic, eye it is in front.

Light coming from a point less than a hundred feet from the observer, instead of being parallel, is perceptibly divergent; the normal relaxed eye converges it to a point farther back than the retina, and for distinct vision it is necessary that the focal length of the lens be changed. This is accomplished by a muscular effort that causes the crystalline lens, which is

elastic, to change its curvature, a process that is called by physiologists **accommodation.** Young children can accommodate their eyes to distinct vision at a distance of less than three inches, but the power is gradually lost, the "near point" being on the average four inches from the eye at the age of twenty, and nine inches at forty, while in old age it recedes to infinity and the power of accommodation is lost. This wonderful ability of the lens of the eye to change its focal length is not possessed by any artificial optical instrument.

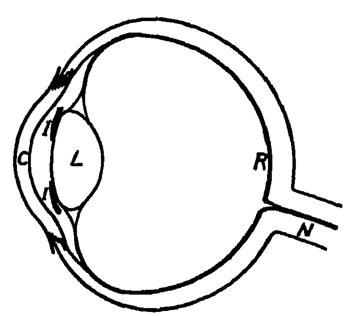


Fig. 38. *Diagram of the Eye:* C, *Cornea;* I, *Iris;* L, *Crystalline Lens;* R, *Retina;* N, *Optic Nerve.*

The opening through which light enters the eye, which appears as a black circular hole at the center, is called the **pupil;** it is surrounded by the **iris,** the colored portion of the eye, which has the power of contracting so as to change the size of the pupil. In very bright light the pupil is reduced to a mere pinpoint, while in darkness it is enlarged to its maximum diameter, which averages about a third of an inch. In the observation of faint stars, it is probable that the pupil has about this diameter. This property of the eye is imitated in most camera lenses by the **iris diaphragm.**

**Simple Refracting Telescope.** The action of an astronomical refracting telescope, in its simplest form, is shown in Figure 39. The large lens, which receives the rays from the star, is the objective, and the small lens is the eyepiece. The objective brings the originally parallel rays together at its principal focus, forming an image of the star which can be

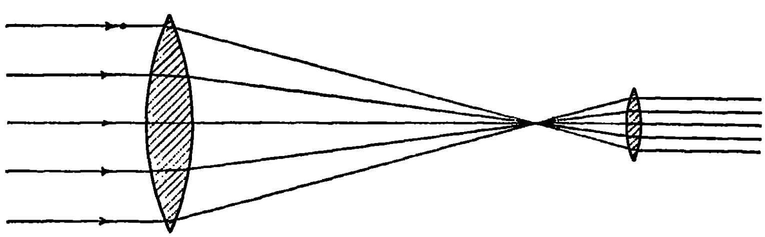

Fig. 39. *Simple Refracting Telescope.*

seen as a bright point on a white screen or registered as a fine dot on a photographic plate held in the focal plane. In the photographic telescope, which is essentially a camera of great focal length, the objective is thus the only optical part needed. If not stopped by a screen or plate, the light

proceeds through the focus in straight lines which diverge beyond the focus. An eye placed in this cone of diverging rays receives light from the star, but in directions as if the star were a bright point situated at its image, so that the observer seems to see the star in the tube of the telescope, just before his eye—a rather interesting observation. In order, however, to receive in the pupil (never more than one-third inch in diameter) all the light that has come through the objective, the eye must be placed very close to the focus, closer than its power of accommodation could allow for. To obviate this difficulty the eyepiece is added.

The astronomical eyepiece is essentially a short-focus convex lens which, for normal vision, is placed at a distance from the focal plane of the objective equal to its own focal length, so that the distance between the two lenses is equal to the sum of their focal lengths. The property of the lens by which it converges parallel rays to its principal focus is thus reversed, and the rays that diverge from the image of the star emerge from the eyepiece in a parallel beam. If the focal length of the eyepiece is small enough, so that the lens is placed near the apex of the cone of rays, the emergent beam is narrower than the pupil of the eye, which thus may receive all the light transmitted through the large objective.

The eyepiece is ordinarily supported in a draw-tube that slides within the tube supporting the objective so that the instrument may be "focused" for the eyes of different observers by changing the distance between the objective and the eyepiece.

**The Telescope as a Pointer.** One difficulty in using sights or pointers for determining the direction of a star or other object, as was done in the ancient astrolabes, arose from the fact that the eye cannot be accommodated for clear vision of the nearby sight and the distant object at the same time. This difficulty is eliminated in the telescope by placing in the common focus of the objective and eyepiece a pair of crossed spider threads or other form of reticle and illuminating it artificially. The light reflected from the threads, since it diverges from the focus of the eyepiece, is made parallel by the latter, and so the threads are seen not in their real position just before the eye, but as exquisitely fine lines drawn on the surface of the sky, among the stars. By using the intersection of the threads as a fiducial point the direction of the line of sight is thus made definite.

**Rays from Two Stars; Magnifying Power.** Suppose the telescope is directed to a close pair of stars, one above the other, and let the parallel rays *AAA* come from the upper of the two stars and the rays *BBB* from

the lower (Figure 40). The rays that pass through the center of the objective, like the ray passing through the plane plate in Figure 32 (page 43), are undeviated, and this central ray from the upper star meets all the others from that star in the focal plane at *a*, while the rays from the lower star are focused at *b*. After diverging from the image, the rays from each star are bent toward the thick central part of the eyepiece and emerge parallel to

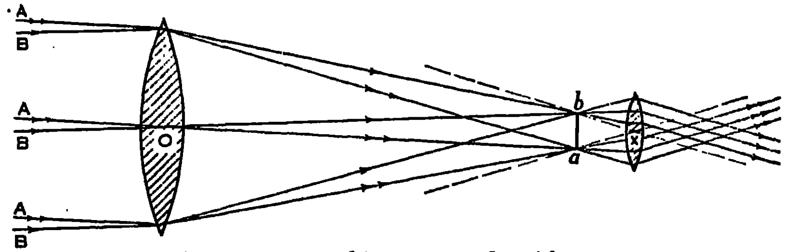


Fig. 40. *Magnifying Power of a Telescope.*

the line that joins the image with the center of the lens. An eye placed behind the eyepiece thus sees star *A* in the direction *xa* and star *B* in the direction *xb*. One effect of the telescope is therefore to invert the image, the upper star appearing below and the lower one above. Though objectionable in viewing terrestrial objects, this inversion causes no inconvenience in studying heavenly bodies, whose position with respect to a vertical line changes in any case with the hour angle.

The angular separation of the two stars, as seen by the naked eye, is the angle *AOB* which is equal to *aOb*; as seen in the telescope, their separation is *axb*, which is larger. In the figure the angles are much exaggerated—two stars to be visible at the same time in a telescope must be more nearly in the same direction than here represented; hence, in practice, the angle *AOB* (or *aOb*) is small, and its value in radians may be taken as the distance *ab* divided by *Oa* or *Ob*. Similarly, the angle *axb* may be taken as *ab* divided by *xa* or *xb*. But *Oa* is equal to the focal length, *F*, of the objective and *xa* equals the focal length, *f*, of the eyepiece; hence, the **magnifying power**, which is the ratio of the apparent distance between the two stars as seen in the telescope to their apparent distance as seen by the unaided eye, is

$$M = \frac{ab}{f} \div \frac{ab}{F} = \frac{F}{f}.$$

That is, the magnifying power of a telescope is equal to the ratio of the focal length of the objective to that of the eyepiece. Since the eyepiece

is small and inexpensive compared to the objective, most telescopes are provided with a number of eyepieces of different focal lengths and therefore of different magnifying powers.

It may be seen in Figure 40 that $M$ is equal also to the ratio of the diameter of the beam of starlight that enters the objective to that of the beam that emerges from the eyepiece, for these diameters are the bases of similar triangles having a common vertex at the principal focus and the focal lengths $F$ and $f$ as their respective altitudes. The lower the magnifying power, the larger is the emergent beam; if too low a power is used, the beam will be too large to enter the pupil of the eye and the full aperture of the objective will not be utilized. If the maximum diameter of the pupil is assumed to be a third of an inch, the minimum useful magnifying power is seen to be three times the diameter of the objective in inches.

**The Filar Micrometer.** An important accessory to a visual telescope is the **filar micrometer**, an instrument used for measuring small angular distances. A rectangular frame, $A$ (Figure 41), is made to slide over a larger frame, $B$, by turning the fine-threaded screw $S$, which has a graduated head for reading small fractions of a revolution. The springs $P$, $P$ take up the backlash of the screw. Across each frame is stretched a fine thread of spider web, $T$, $T$, and these lines are illuminated by the little electric lamp $L$. The two frames are arranged to rotate about the center of the circular plate $C$, the edge of which is graduated to degrees read by the index $I$ for determining the position angle (page 354) of the lines. A positive eyepiece is attached to the frame $A$ and focused on the spider lines, and the plate $C$ is attached to the tube of the telescope so that the lines lie in the focus of the objective. The parallel spider lines then appear as if drawn in light upon the sky, and the distance between the images of two close stars may be measured by setting one line on each star and reading the graduated head of the screw.

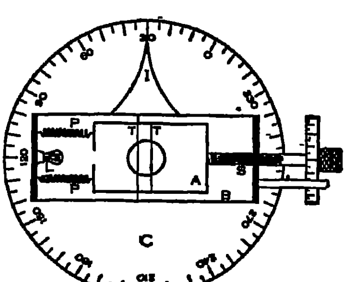

Fig. 41. *Filar Micrometer.*

To reduce this distance to seconds of arc, one must multiply it by the **micrometer constant**, which is the number of seconds that corresponds to one complete turn of the screw. This constant may be determined by setting the lines parallel to an hour circle, separating them by a known number of turns, and finding the time required for a star on the equator to pass from one line to the other. The number of

seconds of *time* in this interval, multiplied by 15, is the number of seconds of *arc* between the spider lines.

**Accessories for Celestial Photography.** For photographing any celestial objects except the brightest ones, long exposures, often of many hours' duration, are necessary. To follow the diurnal motion during this time, the photographic telescope or camera must of course be equatorially mounted and clock-driven, but even then the position of the image upon the plate will be changed by the effects of atmospheric refraction, bad seeing, and the imperfections of the clock unless these errors are corrected by the observer. Short-focus cameras are often attached rigidly to a **guiding telescope** provided with crossed spider lines on which the image of the object being photographed is kept by the observer, using the slow-motion devices of the telescope. With long-focus instruments, a **double-slide plate carrier** is often used. The plate holder is clamped to a rigid frame which may be moved parallel or perpendicular to the equator by fine screws in the hands of the observer. A positive eyepiece fitted with a pair of crossed spider lines is clamped firmly beside the plate, so that the intersection of the lines may be held upon the image of a "guide star" located just outside the limit of the field that is being photographed. The difference in the use of the guiding telescope and the double-slide plate carrier is this: in the former the errors of following are corrected by moving the whole instrument, while in the latter only the plate and eyepiece, with their supporting frame, are moved.

**Resolving Power.** Because light consists of waves, of sensible though exceedingly small length, the rays that meet at the focus of a lens or mirror interfere (page 164) and produce a star image that is not a geometric point but a small, round disk surrounded by a series of concentric dark and bright rings, theoretically infinite in number, but growing fainter so rapidly with distance from the center that usually only one or two are visible, and these only with a high magnifying power. The reason for this **diffraction pattern** is set forth in works on physical optics, but its discussion is beyond the scope of this book.

If two stars are so close together that their images overlap, they appear as a single object, although perhaps somewhat elongated, and an increase of magnifying power serves only to make the combined image larger without separating the stars. If the images could be made smaller, the two stars might be seen separated, or "resolved," by the telescope. Now, it is a fact that the larger the objective of the telescope the smaller is its

image of a star, the diameter of the image being inversely proportional to that of the objective; hence, the **resolving power** of a telescope, or its ability to separate close stars, is directly proportional to the diameter of the objective. The angular distance of the closest pair of stars that can be resolved in a one-inch telescope is, according to Dawes, 4.5 seconds of arc; hence, in a telescope $a$ inches in diameter, the minimum resolvable distance is

$$\frac{4''.5}{a}.$$

One might infer that the naked eye could resolve a pair of stars that were only $4''.5 \div \frac{1}{3} = 13''.5$ apart; actually, the minimum distance for resolution by the eye is much greater than this, the two stars of $\epsilon$ Lyrae, $3\frac{1}{2}'$ apart, being a pretty good test for acute vision. The reason for this lies not in the lenses of the eye but in the structure of the retina, the sensitive elements of which are the microscopic "rods and cones." If a complete cone lies between the images of two stars, untouched by either, the stars are seen separately; if the images are too close for this to occur, the observer is conscious of but a single point of light. The action of a photographic plate, which is granular in structure, is in this respect analogous to that of the retina.

The resolving power of a telescope sets a practical limit to the magnifying power that can advantageously be applied to it, for there is no advantage in enlarging the image of a star beyond the point where it is easily perceived as a disk. Assuming that the eye can perceive a disk 180″ (a little less than the separation of $\epsilon$ Lyrae) in diameter, the maximum useful magnification in a telescope would be found by

$$\frac{4''.5}{a} \cdot M = 180'', \text{ whence } M = 40\,a$$

or the greatest practicable magnifying power is forty times the diameter of the objective in inches. Greater magnification than this is in fact seldom used. It has already been shown, on the other hand (page 48), that the *minimum* useful magnifying power is $3\,a$.

**"Bad Seeing."** One of the most obvious characteristics of a star, and one which appeals to poets far more than to astronomers, is its twinkling. This is a rapid fluctuation in the star's brightness, accompanied by minute and equally rapid changes of its apparent position and sometimes by variations of color. It is caused by motions in the air above us and is analogous to the effect that would be produced if we should view the star from the bottom of a clear lake, the surface of which was agitated by waves. The telescope magnifies the twinkling, so that it is usually great

enough to interfere with the observation of fine detail or of stars that are close together. This condition is called **bad seeing.** Its effects are worse in a large telescope than in a small one because the larger objective receives light from a wider air-path in which the total disturbance is greater. It often happens that a great telescope is rendered practically useless by bad seeing on a perfectly cloudless night. Figure 42 shows two photographs of the "trapezium" of stars in the Orion nebula, made with the 100-inch Hooker telescope. The one on the right, made in fairly good seeing, shows two faint stars whose existence would hardly be suspected from the evidence of the other picture, which was made in poor seeing.

To the unaided eye, a planet usually does not seem to twinkle. This is because its angular diameter is appreciable, and while one point of its surface increases in brightness, that of another diminishes, keeping the total effect about constant. Bad seeing, however, is sufficiently evident in telescopic observation of planets, causing the fine detail to run together and the whole image to become blurred.

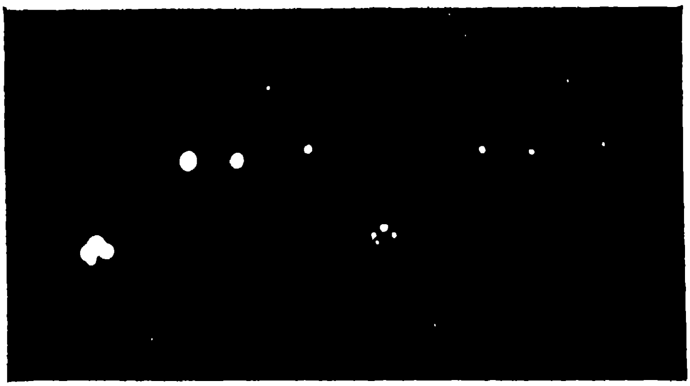

Fig. 42. *Stars Photographed in Poor and in Good Seeing.*

Since the aerial disturbances that cause bad seeing are worse in some localities than in others, this factor is very important in choosing a site for an observatory. Other desiderata are a large number of clear nights in the year and freedom of the air from dust and haze. A high altitude, to place the observer above a part of the air, a dry climate to assure cloudless skies, and distance from the smoke and artificial lights of cities are all desirable.

**Brightness of the Image.** The **light-gathering power** of a telescope may be defined as the ratio of the amount of light from a given star entering

the objective to the amount entering the pupil of the unaided eye. The amount of light that can pass through a given aperture is proportional to the area of the aperture, and therefore to the square of its diameter; and hence, calling the diameter of the objective $a$ and that of the pupil $b$, we have for the light-gathering power,

$$L = \frac{a^2}{b^2}.$$

If we let $b = \frac{1}{3}$ inch, the maximum diameter of the pupil, we have for the light-gathering power of a telescope of $a$ inches aperture,

$$L = 9\,a^2.$$

The stars are all at such prodigious distances that, although their real diameters are very great, their angular diameters are less than that of the diffraction image produced by the objective of any ordinary telescope; and, except when excessive magnification is used, the image is too small to be seen as a disk, and so the effective size of all star images on the retina is the same. The apparent brightness of a star as seen in a telescope as compared with its brightness as seen by the unaided eye depends, therefore, only on the light-gathering power. A ten-inch telescope makes a star appear 900 times, and a hundred-inch telescope 90,000 times, as bright as it appears to the unaided eye. Large telescopes thus disclose millions of faint stars that without their aid would be invisible.

The brightness of the image of a planet, nebula, or other object that presents a perceptible surface depends upon the focal length of the objective as well as upon its diameter; for the brightness is diminished by enlarging the image, and the diameter of the latter is proportional to the focal length, as may be seen in Figure 40. The area of the image varies directly, and its brightness inversely, as the square of the focal length. The focal length of the objective being called $F$ and its linear aperture $a$, the brightness of the image of a planet or nebula in the focal plane is therefore proportional to $a^2/F^2$. The "speed" of an objective for photographing nebulae and planets, or for ordinary landscape and portrait photography, is proportional to this ratio, while for photographing stars it is proportional merely to $a^2$. If the telescope is used visually, the area of the image on the retina is proportional to the magnifying power, and its brightness to $a^2/M^2$.

It can be shown that the brightness of a widely extended surface, which more than fills the field of view, cannot be increased by any optical device.

Such an object is the sunlit sky. The reason we cannot ordinarily see the stars in the daytime is that the background of sky is too bright to afford sufficient contrast to the light of the star. The telescope, by increasing the apparent brightness of the stars without changing that of the sky, enables us to see the brighter stars in daylight without difficulty.

**Chromatic Aberration.** It has already been pointed out (page 43) that when light is deviated by refraction, it is dispersed into its component colors, the red being deviated the least and violet the most of the colors to which the eye is sensitive. A prism thus produces a spectrum. The dispersion that takes place in a simple convex lens produces a series of colored images of a star as shown in Figure 43, the violet one being nearest the lens and the red one farthest removed. With an eyepiece focused on any one of these images, an observer sees a small bright point of the corresponding color surrounded by a halo that is composed of the out-of-focus light of the other colors. By thus enlarging the image of a star, the colored halo greatly impairs the resolving power of the lens, and it also makes it impossible to judge correctly the color of any object viewed. This defect of the simple lens, by which light of different colors fails to arrive at the same focus, is called **chromatic aberration.**

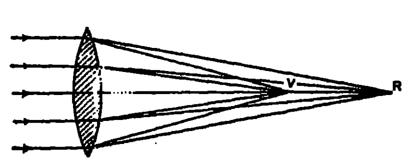

Fig. 43. *Chromatic Aberration.*

Soon after Galileo's application of the telescope to the study of the sky, it was found that chromatic aberration could be mitigated by making the focal length of the objective very great in comparison with its diameter, and some telescopes of the seventeenth century attained fantastic lengths, up to two hundred feet. These long telescopes must have been exceedingly unwieldy, and their use led to no results commensurate with their size.

**Achromatic Lenses.** In 1758 John Dollond, an English optician, patented a method of reducing the effects of chromatic aberration by combining two lenses made of different kinds of glass. The glasses ordinarily used for this purpose are called **crown** and **flint** glass, names derived from the manner in which the glasses were formerly made. Their principal constituents are silica (sand) and sodium sulphate or carbonate or potassium carbonate, to which is added lime in making crown glass and lead in making flint glass. Modern varieties of optical glass intended for special purposes contain also a great variety of other substances.

The refractive index of flint glass is greater than that of crown, and its

dispersive power—i.e., the difference of its refractive index for light of different colors—is also greater. By properly combining a convex crown lens with a concave flint one, a compound objective is formed which will bring to the same focus rays of light of any *two* colors that may be chosen. Such a lens is shown in Figure 44. The colors to which the eye is most sensitive are those near the red end of the spectrum, and so objectives

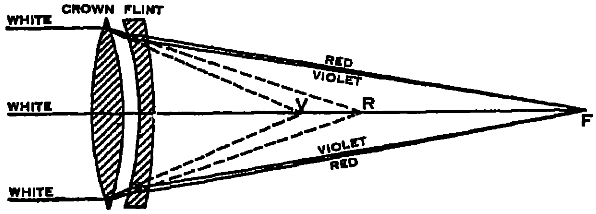


Fig. 44. *Achromatic Objective.*

intended for visual use are made to combine the red and green rays; but the ordinary photographic plate is more sensitive to the blue end of the spectrum, and so photographic objectives are made to combine the blue and violet. Such compound lenses are called **achromatic** (color-free), but the images they produce are not wholly free from color, since the light of all colors except two is focused at different points. However, the colored halo around the image of a star produced by such a lens is not nearly so large or troublesome as in the case of a single lens. With some of the special modern glasses it is possible to produce a compound lens that is very nearly achromatic over the whole range of the visible spectrum, but these glasses have not yet been produced in blocks large enough to make a lens of very great size.

The compound lens may have the further advantage of eliminating spherical aberration, a defect possessed by every simple lens that has a spherical surface, as well as by every spherical mirror. The caustics of a convex lens and a concave lens are complementary in form, and in achromatic telescope objectives they are made to oppose one another.

**Eyepieces.** A simple convex lens used as an eyepiece introduces color and distorts the image unless the object is at the center of the field. The eyepieces of modern telescopes are usually composed of two lenses, and are usually of either of two forms, the **negative**, or **Huyghenian**, and the **positive**, or **Ramsden**, illustrated in Figures 45 and 46, respectively. In the Huyghenian eyepiece, the rays from the objective are intercepted by the first of the two lenses before reaching a focus, and cross between the lenses; in the Ramsden form they reach their focus before encountering either lens. The Huyghenian eyepiece gives better definition at the

center of the field, but of course cannot be used with a reticle; the Ramsden can be used with a reticle and has a wider field.

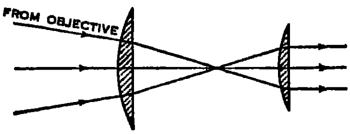


Fig. 45. *Negative Eyepiece.*

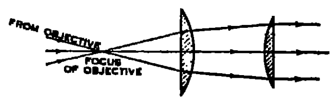


Fig. 46. *Positive Eyepiece.*

Galileo employed in the first telescopes a concave eyepiece placed between the objective and its focus. Such an eyepiece gives an erect image, but it cannot be used with a reticle and its field of view is very small if it is made of any considerable magnifying power. It is used nowadays in low-priced opera glasses, in which a high power is undesirable and inversion of the image would be objectionable.

**Relative Advantages of Refractors and Reflectors.** As compared to lenses, mirrors have the great advantage of perfect achromatism. Moreover, for an instrument of given size, the cost of a mirror is less than that of an achromatic lens, since in the former there is only one surface to bring to an accurate figure whereas in the latter there are four, and the glass of the mirror need not be of so fine a quality as that of the lens. On the other hand, the reflecting telescope is more sensitive to changes of temperature, its reflecting surfaces must be renewed at intervals, and it is likely to be more unwieldy. The fact that the weight of a lens can be supported only at its edge sets a practical limit to the size of refracting telescopes; no such limit exists in the case of reflectors, as all large mirrors are supported at the back as well as at the edge. In instruments of moderate size, the choice between a refractor and a reflector is thus largely a personal concern of the owner, but the largest telescopes are almost of necessity reflectors.

**Telescopes for Wide-Field Photography.** The field of view of either the simple achromatic lens or the parabolic mirror is very limited: the images of stars a short distance from the optic axis are not points but short lines or comma-shaped dots, their alignment is distorted, and they lie upon a curved focal surface to which a flat photographic plate cannot be fitted. The best lenses for photographing a wide field, whether of stars or of terrestrial objects, consist of three or more separate parts, the optical properties of which are connected by highly complicated mathematical relations. Such are the "anastigmats" used in motion-picture and press photography, the Cooke-Taylor triplet, and the four-lens objective of Ross. Combinations of non-parabolic mirrors for a similar effect have been made by Ritchey and Chrétien, and by Cogshall, Smiley, and others.

A great advance in telescope design was begun by Schmidt in Germany in 1931 and has been rapidly carried on in America. The Schmidt camera combines the principles of the reflector and refractor and consists essen-

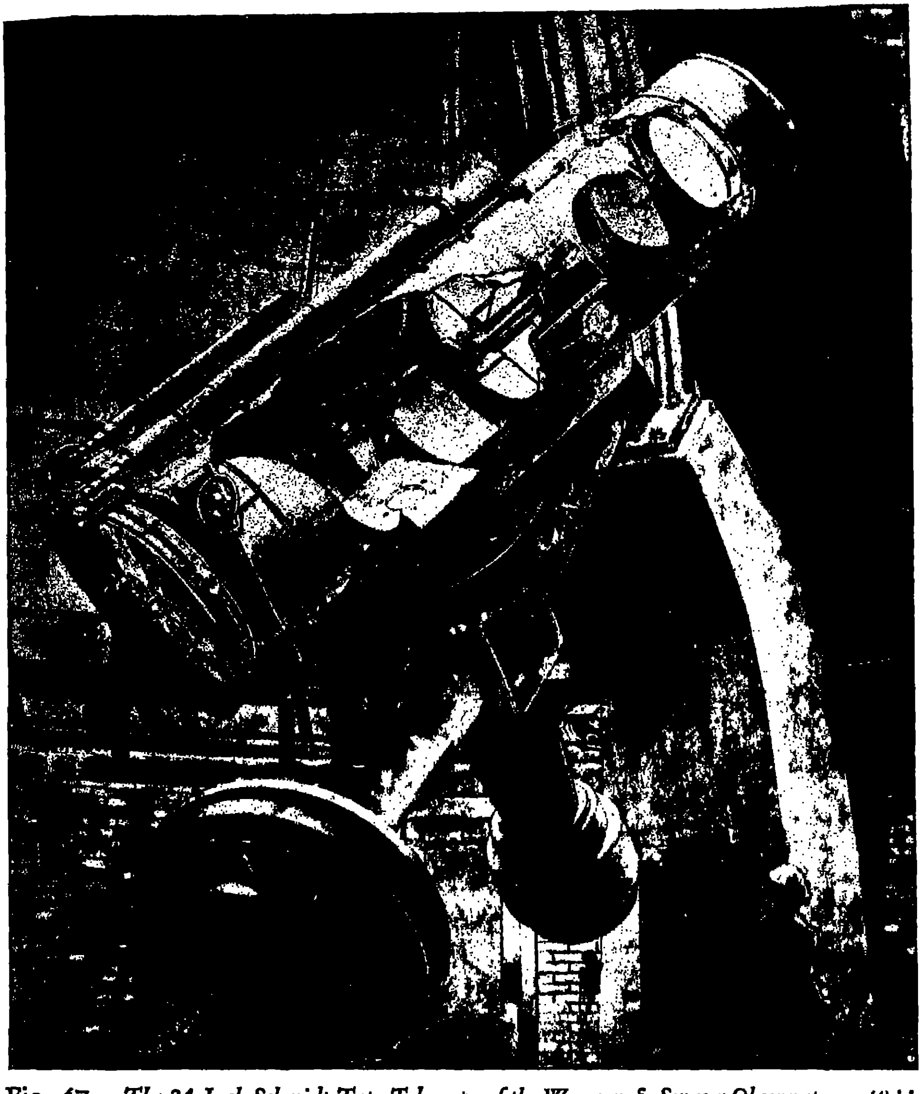

Fig. 47. *The 24-Inch Schmidt-Type Telescope of the Warner & Swasey Observatory, 1944.*

tially of a concave spherical mirror and a specially figured lens. The mirror is made with a large ratio of aperture to radius, affording high photographic speed, and its spherical aberration is corrected by the lens, which is so thin and so nearly flat as to introduce but very little chromatic aberration.

To the unaided eye, indeed, the lens appears like a sheet of excellent thin plate glass and so it is called the **correcting plate.** Its center coincides with the center of the sphere; and the principal focus of the combination, where the center of the photographic film or plate is placed, is in the middle of the tube, halfway between the plate and the mirror. Guiding is done with an auxiliary telescope if at all; but with a driving clock of recently improved form and in the short exposures which the Schmidt camera and modern rapid photographic materials permit, guiding may be dispensed with. Figure 47 is a phantom drawing by Russell Porter, telescope designer, which shows both the internal and the external features of a Schmidt telescope equipped with an objective prism (page 322).

**Observatory Domes.** A telescope should have an unobstructed view toward the sky when in use and should be protected from the weather when not in use. In the best form of observatory the wall surrounding the equatorial telescope is surmounted by a hemispherical dome which can be rotated on a level circular track and which is provided with an opening extending from the base to a little beyond the vertex; this opening is closed by a shutter. As the telescope turns on a polar axis and the dome on a vertical axis, their motions are not usually synchronized; the dome remains stationary while the telescope, in its diurnal motion, moves over the width of the shutter.

**The Largest Telescopes.** The largest *refractor* is that of the Yerkes Observatory of the University of Chicago, completed in 1895. The objective is 40 inches in diameter and 64 feet in focal length. The second largest is the 36-inch refractor (Figure 20) of the Lick Observatory of the University of California, situated on the top of Mount Hamilton. Both these telescopes were designed for visual observations, but are used also for photographic purposes by employing special plates. There are many refractors, both visual and photographic, of 30 inches aperture and smaller.

Much larger still are the great modern *reflectors,* of which the largest yet put in use is the Hooker telescope of the Mount Wilson Observatory near Pasadena, completed in 1918. It has a parabolic mirror of 100 inches diameter and 500 inches focal length, which is 13 inches thick and weighs 5 tons. It is sometimes used with its natural focal length (Newtonian form), and sometimes with either of two hyperbolic secondary mirrors giving it an equivalent focal length of 135 feet (Cassegrainian form) or 250 feet (coudé). The moving parts of the telescope weigh 100 tons, and this weight is supported principally by two steel drums, attached to the polar axis, which float in troughs of mercury. The instrument is of course

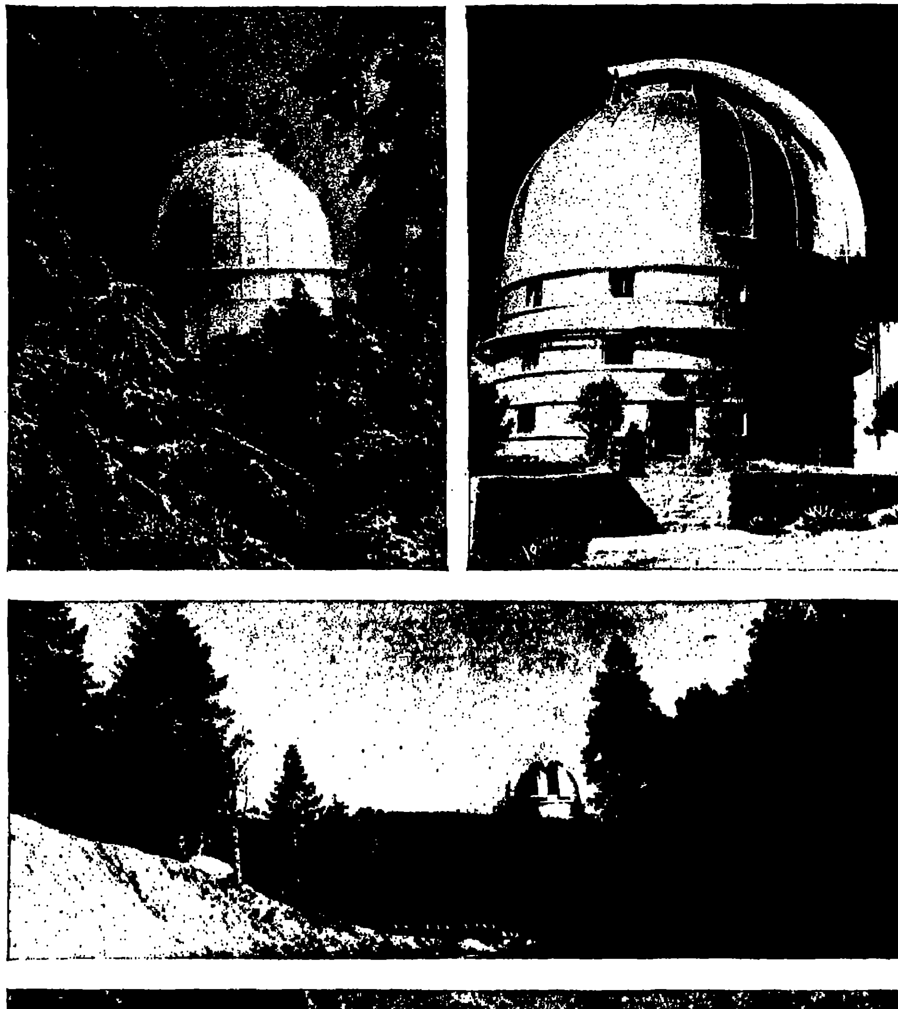

Fig. 48. *Some Great American Observatories: (Upper left) Dome of the 100-inch Hooker telescope of the Mount Wilson Observatory (photograph by Edison R. Hoge); (upper right) McDonald Observatory of the University of Texas (photograph by Paralto Studios); (center) Dome of the 200-inch telescope on Palomar Mountain (photograph by T. V. Watterson); (bottom) Lick Observatory (photograph by Martha E. Stahr).*

too massive to be moved by hand; hence fifty electric motors, controlled by switchboards near the eyepieces and at the assistant's desk, are provided for operating the telescope, observing platforms, and dome. The Mount Wilson Observatory, in addition to the 100-inch, possesses a fine 60-inch reflector.

Almost completed at the time of this writing (1945) but delayed by war is the vast Astrophysical Observatory of the California Institute of Technology, on Palomar Mountain, about 50 miles from San Diego. Its principal instrument is a 500-ton reflector with a parabolic mirror of pyrex 200 inches in diameter, 55 feet in focal length, and 14½ tons in weight. This observatory was originated by the late Dr. George E. Hale, as were the Yerkes Observatory (1895) and the Mount Wilson Observatory (1904).

Other large reflecting telescopes are the 82-inch at the McDonald Observatory, Mount Locke, Texas; the 74-inch at the Dunlap Observatory, Toronto, Canada; the 72-inch at the observatory of the Earl of Rosse, Parsonstown, Ireland (no longer used); and the 72-inch at the Dominion Astrophysical Observatory, Victoria, Canada. There are many with apertures up to 60 inches.

On Palomar Mountain there will be a Schmidt camera with a 72-inch spherical mirror and a 48-inch Schmidt plate. Schmidt cameras with apertures of 24 inches are located at the Oak Ridge station of the Harvard College Observatory at Harvard, Massachusetts; the Warner & Swasey Observatory at Cleveland, Ohio; and the Mexican National Observatory at Tonanzintla; and there are many smaller Schmidts.

## EXERCISES

1. The objective of a certain telescope has an aperture of 12 inches and a focal length of fifteen feet. An eyepiece frequently used with it has a focal length of one inch. What is the magnifying power?

*Ans.* 180

2. What is the least apparent distance of the components of a double star which can be seen clearly separated in the above telescope?

*Ans.* $0''.375$

3. How does the brightness of a star as seen in the above telescope compare with its brightness as seen by the naked eye?

*Ans.* The telescope increases the brightness 1296 times

4. One telescope has an aperture of 20 inches, and the focal lengths of its objective and eyepiece are respectively 25 feet and one inch; the corresponding numbers

for a second telescope are 60 inches, 25 feet, and ½ inch. How do the two telescopes compare in (a) magnifying power, (b) resolving power, (c) the brightness of the image of a star, and (d) the brightness of the image of a planet (observed visually)?

*Ans.* The properties of the second as compared to the first are: (a) 2; (b) 3; (c) 9; (d) 9/4

5. If a fly alights on the objective of a telescope which is directed to the Moon, will an observer at the eyepiece seem to see the fly upon the Moon?

# CHAPTER 3

## THE EARTH AND THE SKY

**General Description of the Earth.** The Earth is one of the smallest of the nine principal planets that revolve around the Sun. In form it is almost a perfect sphere, 7920 miles in mean diameter (exclusive of the atmosphere); but the equatorial diameter is about 27 miles longer than the polar, giving the planet more nearly the form of an oblate spheroid. The mass of the Earth is $6 \cdot 10^{21}$ (6 followed by twenty-one ciphers) metric tons. Its mean density, as compared with water, is 5.52. This great ball rotates on its axis in twenty-four sidereal hours, and moves in its vast orbit around the Sun at an average speed of eighteen miles, or thirty kilometers, a second.

To the casual observer the surface of the Earth, and especially that of the sea, appears flat, and until its circumnavigation by the fleet of Magellan in 1522 it was generally believed really to be flat; yet throughout history, beginning with ancient Greece, at least a few well-informed thinkers have believed the Earth to be a globe. Two facts which prove the Earth's approximate sphericity, and which may be verified by any careful observer, are:

Fig. 49. *Proof of the Earth's Convexity.*

1. The shadow of the Earth, seen on the Moon during a lunar eclipse, is always sensibly circular, whatever may be the face of the Earth that is turned toward the Sun.
2. When a ship moves away from a stationary observer, the first part to disappear is the hull and the last is the highest point of the mast or funnel—showing that the surface of the water is convex (Figure 49).

The Earth is a huge magnet, having in each hemisphere a magnetic pole about 20° distant from the geographic pole.

The Earth possesses a **lithosphere**, which is the main body of the

planet; a **hydrosphere**, consisting of the water on its surface; and an **atmosphere**, the gaseous envelope that surrounds them both.

The outer portion of the lithosphere is a solid crust, on the surface of which we live. It consists mainly of rocks, the principal constituents of which are oxygen and silicon (always in chemical combination), with smaller quantities of most of the other chemical elements. The average density of the crust is considerably less than that of the Earth as a whole, from which fact we may infer that the interior of the lithosphere is compressed by the great weight of the crust. This compression, however, is regarded by geophysicists as insufficient to explain all the difference in density, and by many it is believed that the central core of the Earth is composed of solid nickel-iron having at the center a density of about 10. This view is supported by several lines of evidence, including the magnetic properties of the Earth and the analogy of iron meteorites which are believed by some to be fragments of a cosmic body. Such phenomena as volcanoes and hot springs show that, at no great distance below the surface, the temperature is very high, and it is believed that the main body of the lithosphere is at a temperature above the melting point of rocks at ordinary pressures. It might be expected that the interior of the Earth would in these circumstances be liquid, and until late in the nineteenth century this was thought to be true; but the study of the speed with which earthquake shocks are transmitted through the body of the Earth, and other evidence as well, show that the lithosphere is both elastic and rigid like a solid, the rigidity being between that of glass and that of steel.

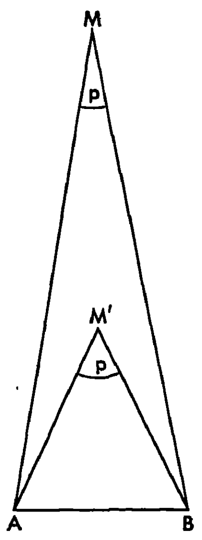

Fig. 50. *The Meaning of Parallax.*

The hydrosphere consists of all the oceans, lakes, and rivers, and covers about four-fifths of the surface of the lithosphere. As the molecules of a liquid are free to yield to whatever forces are applied to them, the surface of the ocean is (except for the slight effects produced by winds and tides) a **level** surface, everywhere normal to the direction of gravity. It is the form of this surface that we mean when we speak of the shape of the Earth.

**Parallax; Compared to Celestial Distances, the Earth Is Small.** When an object $M$ (Figure 50) is viewed from two different points $A$ and $B$, it appears in different directions. This difference of direction is so important in the study of the

universe that astronomers have a name for it: **parallax.** The parallax of $M$ as seen from $A$ and $B$ is the angle $AMB$. It is also the apparent distance between $A$ and $B$ as seen from $M$. Any point $M'$ which is nearer than $M$ to the line $AB$ has a larger parallax. Expressing the same thing from the point of view of $M$ and $M'$, we may say, as on page 11, the nearer an object (such as the line $AB$), the larger it looks. If the parallax $AMB$ is $1^\circ$, the distance $AM$ or $BM$ is about 57.3 times $AB$; if the parallax is $1'$, the distance is nearly 3438 times $AB$; and if the parallax is $1''$, the distance is very nearly 206265 times $AB$.

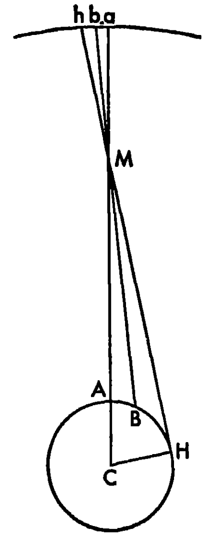

Fig. 51. *Geocentric Parallax.*

Let the circle centered at $C$ in Figure 51 represent the Earth, $M$ a not too distant heavenly body, and the arc at the top a part of the celestial sphere. Let one observer be stationed at $A$ on the line $CM$; then (assuming the Earth to be spherical) $M$ will appear at his zenith, $a$. Let another observer be stationed at $B$. For him, $M$ will appear at $b$; and the parallax, $AMB$ or $aMb$, is in this case called the geocentric parallax of $M$ as seen from $B$, since it is the difference of direction of $M$ from $B$ and from the center of the Earth. For an observer at $H$, where $M$ appears on the horizon, the geocentric parallax of $M$ is the largest possible and is called the **horizontal geocentric parallax.** It is the apparent semi-diameter of the Earth as seen from $M$.

The Earth's rotation continually carries any observer into different positions with respect to the line $CM$, permitting determinations of the geocentric parallaxes of such bodies as are near enough. It is found that all geocentric parallaxes are small, showing that, compared to the Earth's diameter, the distances of all heavenly bodies are great. The mean horizontal geocentric parallax of the Moon is greater than that of any other known heavenly body, being just over $57'$; therefore, the Moon's average distance from the Earth is $3438 \div 57 = 60+$ times the radius of the Earth, or about 239,000 miles. The Sun's mean horizontal geocentric parallax is very nearly $8''.8$ and its distance is 23,440 Earth-radii, or about 93 million miles. The geocentric parallaxes of the stars are so small as to be utterly imperceptible, and compared to stellar distances the Earth is vanishingly small.

**The Earth's Atmosphere.** The atmosphere is a mixture (not a chemical combination) of a number of different gases held to the surface

of the Earth by gravitational attraction. The lower portion of the air is about three-fourths nitrogen and one-fourth oxygen, with small quantities of water vapor, carbon dioxide, and argon. Suspended in the lower air is a considerable amount of solid material in the form of dust and smoke, and usually also liquid water in the form of the droplets that make up the haze and lower clouds; the higher clouds are often composed of ice crystals—particles of solid water. The upper air is believed to be made up of the light gases—mostly hydrogen and a small amount of helium.

The distinction between solids, liquids, and gases has to do with the arrangement of the molecules. In a solid, the molecules resist any force that tends to change their relative positions or distances, so that the body possesses **incompressibility** and **rigidity**. In a liquid, the molecules pass freely over one another, but resist any tendency to change the distance between adjacent molecules. A liquid thus possesses incompressibility but not rigidity. The molecules of a gas constantly move about among one another, each one darting in a straight line until it encounters another molecule or the wall of the containing vessel. The speed of a molecule of gas is greater the greater the temperature, and the length of its free path is greater the less the density. Increasing the pressure on a given mass of gas both decreases its volume and increases its density by diminishing the free paths of the molecules. If the pressure is relieved, the molecules at once spring apart and the volume increases, decreasing the density. Most substances can be made to pass successively through the three states by varying the temperature or the pressure. Thus, in air at ordinary atmospheric pressure, at a temperature of 0° Centigrade, solid water (ice) is transformed into liquid, and at 100° Centigrade liquid water boils, i.e., is rapidly transformed into an invisible gas called water vapor.

The weight of each layer of air produces a pressure upon the portion of the atmosphere that lies below it, and the total pressure and density are therefore greatest at the level of the sea and diminish with increasing height. The total weight of the atmosphere is some $10^{15}$ tons and the pressure at sea level is about fifteen pounds to every square inch of surface; but we are not ordinarily conscious of this pressure because it is the same in all directions. When a part of the air in a vessel is artificially removed, the atmospheric pressure forces other air to take its place. Philosophers before the time of Galileo were unaware of the pressure produced by the weight of the air, and attributed its effects to the false but famous principle, "Nature abhors a vacuum."

**Illumination and Color of the Sky; Twilight.** When light passes through a medium which contains particles that are small compared with the wave length of the light, a part of the light is scattered by the particles in all directions, and the ratio of the intensities of the scattered and incident lights varies inversely as the fourth power of the wave length. The mole-

cules of the air, and also the particles of dust suspended in it, have this effect upon sunlight and starlight; and so the light that reaches the eye directly through the air has been deprived of some of its shorter waves (composing the violet end of the spectrum), and the heavenly bodies appear of a redder or yellower hue than they would if the Earth had no atmosphere. Just before sunset, the sunlight passes through a greater depth of air than near the middle of the day, and its redness is increased both by this cause and by the greater dustiness of the air at that time of the day. The brilliant colors of sunset clouds are due to their illumination by light that has passed through different depths of air. On the other hand, when we turn to directions nearly at right angles to that of the Sun, we receive light that has been scattered by particles in the air, and which is therefore blue or, in the pure air of high altitudes, a deep violet. The illumination and color of the sky are thus due to the scattering of light by the small particles of the air. If the Earth had no atmosphere, the sky would be black and the stars could be seen at all times, day or night.

**Twilight** and **dawn** are the names applied to the illumination given to the sky by the Sun when it is below the horizon. Just after sunset, the Sun is still shining on the air above our heads, and the entire sky is still bright; but this brightness rapidly diminishes, especially near the eastern horizon, where, if the sky is very clear, the dark shadow of the Earth (known as the **twilight arc**) may be seen mounting the sky as the invisible Sun descends. Twilight ends when the depression of the Sun below the horizon, as measured on a vertical circle, is 18°. For an observer at the Earth's equator, where the diurnal motion takes place vertically, the duration of twilight is only about $1^h\ 12^m$; but it varies with the observer's latitude and the Sun's declination, both of which modify the angle made by the Sun's diurnal path with the horizon, and in summer, in latitudes greater than $48^\circ\!.5$, twilight lasts all night.

**The Depth of the Atmosphere.** Since the pressure produced by the weight of the air diminishes with increasing height, the density also diminishes in the same way, and it is difficult to locate the boundary of the atmosphere if indeed any definite boundary exists. It is possible, however, to determine approximately the height to which air of a certain density extends. The tops of the highest mountains are about five miles above sea level, and here the air is too rare to support life indefinitely or to retain a moderate temperature by the greenhouse effect (page 172). Man has ascended about thirteen miles (Stevens and Anderson, in the closed cabin

of the helium-filled balloon *Explorer II*, 1935). Sounding balloons, carrying recording meteorological instruments, have risen much higher and their records show that, at heights greater than about seven miles, the temperature is constant at about $-55°$ C. This region, called the **stratosphere**, is largely free from ordinary clouds and dust. The fact that twilight persists until the Sun is 18° below the horizon shows that the air is dense enough to scatter light perceptibly at heights above forty miles.

This result is obtained as follows: In Figure 52 let the two concentric arcs represent the surface of the Earth and that of the highest reflecting layer of atmosphere, $C$ the center of the Earth, $O$ the observer, $OH$ the plane of the observer's horizon; and $AS$ the direction of the Sun at the end of twilight. Twilight ends when the Sun ceases to shine at $A$, and observation shows that this occurs when the angle $HAS$ is 18°. This angle is equal to $OCB$ since their sides are respectively perpendicular, and $OCB$ is bisected by the line $CA$, whose length exceeds the radius $R$ of the Earth by the height $h$ of the atmosphere. In the right triangle $OCA$ we have

$$\frac{(R + h)}{R} = \sec 9°$$

or

$$h = R(\sec 9° - 1).$$

The secant of nine degrees is 1.0125 and the radius of the Earth is 3960 miles, and so $h$ by the above formula is $3960 \times 0.0125 =$ 49 miles. The rays $AO$ and $AS$, however, are slightly curved downward by refraction (see below), and the actual depth of the reflecting atmosphere is less than that given by the formula.

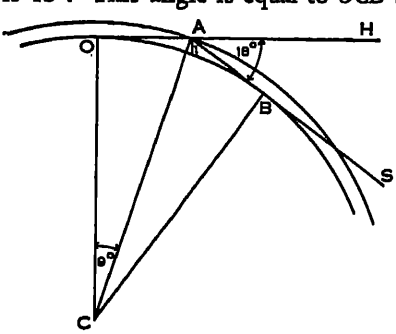

Fig. 52. *Depth of the Atmosphere.*

The light of "shooting stars" or meteors is due to the heat generated by their swift passage through the air (page 292), and these bodies are often seen at heights exceeding one hundred miles, showing that the air extends to that height. Finally, the arcs and streamers of the polar aurora (page 198), which is believed to be due to the impact of free electrons from outer space upon the particles of the air, have been observed by Störmer in Norway at heights as great as six hundred miles.

**Atmospheric Refraction and Dispersion.** When a ray of light from any heavenly body encounters the atmosphere it obeys the law of refraction and is bent toward the normal to the surface. The index of refraction of air increases with the density, and so the bending increases as the light passes downward, and the ray follows a curved path through the air, as shown in Figure 53, where a ray from the star $S$ reaches the observer at $O$.

We "see" an object in the direction from which its light enters the eye, and so the star appears to be at $S'$, in the direction of a tangent drawn to the curve at $O$. The effect of atmospheric refraction is thus to make the heavenly bodies appear slightly higher in the sky than they really are, and a "refraction correction" must accordingly be applied to all measured altitudes.

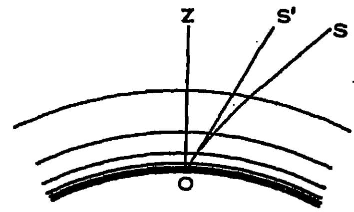


Fig. 53. *Atmospheric Refraction.*

If the body is observed at the zenith, the incident ray coincides with the normal and the refraction is zero. It is greatest at the horizon, and varies nearly as the tangent of the zenith distance, an approximate formula for the refraction correction being

$$r = 60''.7 \tan \zeta$$

where $\zeta$ is the zenith distance. This formula obviously breaks down at the horizon, where it would give an infinite displacement of the star, and in fact is of little value at altitudes below 25°. The exact calculation of atmospheric refraction must take into account the barometric pressure and temperature of the air, and is very complicated.

At the horizon, under average conditions, the refraction is about 35′, which is greater than the apparent diameter of the Sun or Moon. The result is that we see these bodies while they are still entirely below the true horizon; thus refraction has the effect of lengthening the day and shortening the night. Near the horizon, the refraction changes very rapidly, and at an altitude of half a degree it is about 6′ less than at the horizon. For this reason, the upper limb of the rising or setting Sun is elevated 6′ less than the lower, and the disk appears noticeably flattened on the under side.

Violet light is refracted more than red by air, just as by glass, but the dispersion is much less. It is noticeable, however, in the case of a star near the horizon, especially if viewed through a telescope, when the image appears as a short spectrum. The effect is not conspicuous in ordinary telescopes except at altitudes of 5° or less, but in the modern giant reflectors it affects the quality of star images when the star is as high as 30°.

**The Transmission of Radio Waves.** Radio waves, which differ from light waves in respect to length only, are regularly transmitted to great distances over the curved surface of the Earth. To explain their departure from rectilinear propagation, it is commonly supposed that such waves, leaving the sending station in a direction which slants upward, are turned downward by the **ionosphere**, a portion of the atmosphere that is ionized by the ultra-violet radiation of the Sun. There are two

principal layers of the ionosphere, the Kennelly-Heaviside layer, located about 60 to 100 miles above the ground, and the Appleton layer, at a height of 120 to 300 miles. The former deflects radio waves of frequencies up to 3000 kilocycles, and the latter the shorter waves of higher frequency. Much of the fluctuating quality of radio reception is attributed to variations of ionization between day and night, at different seasons, and at different times in the sunspot cycle.

**Experimental Evidence of the Earth's Rotation.** That the Earth rotates on an axis was taught as long ago as 395 B.C. by Herakleides of Pontus, but the doctrine was not accepted by many of the early philosophers, and until after the work of Copernicus (1543) it was generally believed that the Earth was stationary and that the diurnal motion of the celestial sphere was a real motion in which all the heavenly bodies took part. With the expansion of ideas regarding the vast distances of these bodies, especially the stars, the simplicity of the view of Herakleides as opposed to the complexity of its alternative became evident, and throughout the past three centuries the rotation of the Earth has not, by intelligent persons, been seriously questioned.

In 1851 Foucault performed a famous experiment which for the first time demonstrated the Earth's rotation without the use of any point of reference outside the Earth itself. He suspended in the dome of the Pantheon at Paris a pendulum consisting of a heavy iron ball attached to a slender wire more than two hundred feet long, the upper end of which was pivoted on a small round point under the top of the dome. The pendulum was thus free to swing in any azimuth, and a rotation of the support could not readily be transmitted to it. Such a pendulum, if left undisturbed, continues to swing for several hours very nearly in the same plane. At the bottom of the ball was fixed a pin which, as the pendulum swung, just touched the surface of a circular ridge of sand heaped upon a table beneath, thus showing by a mark in the sand the direction of the plane of vibration. If the Earth did not rotate, the pin would continue to cut the sand in the same place; but Foucault found, and showed to a great crowd of spectators, that the sand was cut in a fresh place at each swing, the floor of the building visibly turning under the pendulum. The experiment has been repeated many times and in many places, including some in the southern hemisphere, where the apparent rotation is, as would be expected, in the direction opposite that observed in the northern.

If the experiment were performed at the Earth's pole, the rate of apparent rotation of the plane of vibration would be the same as the real rate of the Earth—one complete turn in twenty-four hours. At the equator, there would be no apparent rotation

at all, because the rotation of the plane of the pendulum must take place around a vertical axis, and a building located at the equator does not rotate in this way, but is carried bodily in a circle around the center of the Earth. The motion of a building located between the pole and the equator may be regarded as a combination of rotation and revolution, the former predominating the more the pole is approached; and it may be shown by a principle of dynamics, known as the principle of composition of angular velocities, that the time of an apparent rotation at a place whose latitude is $\phi$ is $24^h \div \sin \phi$. At the latitude of Paris the time of a complete rotation is about 32 hours.

There are other ways in which the rotation of the Earth has been detected independently of observations of the stars; perhaps the most important of these is by the action of the gyroscopic compass which is used on many large ships and which depends upon the directive effect of the Earth's spin upon the axis of a rapidly rotating massive wheel.

**Geographic Coördinates.** The position of any place on the surface of the Earth is completely described by stating its coördinates in the geographic system—that is, its longitude and latitude. For example, if a ship in distress sends out a radio *SOS* call accompanied by a statement of its longitude and latitude, its distant rescuers know at once what course to pursue in order to reach the spot. Since the Earth is not a sphere, it is best to attempt to define these coördinates not as arcs of circles as we defined the coördinates in the various celestial systems in Chapter 1, but rather as angles.

The points where the Earth's axis pierces the surface of the spheroid are called the north and south terrestrial **poles.** The surface is cut along **meridians** by planes that intersect one another along the axis, and on **parallels of latitude** by planes perpendicular to the axis. The parallel of latitude that lies midway between the poles is the terrestrial **equator.**

The **longitude** of a point on the Earth's surface is the angle between the plane of its meridian and the plane of the meridian of some place chosen as a standard—called the **prime meridian.** Civilized countries have agreed to use as a standard the meridian of the Royal Observatory at Greenwich, England, which was founded in 1675 primarily for the purpose of providing data for the determination of longitudes. Longitude is counted either in degrees or in hours, and either east or west of Greenwich up to 180°.

It should be noted that longitude is defined as a *dihedral angle*, or wedge angle, and that it is measured by the arc of either the terrestrial or the celestial equator included between the two planes.

The **astronomic latitude** of a place is the angle between the plane of the equator and the direction of gravity at the place. The **geocentric latitude** is the angle between the plane of the equator and a straight line passing from the place to the center of the Earth. The difference between them is caused by the oblateness of the Earth, and amounts at its greatest to about 11′. The two kinds of latitude are illustrated in Figure 54, in which the departure of the Earth's form from a sphere is of course greatly exaggerated, and in which the astronomic latitude is indicated by $\phi$ and the geocentric by $\phi'$.

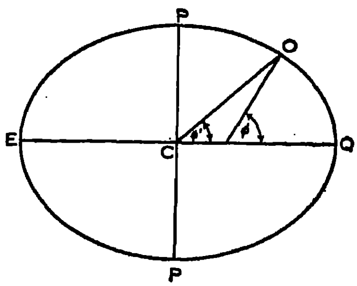


Fig. 54. *Astronomic and Geocentric Latitude.*

Reference is sometimes made to a **geographic latitude,** defined as the angle between the plane of the equator and a normal to the standard spheroid. It differs from astronomic latitude only by the effects of local deviations of the direction of gravity caused by the attraction of mountains, etc.—never by more than 30″ or 40″.

**The Astronomic Latitude of the Observer Equals the Altitude of the Celestial Pole.** In Figure 55, let $O$ be the place of an observer on the Earth's surface, $C$ the center of the Earth, and $CP$ the Earth's axis. The plane of the paper will then be the plane of the observer's meridian, and this meridian will itself be represented by the nearly circular ellipse drawn through $O$ with center at $C$. Draw $CE$ perpendicular to $CP$; by definition, $CE$ is then the intersection of the plane of the equator with the plane of the meridian. Let $ZO$ be the direction of gravity at $O$ and let it intersect $CE$ at $Q$. By definition, the angle $ZQE$, or $\phi$, is the astronomic latitude of $O$, and $OZ$ is directed to the observer's zenith. Draw $OH$ perpendicular to $OZ$; by definition it is directed to the observer's horizon. Draw $OP'$ parallel to $CP$; it meets the infinitely distant surface of the celestial sphere at the same point as does $CP$, and is therefore directed to the celestial pole.[1]

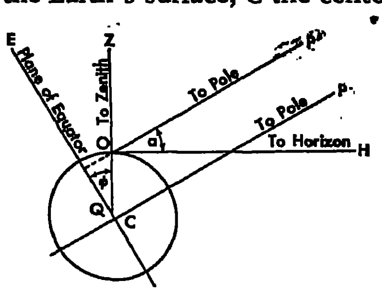


Fig. 55. *Equality of Latitude and Pole Height.*

[1] Because the pole is the "vanishing point" of the two lines. Suppose a line drawn from $O$ to any point $X$ of $CP$ within a finite distance of $O$; it would make a certain angle $\theta$ with $OP'$. Now let $X$ recede along $CP$. As it does so, $\theta$ grows smaller, approaching zero as a limit. When $X$ reaches the surface of the celestial sphere, $\theta = 0$ and the line is parallel to $CP$, coinciding with $OP'$.

The angle $P'OH$, or $a$, is subtended on the celestial sphere by the arc of a vertical circle included between the pole and the horizon, and is therefore the altitude of the pole. But the sides of the angles $a$ and $\phi$ are respectively perpendicular and therefore the angles are equal, and so we have the important relation: *The observer's astronomic latitude is equal to the altitude of the celestial pole.* It is equal also to the *declination of the zenith*, which is the arc of the meridian subtending the angle $EQZ$. The celestial equator is tilted to the horizon at an angle equal to the complement of the observer's latitude.

**Right, Parallel, and Oblique Spheres.** At the Earth's equator, the latitude is zero and both celestial poles lie upon the horizon. The celestial equator coincides with the prime vertical, all diurnal circles intersect the horizon at right angles, and each star is above the horizon just twelve sidereal hours (refraction being neglected) and below for the same length of time. This aspect of the heavens is called the **right sphere.**

The term *right ascension of a star* originated from its being defined as the ascension (altitude) of the vernal equinox at the moment of the star's rising in a right sphere. The reader will perceive that this definition is equivalent to those given earlier in this book.

At the Earth's pole the latitude is 90° and the celestial pole appears at the zenith. The celestial equator coincides with the true horizon and each star sails around the sky on an almucantar, parallel to the horizon. This appearance of the sky is called the **parallel sphere.**

In intermediate latitudes, the diurnal motion is oblique to the horizon as it was described in Chapter 1, the celestial pole being more elevated the greater the latitude. This state of affairs is called the **oblique sphere.**

As the observer travels toward the pole, the circles of perpetual occultation and apparition (page 20) enlarge, approach each other, and become more nearly level until at the pole itself they coincide with the equator and horizon. At the pole, the stars of half the sky never set, and those of the other half never rise.

As the observer approaches the equator, the circles of perpetual occultation and apparition contract and move apart and finally become mere points coinciding with the poles. If the observer crosses the equator, they exchange places and the direction of the diurnal motion becomes reversed with respect to the horizon, being clockwise in the northern hemisphere and counterclockwise in the southern.

**The *ZPS* Triangle.** As a plane triangle is made up of straight lines, a **spherical triangle** is made up of arcs of great circles on the surface of a sphere. The sides as well as the angles of a spherical triangle are measured in degrees, and by the formulae developed in spherical trigonometry it is possible to compute any of the six parts (three sides and three angles) when any three are given.

Many important problems in spherical astronomy require a transformation of coördinates in the horizon system to coördinates in the equator

system, or vice versa, and involve the solution of the triangle whose vertices are at the zenith, the pole, and some star—commonly called the *ZPS* triangle. It is easily seen (Figure 56) that the sides and angles of the *ZPS* triangle are as follows:

Side $ZP = 90° - \phi$, the complement of the observer's latitude
Side $ZS = 90° - h$, the complement of the star's altitude
Side $PS = 90° - \delta$, the complement of the star's declination
Angle $Z$ = the star's azimuth reckoned from the north point
Angle $P$ = the star's hour angle, negative if east of the meridian

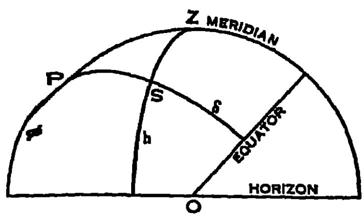


Fig. 56. *The* ZPS *Triangle.*

The angle $S$ (not previously defined) is called the parallactic angle.

**Determination of the Latitude of an Observatory.** The *geocentric* latitude of a place cannot be determined by direct observation, but is computed from the astronomic latitude by means of a knowledge of the figure of the Earth. There are many ways, of varying degrees of accuracy and convenience, by which the astronomic latitude may be determined, each depending upon a measurement of the altitude of the pole or its equivalent, the declination of the zenith. Of these, we shall here describe only one, the **method of circumpolar stars.**

This method consists of determining the altitude of the pole by observations with the meridian circle. In Chapter 1 we described this instrument and also the method by which the "polar reading" is found from observations of a circumpolar star at its transits above and below the pole (page 34). In addition to this, for finding the latitude the exact reading of the circle must be obtained when the telescope is pointed to the horizon. The altitude of the pole (that is, the latitude) is then found at once by subtracting the two readings.

It is not possible to locate the horizon accurately by a direct observation, and so recourse is had to the **nadir reading,** which is exactly 90° from the horizon reading, and which is obtained as follows: A dish of mercury is placed below the axis of the meridian circle and the telescope is directed downward toward it. The light reflected from the lines of the reticle (which must be illuminated from the side next the eye) and passing downward is made parallel by the objective, reflected by the bright surface of the mercury, and focused again by the objective in the plane of the reticle. The observer adjusts the instrument so that the reticle coincides with its own reflected image, in which case the line of sight is exactly vertical, i.e., the telescope is directed exactly to the nadir; for, by the law of reflection, the incident and reflected

rays then both coincide with the normal to the mercury surface, and this surface is precisely level because the mercury is a free liquid.

The polar and nadir readings of every meridian circle in regular use are determined many times, so that the latitude of the instrument is well established. Latitudes determined from a good series of observations of this kind are reliable to about $0''.01$, that is, to about one foot as measured on the Earth's surface.

As compared with other accurate methods, the method of circumpolars has the advantage that it is independent of previous observations, as it is not even necessary to identify the star used for determining the polar reading; but it cannot be used at observatories near the equator and has the further disadvantage of requiring two observations twelve hours apart. For rapid and accurate determinations of latitude, such as are needed, for example, in boundary surveys, the method introduced in 1845 by the American engineer Talcott is extensively used. It requires the use of a special instrument known as a **zenith telescope,** and an accurate independent knowledge of the declinations of the stars observed.

**Variation of Latitude.** It was shown by the Swiss mathematician Euler in the latter part of the eighteenth century that if the Earth were perfectly rigid and were rotating about an axis not coinciding with its axis of symmetry (shortest diameter), then its axis of rotation must itself rotate about the axis of symmetry in a period of about 305 days. This does not mean that the Earth's axis of rotation should change its position with respect to the stars (though it does this, too—see Precession and Nutation, pages 115 *et seq.*), but that the Earth should so move that its poles must wander slightly upon its surface. Since the latitude of any place is reckoned from the plane of the equator and the equator is fixed with respect to the poles, any such wandering of the poles must result in a variation of latitude.

A variation of latitude was actually detected about a century after Euler's work (1888) by Küstner and by Chandler. Chandler found from a laborious investigation of a great quantity of observations that the motion of the pole could be regarded as the resultant of two motions, one of these being in an ellipse with a period of a year and the other in a circle with a period of 428 days. The actual motion is very complicated but very slight, the greatest departure of the pole from its mean position being less than forty feet and resulting in a total variation of latitude of only about $0''.6$.

The annual component of this motion of the Earth is probably due to meteorological causes such as the deposition of snow in one hemisphere in one half of the year and in the other during the other half. The other component may be due to the way in which the Earth originally started rotating, or possibly to some event in its past history that modified its rotation; if so, the difference between the 428-day period and Euler's theoretical 305 days is to be explained by the Earth's not being perfectly rigid.

**Time and Longitude.** As the subject of longitude is intimately connected with that of time, it is appropriate that we review and extend what was said in Chapter 1 about the latter before discussing methods of deter-

mining the former. By the word *time* is here meant the number of hours and subdivisions that indicate the time of day; no discussion of the recondite subject of Time in the abstract or of its relations to Space is intended. Astronomically speaking, the time of day is the hour angle of some reference point in the sky. Astronomers and navigators are accustomed to count the hours of the day from 0 to 24 instead of dividing the day into twelve A.M. hours and twelve P.M. hours.

**Sidereal time** is the hour angle of the vernal equinox, or the right ascension of the meridian. The method by which it is determined by observations of stars with the transit instrument has already been described in Chapter 1 (page 31). This is the most accurate method known for determining time.

**True or apparent solar time** is the hour angle of the Sun.[2] It is the time shown by a correctly adjusted sundial and differs from sidereal time by the right ascension of the Sun.

The apparent motion of the Sun in the ecliptic, caused by the Earth's orbital revolution, is not uniform, and even if it were, the Sun's right ascension would not increase uniformly because the right ascension is counted along the equator, which makes an angle with the ecliptic of 23½°. The **mean sun** is a fictitious sun that moves in the celestial equator with the mean speed with which the true Sun moves in the ecliptic. **Mean solar time** is the hour angle of the mean sun.

**The equation of time** is the difference between mean and apparent solar time. It amounts, at its maximum, to about sixteen minutes. The right ascension of the Sun and the equation of time depend upon the position of the Earth in its orbit and can be computed for any given instant from a knowledge of the position, form, and size of that orbit. They are given for every day of the year in the almanacs and ephemerides published by the principal governments, such as the *American Ephemeris and Nautical Almanac.*

In Figure 57, the little Earth and the celestial sphere are centered at C; an observer is at $O$; and $G$ is the Royal Observatory at Greenwich. $O'$ and $G'$ are the points where the respective meridians of $O$ and $G$ cut the terres-

[2] These definitions refer to astronomical time, which is reckoned from noon. The civil day begins at midnight, and to obtain civil time it is necessary to add 12 hours to astronomical time. In 1925, the principal national almanacs, which up to that year had used the astronomical day, agreed to discontinue its use and to adopt the civil day; but as it is inconvenient for astronomers to change the date in the middle of their night's work, the records of many great observatories are still kept in astronomical standard time, i.e., time counted from standard noon.

trial equator. The longitude of $O$ is the angle $O'CG'$ which is measured by either the arc $O'G'$ of the terrestrial equator or the arc $O''G''$ of the celestial equator. Let $S$ be any point in the sky, and let $T$ be the point where the hour circle of $S$ cuts the celestial equator. Then $G''T$ and $O''T$ are the

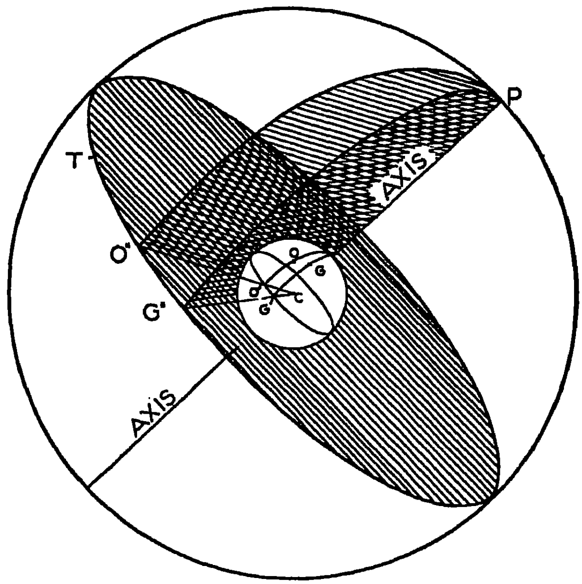


Fig. 57. *Longitude and Time.*

Greenwich hour angle and the local hour angle, respectively, of $S$, and the longitude of $O$ is the difference of these hour angles. If we let $S$ or $T$ be one of the reference points used for indicating time (that is, the true or mean Sun or the vernal equinox), then we see that the longitude of $O$ is the Greenwich time minus the local time, whether it be apparent solar, mean solar, or sidereal. Observers in different longitudes therefore have different local times at the same instant, and the difference in time is the same as the difference in longitude.

Prior to 1883, the affairs of people were adjusted to their local solar time, as was natural before the advent of rapid communication. With such a system a traveler in the United States, if he were to keep the correct time, had to change his watch at the rate of about a minute for every fourteen miles traveled eastward or westward. Each railroad had its own arbitrary time system in which trains were scheduled throughout the length of a line or division, and in most of the towns along the way two kinds of time were recognized—"Sun time" and "railroad time."

In November, 1883, the railroads of the United States adopted a system of **standard time** by which four standard meridians were established in longitudes exactly five, six, seven, and eight hours west of Greenwich respectively. The country was divided into four belts having these meridians approximately central, and the railroads of each belt agreed to use the time of the corresponding meridian. The four times were named Eastern, Central, Mountain, and Pacific Standard Time, respectively. The boundaries of the belts are irregular, the change from one standard to another being ordinarily made at an important city or at the end of a railway division. The system was legalized by Congress in 1884, and standard time based similarly on Greenwich mean time has since been adopted in all the important countries of the globe. In efforts to "save daylight" or from considerations of economy or convenience, many changes (usually of one hour each) have been made in the official time of certain cities, states, or entire nations, but in general the time adopted has differed from Greenwich mean time by an integral number of hours.

**The Date Line.** Traveling westward from Greenwich, one passes through regions where the time is increasingly *less* than Greenwich time, and traveling eastward he finds the time increasingly *greater* than Greenwich. Just east of the 180th meridian the standard time is twelve hours less than Greenwich, and just west of that line it is twelve hours greater. These two regions thus have times differing by a whole day, and in passing from one to the other it is necessary to change the date. Suppose a ship sailing eastward arrives at this **date line** on the evening of December 25; the date in the log is changed to December 24 and the next day the crew may claim a second Christmas dinner. A ship that reaches the date line on a westward voyage just before midnight on December 24 misses Christmas, for the date is changed to December 25 and shortly after, at midnight, it becomes December 26. Were it not for this change of date, a person traveling around the globe would find upon his return that his reckoning differed by a day from that of his friends at home. The date line does not follow exactly the course of the 180th meridian, but, like the boundaries of the time belts in the United States, is somewhat irregular.

**Time Signals.** The determination and distribution of official time is now a Government service. At the U. S. Naval Observatory at Washington, a knowledge of sidereal time is maintained by clocks whose error and rate are frequently determined by observations of meridian transits of stars, the clocks and meridian instruments being of the most accurate type.

Sidereal time at any moment is converted into Greenwich civil time by use of the known longitude of the observatory and of the right ascension of the mean Sun and other data tabulated in the *American Ephemeris* (these data having been computed with a knowledge of the Earth's motion derived from many previous astronomical observations). An electric distributing clock, controlled not by a pendulum but by a rapidly vibrating crystal similar to those that control the frequency of radio waves at broadcasting stations, is thus kept running on Greenwich civil time with an error never greater than a few thousandths of a second. At stated times every day this clock broadcasts signals over the radio and telegraph lines which may be received as clicks of a telegraph sounder or as briefly sustained notes in a radio receiver. These signals are usually emitted during the final five minutes of the hour and are arranged in the pattern shown in Figure 58, certain seconds being omitted and the final signal, denoting the beginning of the new hour, being a dash. If the person receiving the signals knows his Greenwich or standard time within an hour, he may thus obtain his clock error within a small fraction of a second. From the U. S. Bureau of Standards, time signals of a slightly different pattern are broadcast on several different wave-lengths continuously.

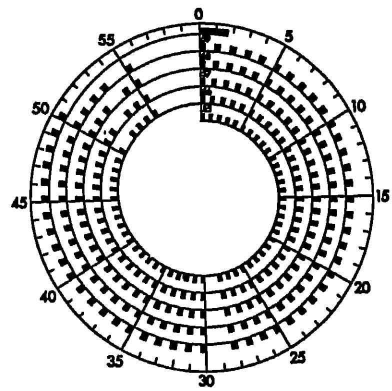

Fig. 58. *Pattern of Washington Radio Time Signals (by C. H. Smiley).*

Similar services are maintained by other governments, and the time is broadcast from many powerful and widely distributed stations.

**Determination of the Longitude of an Observatory.** To determine the longitude of a place, it is now necessary only to receive the broadcast time signals, to determine from observation the local mean solar time of their reception, and to subtract the local and Greenwich times. This usually involves transit observations of stars and the use of a clock whose rate can be relied upon for the interval between the observations and the reception of the signals. The possible sources of error are the small error of the distributing clock; the time of transmission of the signals, which is exceedingly small since radio waves are propagated with the speed of light; and the error of the local time determination.

Until recently, the problem was much more difficult because time could not be easily or accurately transmitted. Use was sometimes made of the eclipses of Jupiter's satellites and of occultations of stars by the Moon, events which could be predicted somewhat accurately in terms of Greenwich time and could be observed by local time. At sea, Greenwich time is carried by ships' chronometers which are rated and set before leaving port and which, before the advent of the radio, had to be depended upon to run true throughout long voyages. Before the invention of the chronometer and the telescope, the determination of longitude was indeed a formidable problem, and the uncertainty of the results rendered many of the geographic maps of those days egregiously erroneous.

**Measurement of the Size of the Earth.** The first intelligent estimate of the size of the Earth of which we have any record is that of Eratosthenes of Alexandria (*c.* 250 B.C.), who used a method of which the underlying principle is essentially that of the best modern determinations. He found that at Syene, a place almost directly south of Alexandria, at noon of the day of the summer solstice, the Sun cast no shadow in the bottom of a well and was therefore in the zenith; while at the same time its zenith distance as measured at Alexandria was about 7°.2, or one-fiftieth of a circumference. Since the Sun is so distant that its rays reaching the two places are sensibly parallel, this angle, $\theta$, is equal to the angle between vertical lines drawn at the two cities, as is evident in Figure 59. If the Earth is spherical, as Eratosthenes believed it to be, these lines meet at its center and the angle is measured by the arc $SA$, which is therefore one-fiftieth of the circumference of the Earth.

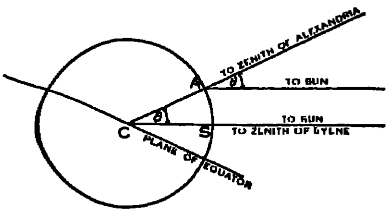

Fig. 59. *Eratosthenes' Determination of the Size of the Earth.*

Using the value 5000 stadia as the distance between the two cities, Eratosthenes calculated that the Earth's circumference was 250,000 stadia, a value that we are not in position to dispute because we do not know the length of his stadium.

The angles made by the verticals $CA$ and $CS$ with the plane of the equator are, by definition, the astronomic latitudes of the two places of observation and hence the angle $ACS$ is the difference of these latitudes. In modern measurements of the Earth, the latitudes of the two stations are determined with the greatest possible precision, usually by the employment of the zenith telescope; and the length of the arc of the Earth's meridian included between their parallels of latitude is determined by the process of triangulation.

The method of triangulation, which is in constant use by surveyors and civil engineers, depends upon the fact that, given the length of one side of a triangle and the values of two of its angles, one may compute the lengths of the two other sides. Suppose that the latitudes of the points $A$ and $B$ (Figure 60), several hundred miles apart and corresponding to Alexandria and Syene in the case of Eratosthenes, have been accurately measured by astronomers. To make use of these points in the determination of the size of the Earth, it is necessary to know the distance $aB$, in miles or kilometers, as measured over the curved surface of the Earth between the parallels of latitude of the two stations. The direct measurement of so great a distance would involve enormous difficulties, but in the method of triangulation the only direct measurement of length that is necessary is that of a comparatively short *base line*, as $HK$, the remaining measurements being of angles only. The base line is laid off on carefully leveled ground, its ends are indicated by permanent marks on stone or concrete, and its length is carefully measured with steel tape of known temperature. The accuracy of the measurement, in modern triangulations such as are made by the United States Coast and Geodetic Survey, is so great that the error does not exceed a few millimeters in a measured length of ten miles.

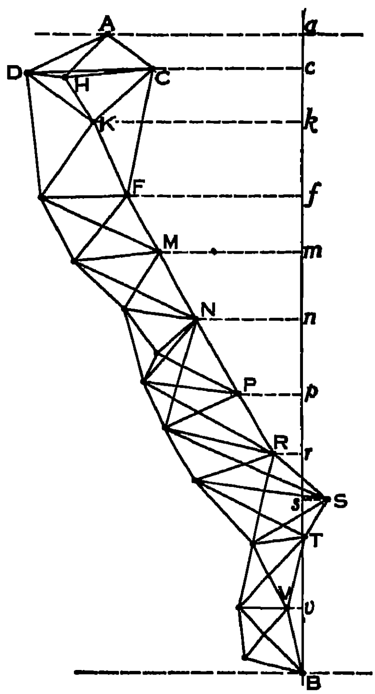


Fig. 60. *A Triangulation.*

In the remaining operations, points $C$, $D$, $F$, etc., are chosen so that each is visible from at least two other points of the series, and each is as definitely marked as are $H$ and $K$. The lines joining these points thus form a network of triangles joining $A$ and $B$, the angles of which, and also the azimuths of many of the lines, are measured with accurate theodolites. For example, the theodolite is set up over the point $K$ and its telescope is directed first to $H$ and then to $C$; the difference of the readings of the horizontal circle is the angle $HKC$. The angle $KHC$ is similarly measured with a theodolite set over the point $H$. Having thus the base and two angles of the triangle $KHC$, the surveyor may compute the distances $HC$ and $CK$. These then serve as bases for solving the triangles $HAC$ and $CKF$, the angles of which are likewise measured with the theodolite; and so on throughout the chain of triangles from $A$ to $B$.

The azimuth of any line, as $KC$, may be found by comparing its direction with that of a close circumpolar star. A simple method is to set the theodolite over $K$ and direct its telescope first to $C$ and then to Polaris at the time of its meridian passage, when the star is due north; the desired azimuth is 180° plus the difference of the readings of the horizontal circle. The length $aB$, which is the north-south distance between the parallels of latitude of $A$ and $B$, is the sum of the projections $ac$, $ck$, $kf$, etc., of the lines $AC$, $CK$, $KF$, etc., upon the meridian, the length of each

projection being that of the corresponding line multiplied by the cosine of its azimuth. The distance *aB* divided by the number of degrees in the difference of latitude between *A* and *B* is the mean length of a degree; and, on the assumption of a spherical Earth, this number multiplied by 360 is the Earth's circumference. Obviously, the process of triangulating over a large area is long and laborious; but as it is essential for making accurate surveys and maps, it is a process to which governments give generous support. Most of the area of the United States and of other leading countries has been surveyed in this manner, primarily for utilitarian purposes, and the determination of the Earth's size and form has been carried on incidentally. As longitudes may now be determined about as accurately as latitudes, arcs of the Earth's surface lying in any direction may be utilized.

Triangulation was first used in this problem by Snell, who in 1617 made a series of measurements in the flat country of Holland from which he deduced a length of the degree of about sixty-seven miles, a value later changed to about sixty-nine miles by one of his pupils, who found an error in the original calculations. In 1671 Picard, from measurements made near Paris, derived a length of about sixty-nine miles, and it was this determination that enabled Newton to verify the law of gravitation (page 236).

**The Form of the Earth.** If the Earth were stationary, with no force but gravitation acting on its parts, it would have the form of a perfect sphere; and the length of a degree of latitude would be everywhere the same. Owing, however, to the Earth's rotation, every particle of its substance (except those exactly on its axis) revolves in a circle which is centered on a point on the axis and which lies in a plane parallel to the equator. In Figure 61, let *C* be the center of the Earth, *P* its pole, and *Q* a point on its equator. A particle at *M* revolves in a circle of radius *MN* with center at *N*. Now, motion in a circle results in a tendency of the moving body to fly away from the center, a tendency which is known as centrifugal acceleration, and which is shown in works on physics to be proportional directly to the radius of the circle and inversely to the period of revolution. It is zero for particles on the Earth's axis and a maximum for particles on the equator. The centrifugal acceleration at *M* may be represented by the arrow *MS*.

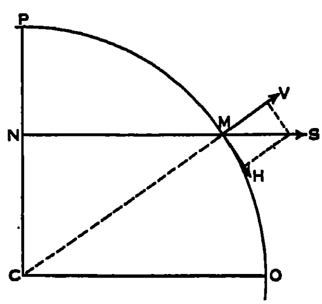

Fig. 61. *Effect of Earth's Rotation on Its Form.*

Although, because of the slowness of the Earth's rotation, the centrifugal force acting on a body of ordinary mass is small (force is equal to mass times acceleration), the Earth is so enormous that the aggregate of the forces acting on all its parts is greater than any known material could

withstand, and it would fly apart like an overstrained flywheel were it not held together by the mutual gravitation of its parts.

Suppose the Earth were spherical and homogeneous. The resultant gravitational attraction of all its parts upon $M$ would be directed to the center $C$. The centrifugal acceleration $MS$ is, according to the principle of the parallelogram of forces (page 230), the equivalent of two accelerations, represented by the sides $MV$ and $MH$ of a rectangle of which $MS$ is the diagonal. The first of these is directed away from $C$ and is overpowered by the gravitational attraction, its only effect being to reduce the weight of the body $M$; but the component $MH$ acts at right angles to the gravitational attraction, tending to make the body slide along the surface of the sphere toward the equator, and has only the rigidity of the planet to resist it. The result is that the Earth, instead of being an exact sphere, is bulged at the equator and has approximately the form of an **oblate spheroid.**

An oblate spheroid is the solid figure generated when an ellipse (page 109) is rotated about its minor axis. Rotation of an ellipse about its major axis generates a **prolate spheroid.** The former is shaped somewhat like an orange or pumpkin, the latter like a football or an airship.

The effect of the Earth's rotation upon its form is illustrated by the simple apparatus shown in Figure 62.

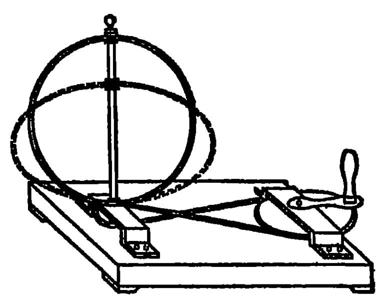

Fig. 62. *A Rotating Sphere Becomes Oblate (From Young's* Manual of Astronomy).

The mean surface of the Earth (such minor irregularities as mountains and valleys being neglected) is a **level surface,** that is, a surface which is at every point normal to the direction of gravity; and so vertical lines at two places near the equator must converge to a point $C_1$ (Figure 63) which lies between the surface and the Earth's center, while verticals near the pole meet at $C_2$, beyond the center. For a difference of latitude of one degree, therefore, the distance between two stations near the pole must be greater than that between two stations near the equator; and this is what is found by actual measurement.

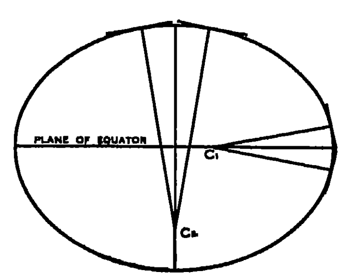


Fig. 63. *Length of a Degree on a Level Spheroidal Surface.*

In the early part of the eighteenth century, Cassini was of the opinion that the opposite was true and that the Earth was prolate; but Picard's arc was extended to 9° in France, and other arcs were measured by French astronomers in Peru (now Ecuador) and Lapland, and by 1745 these measures had established beyond a doubt that the degree of latitude increases in length from the equator to the pole. In the last century extensive arcs have been measured in many different countries and the form and size of the Earth determined with great accuracy.

From the way in which the length of a degree varies in different parts of the Earth's surface the precise form and size of the Earth are computed, but the process is too complicated to be described here. The most reliable measurements do not fit exactly into any mathematical figure. The equator at sea level is almost, but not quite, a circle, and each meridian is very nearly, but not quite, an ellipse, the departures from these curves being much less than a mile. The results of the very extensive measurements of the United States Coast and Geodetic Survey are summed up in Hayford's *Spheroid of 1909*, from which are derived the following dimensions of the Earth:

Equatorial radius, $a = 6378.388$ km. $= 3963.34$ mi.
Polar radius, $b = 6356.909$ km. $= 3949.99$ mi.
Oblateness, $\dfrac{(a - b)}{a} = \dfrac{1}{297} = 0.034$

and the following values of the length of a degree in various latitudes:

| Latitude | Length of a Degree |
|---|---|
| 0° | 68.708 miles |
| 15 | 68.757 |
| 30 | 68.882 |
| 45 | 69.056 |
| 60 | 69.231 |
| 90 | 69.407 |

**Distance of the Sea Horizon; Dip.** The circle of water surface that is visible to an observer at sea is limited by his apparent horizon which, owing to the Earth's curvature, expands if the observer ascends. The approximate relation between the observer's height and the distance of the apparent horizon is simple. In Figure 64, $C$ represents the center of the Earth (considered spherical); $O$ the observer; and $OH$ the direction of the true horizon, at right angles to $OC$. The visible horizon lies at $A$, the point of contact of a tangent to the sphere drawn from $O$. If $R$ is the radius of the Earth in statute miles of 5280 feet each, and $h$ the observer's height in feet, we have in the triangle $OAC$

$$\overline{OA}^2 = \left(R + \frac{h}{5280}\right)^2 - R^2 = \frac{2Rh}{5280} + \left(\frac{h}{5280}\right)^2.$$

$R$ is 3960, so that the first term has the value $2 \times 3960/5280h$; whereas the second, for all values of $h$ which occur on shipboard, is so small as to be negligible. Hence the distance of the sea horizon is given in miles by

$$\overline{OA} = \sqrt{\frac{7920}{5280}h} = \sqrt{\frac{3}{2}h}.$$

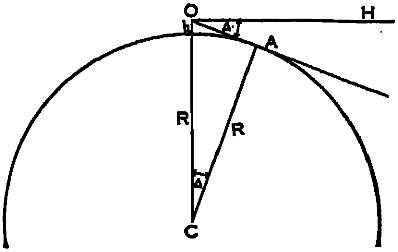


Fig. 64. *Dip and Distance of the Sea Horizon.*

The depression of the visible sea horizon below the true horizon is called the **dip**, and its angular value is a correction that must be subtracted from all measurements of the altitude of a heavenly body in which the sea horizon is used as a line of reference. In Figure 64, the dip is represented by the angle $HOA$, or $\Delta$. It equals the angle $OCA$ since the sides of the two angles are respectively perpendicular. As these angles are always small, their value in radians may be taken as $\overline{OA}/R$, or

$$\Delta \text{ in radians} = \frac{\sqrt{\frac{3}{2}h}}{3960}$$

and (page 10),

$$\Delta \text{ in minutes of arc} = \frac{3438}{3960}\sqrt{\frac{3}{2}h}$$
$$= \sqrt{h}, \text{ approximately.}$$

We thus have the sailors' rough but convenient rules:

1. *The distance of the sea horizon in miles is equal to the square root of three-halves of the observer's height in feet;* and

2. *The dip of the horizon in minutes of arc is equal to the square root of the observer's height in feet.*

For example, from the deck of a ship 25 feet above the water, the circle of visible water is about 6 miles in radius, and the value of the dip is about 5′.

## EXERCISES

1. What are the approximate dip and distance of the sea horizon as seen from a ship's deck 50 feet above the water? From an airplane 10,000 feet above the water?
*Ans.* 7′, 8⅔ miles; 1° 40′, 122 miles

2. What angle is made by the celestial equator with the horizon in latitude 40°?
*Ans.* 50°

3. What angle is made by the ecliptic and the horizon, in latitude 40° north, at sunset on March 21? At sunrise on March 21?
*Ans.* 73°.5; 26°.5

4. Prove that, in latitudes greater than 48°.5, twilight lasts all night on the shortest night of the year.

5. In latitude 40° north, what are the altitude, azimuth, hour angle, and declination of the zenith? Of the north celestial pole?

6. Toward what point of the celestial sphere is the observer carried by the Earth's rotation?

*Ans.* The east point of the horizon

7. The latitude of Wellesley, Massachusetts, is 42° 18′ north. The declination of Capella is + 45° 56′. Show that, at lower culmination (below the pole), Capella is below the horizon of an observer on the streets of Wellesley but might be seen from an aircraft 6000 feet above the town.

# CHAPTER 4

## CELESTIAL NAVIGATION

**Navigation.** One of the most important practical uses of astronomy is found in navigation, the art of ascertaining the position of a ship, aircraft, or caravan upon the surface of the Earth and of determining the direction in which it is desirable to proceed from that position to the intended destination. For clearness of description, modern navigation is often discussed under four divisions: piloting, radio navigation, dead reckoning, and celestial navigation. Although the skilled navigator makes use of all four, applying to each problem the method that is best suited or most easily available, we shall here pass briefly over the first three divisions and give attention mainly to that branch of the art that utilizes astronomy.

**Piloting** makes use of fixed visible or audible landmarks. In marine piloting these are mostly close to shore and may be natural features such as mountain peaks, artificial objects like church spires and water towers, or special aids to navigation such as lighthouses, buoys, and foghorns. Marine piloting also uses soundings to determine the depth of the water. Air piloting makes use of beacons erected along the principal airways, and of such lines as railroad tracks and rivers.

**Radio navigation** was born about 1920 and is still developing. It differs from piloting mainly in that radio waves instead of light or sound are employed—an important difference, since landmarks can be seen over only a few miles whereas radio signals can be received over thousands.

**Dead reckoning** keeps account of the path of the craft by measurements of the direction and speed of its motion and of the time elapsed since departure. Direction is indicated by the **compass**, which may be either magnetic or gyroscopic. The former type, which is more common and much older, is dependent on the Earth's magnetism and must be corrected for **compass deviation** caused by magnetic materials in the ship, and for

**magnetic variation**, the angle between the magnetic and true meridians. It is seldom that a magnetic needle points true north. The **gyrocompass**, which depends for its action on the directive effect of the Earth's rotation upon the axis of a rapidly rotating, delicately balanced, massive wheel, gives directly the true azimuth of the ship's course.

The device used for measuring the speed of a marine craft is called a **log**, the primitive form having consisted of a wooden float attached to a knotted line. The float was thrown overboard and drifted astern as the ship moved, and the speed was found by counting the knots that passed over the rail in an interval that was timed with a sandglass. More effective devices are of course now used at sea and in the air, but it is not our purpose to discuss them. The word log, or logbook, came to be applied also to the record of the ship's progress, which should be faithfully made and kept; and the word knot became the name of the marine unit of speed.

A **knot** is a speed of one nautical mile per hour. A **nautical mile** is the length of a minute of arc measured on the Earth's meridian; on account of the Earth's oblateness this length varies from 6046 feet at the equator to 6108 feet at the poles. For practical purposes it is taken as 6080 feet. Avoid the meaningless phrase "knots per hour."

Piloting cannot be used on the high seas or in flying above clouds or unfamiliar country. Radio navigation depends upon artifices that may fail and it is especially unreliable in time of war. Dead reckoning may be falsified by winds or ocean currents or by the inaccuracies of instruments, and cannot be depended on over very long distances. The final appeal is to celestial navigation; hence the careful navigator observes the heavenly bodies frequently unless weather conditions prohibit it.

**The Sextant.** On a moving craft it is of course impossible to use instruments like those in a fixed observatory, and at sea or above the clouds the only coördinate that can be easily measured is altitude. The principal instrument used for this purpose at sea is the sextant; in the air, the bubble octant. Both are sturdy instruments but so small and light that they are easily held in the hand.

The sextant was invented in 1730 by two men quite independently: Godfrey of Philadelphia and Hadley of London. It is represented diagrammatically in Figure 65. The circular arc, $T$, about 60° long (whence the name), is graduated to half degrees which are numbered as whole degrees; the sextant can measure an angle as large as 120°. At the center of the circle is pivoted the **index-arm** $MN$, which carries the **index-mirror** $M$ and the

index or vernier $N$ which reads usually to 10″. The **horizon-mirror** $H$ is attached to the frame in such a position that when the index reads zero the horizon-glass and index-glass are parallel. The horizon-glass is about twice the height of the index-glass, and only the lower half is silvered, the upper half remaining transparent. At $E$ is a small telescope directed to the horizon-glass.

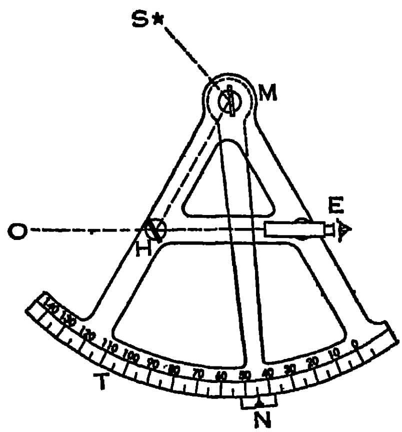

Fig. 65. *The Sextant.*

The sextant may be used for measuring any angular distance, but its most common use is for finding the altitude of the Sun. With the index near zero, the observer holds the instrument so that the telescope points to the Sun, of which two images appear, one seen directly through the clear half of the horizon-glass and the other by reflection, first from the index-mirror to the horizon-mirror, and then from the latter to the eye. Then, keeping the doubly reflected image in the field of view by moving the index-arm with his left hand, the observer lowers the line of sight of the telescope until he sees the horizon through the horizon-glass, and he adjusts the index-arm until, as the instrument is rotated slightly about the axis of the telescope, the arc in which the Sun appears to swing comes tangent to the horizon. The reading of the index is then the angular distance of the Sun's limb from the apparent horizon. To get from this the true altitude of the Sun's center, corrections must be applied for dip of the horizon, atmospheric refraction, parallax (so small as to be usually neglected), and the Sun's semidiameter.

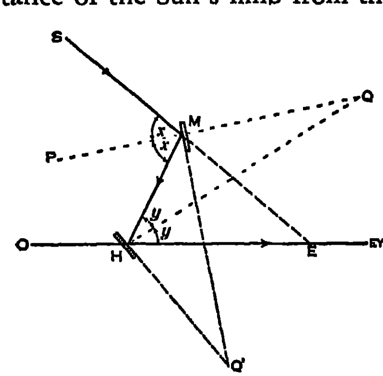

Fig. 66. *Principle of the Sextant.*

That the measured altitude is twice the angle $HQ'M$ (Figure 66) between the mirrors, which is the angle passed over by the index from zero, is demonstrated as follows:

Let $PQ$ and $HQ$ be the normals to the index-glass and horizon-glass, respectively; $OHE$ the line of sight from the horizon; and $SMHE$ the ray of sunlight reflected by the two mirrors to the eye. By the law of reflection the angles marked $x$ are equal

to each other, and the same is true of those marked $y$. But $2x$ is exterior to the triangle $HME$, and therefore the angle $E$, which is the apparent altitude of the Sun's limb, is equal to $2x - 2y$. Similarly, since the lower $x$ is exterior to the triangle $HMQ$, the angle $Q = x - y$. Hence $E = 2Q$. But $Q = Q'$ since the sides of $Q$ are perpendicular to those of $Q'$; therefore $E = 2Q'$. Q.E.D.

**The Bubble Octant.** Because the sea horizon is invisible at night, the use of the sextant in observing stars is limited to the hours of twilight and dawn. In daylight also, the natural horizon is sometimes obscured by fog or otherwise, and this occurs with special frequency in aerial navigation. The direction of the unseen horizon may be determined, however, by means of a bubble in a chamber nearly filled with liquid and having a spherical glass top against which the bubble seeks the highest point, as in the common carpenter's level. This principle was utilized as early as 1894 by Willson, who attached a bubble chamber to a marine sextant.

The modern bubble octant measures angles up to 90°, but the 45° arc from which it takes its name is not a conspicuous part of the instrument. The optical arrangement is such that the observer sees the bubble (electrically illuminated at night) in the field with the heavenly body, and the observation consists in bringing the two images together as those of the Sun and horizon are brought together in the sextant. No correction for dip is required. The octant may also be used with the sea horizon, like the sextant. The use of the bubble is subject to an **acceleration error** resulting from the irregular movements of the craft caused by wind or waves. The cure for this error is to take several readings in rapid succession and adopt the average. In some of the latest instruments the average is indicated mechanically.

**The Chronometer.** The timepiece used in navigation is called a chronometer; it is a large and accurately made watch. The marine chronometer has a dial about 4 inches in diameter and is made to beat half seconds for convenience in use by the eye-and-ear method. It is mounted on gimbals so that it remains nearly level despite the rolling and pitching of the ship, and is kept in a box where it is undisturbed except for daily winding. Many ships carry three chronometers so that, if one acquires an error, the agreement of the other two will disclose it. Radio time signals, however, have made this precaution less important than it was a few years ago. Aircraft chronometers need not be so accurate, since it is not necessary to depend upon them over such long periods of time. Special wrist watches are sometimes used in aerial navigation; most of them have a second-setting device to facilitate setting them with the radio signals.

**Almanacs.** Almanacs for the use of astronomers and navigators are issued by the principal governments and are the result of a prodigious amount of observation, mathematical theory, and computation. The United States government publishes three.

The *American Ephemeris*, published annually a few years in advance, began with the volume for 1855 and is a vast compendium of accurate data for the use of astronomers. For example, it tabulates the right ascension and declination of the Sun for every day and of the Moon for every hour, to the tenth of a second of time and the hundredth of a second of arc, respectively—an accuracy which for the navigator is superfluous and troublesome.

The *American Nautical Almanac* and *American Air Almanac* are for the use of navigators. The *Nautical Almanac*, in one volume per year, gives the Sun's position (for example) to the nearest 0ʹ.1 for every two hours; the *Air Almanac*, in one volume for every four months, gives it to the nearest 1ʹ.0 for every ten minutes. For all the needs of the flyer and most of those of the sailor, the *Air Almanac* has sufficient accuracy, and it is easier to use. On a sheet for each day, the *Air Almanac* tabulates, for each ten minutes of Greenwich civil time (GCT), the Greenwich hour angle (GHA) and declination of the Sun, Moon, and principal planets and, under the heading GHA♈ (Greenwich hour angle of the vernal equinox), the Greenwich sidereal time; also, for each day, the Moon's parallax and semidiameter, the Sun's semidiameter, and other useful data; it gives a diagram showing the relative apparent position of the Sun, Moon, planets, and certain stars along the ecliptic. In a separate table it lists, for 55 of the principal stars (known as navigators' stars), the name, magnitude, SHA, and declination. SHA stands for **sidereal hour angle,** a term that has recently come into vogue. It means the arc of the celestial equator measured westward from the vernal equinox to the star's hour circle, and is reckoned in degrees and minutes; thus, it is nothing else than 360° minus the right ascension.

The Greenwich hour angle of a star is to be found by adding the sidereal hour angle to the Greenwich sidereal time; or, as expressed in the *Air Almanac:*

$$\text{GHA}^* = \text{GHA}♈ + \text{SHA}^*.$$

**Geographic Position of a Heavenly Body.** At any particular instant, each heavenly body is at the zenith of a particular point on the Earth called the geographic position (GP) of the body, or the substellar (subsolar, sublunar, etc.) point. For the purposes of navigation the Earth's oblateness is neglected, and so the GP is regarded as the point where a straight

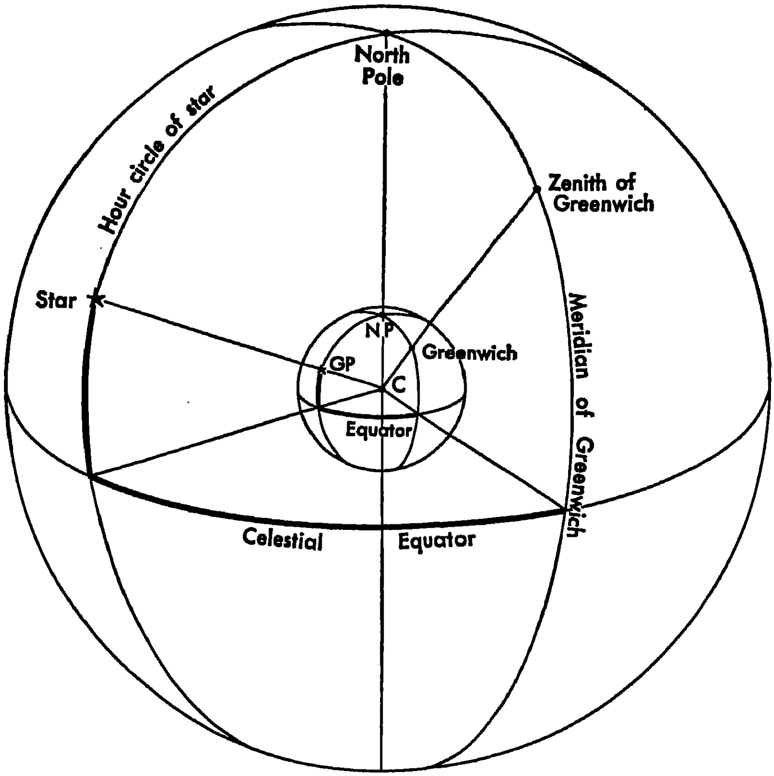


Fig. 67. *The Substellar Point or* GP.

line joining the body and the center of the Earth pierces the Earth's surface (Figure 67). Therefore, the latitude and longitude of the GP are the declination and Greenwich hour angle, respectively, of the body (heavy lines in Figure 67). For any Greenwich civil time (given by the chronometer) these coördinates may be taken from the almanac.

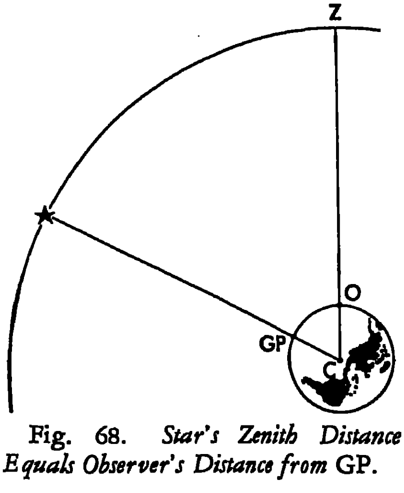


Fig. 68. *Star's Zenith Distance Equals Observer's Distance from* GP.

**Circle of Position.** An observation with the sextant or octant gives, after the appropriate corrections are applied, the star's altitude at the place of observation. Subtracting this from 90° gives the star's zenith distance, and the value of this in minutes of arc equals the dis-

tance in nautical miles on the Earth's surface between the observer and the GP (Figure 68). Therefore, the observer must be somewhere on a circle on the Earth's surface, each point of which is at this distance from the GP. This circle is called a circle of position (Figure 69). Observation of another star determines a second circle of position intersecting the first at two points. The observer must be at one of these points of intersection and, as they are usually hundreds of miles apart, he is not likely to be in doubt as to which one. This point is called a fix. A third observation gives a third circle of position intersecting the others at the fix and affording a check on the accuracy of the work.

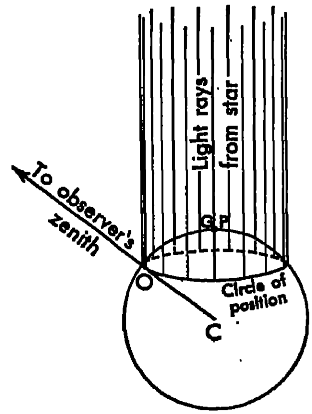

Fig. 69. *Circle of Position.*

**Line of Position.** The fix can be shown by drawing the circles of position on a terrestrial globe; for the instruction of the student this is a valuable exercise, but to attain sufficient accuracy for actual navigation the globe would have to be inconveniently large (on a 12-inch globe representing the Earth, for example, the scale is 1 inch = 573 miles only), and so carefully made as to be expensive. The practical navigator uses a large-scale chart representing only a small part of the Earth's surface within which he believes his ship to be, and draws only short, straight lines representing tangents to the circles of position. These are called lines of position or, in honor of the American navigator who in 1837 discovered their usefulness, **Sumner lines.**

**Determining the Line of Position.** The method commonly used for determining the line of position has evolved from that which was proposed in 1875 by the French navigator, Marcq St. Hilaire. Note that the observer, the Earth's pole, and the GP form upon the Earth's surface a spherical triangle whose sides and angles are equal to those of the *ZPS* triangle in the sky. The navigator knows by dead reckoning the approximate position (DR latitude and DR longitude) of his ship. Using these coördinates (or round numbers approximating them) and the coördinates of the body given by the almanac, he solves the triangle and obtains the altitude and azimuth which the body would have if the DR position were correct. For the purposes of plotting, the error in the azimuth is negligible. The difference, in minutes of arc, between the observed and computed

altitudes (called the altitude intercept) is the distance in miles between the DR position and the Sumner line. Accordingly, the navigator draws the Sumner line on his chart at this distance from the DR position (on the side toward the GP if the observed altitude is larger than the computed), and in an azimuth differing 90° from the computed azimuth of the body.

The solution is greatly facilitated by the use of modern tables. Most valuable are three publications of the United States Hydrographic Office. Of these, the most complete is *Tables of Computed Altitude and Azimuth* (commonly known as HO 214), the product of the solving of some hundreds of thousands of *ZPS* triangles. It fills eight large volumes and gives directly the altitude and azimuth in terms of latitude, declination, and local hour angle. Very compact but not so rapid in use are the tables of Ageton (HO 211) and of Dreisonstok (HO 208), special arrangements of the logarithmic trigonometric functions that are needed for a quick arithmetic solution of the triangle.

**The Fix.** The most satisfactory fix is obtained by observing in rapid succession three stars at moderate altitudes and in azimuths about 120° apart. Generally, because of small errors of observation, the three lines of position will not meet in a point but will form a small triangle; then the most probable position is taken to be the center of the triangle. In the daytime, however, the Sun is often the only observable heavenly body, though the Moon and even Venus may sometimes be used. In navigation by a single body, such as the Sun, a **running fix** is obtained from two observations made a few hours apart, by redrawing the first line of position, parallel to itself, at a distance measured along the course equal to the run of the ship in the interval between observations as found by dead reckoning. The running fix is the point where this advanced line intersects the second line of position.

Even a single line of position, when no fix is obtainable, is valuable as a partial check on dead reckoning and, sometimes, as a means of apprising the navigator of the proximity of danger.

**The Noon Sight.** When a body is on the meridian, as the Sun is at apparent noon, its GP is due south or due north of the observer so that the line of position lies due east and west; its local hour angle is zero; and its altitude is momentarily unchanging. Partly for these reasons and partly because of long-established custom, the noon sight is a regular daily ritual on most seagoing vessels. A few minutes before apparent noon, the navigating officer brings the image of the Sun's limb into coincidence with the horizon in the field of his sextant, and keeps it there by slowly moving the index-arm until the altitude ceases to increase. This moment

of maximum altitude is apparent noon and the Sun is on the meridian. The reading of the sextant, corrected as usual, represents the Sun's meridian altitude.

**Choice of Course.** Unlike trains and automobiles, which must keep to the roads, ocean vessels and aircraft may follow any course whatever between point of departure and destination. For a distance of a few hundred miles or less, the course usually chosen is the **rhumb line** or **loxodrome**, the line on the Earth's surface which makes the same angle with every meridian, and so has the same azimuth throughout its length. On a Mercator chart [1] it is represented by a straight line, but because of the convergence of the meridians it is in fact a spiral, and a craft that followed such a course indefinitely would (unless the angle with the meridian were 0 or 90°) approach the pole as a limit, making around it an infinite number of turns. The shortest distance between two points on the globe is along a **great circle course**, and on journeys of over a thousand miles it is preferable to approximate this line by frequent alterations of course unless it leads into dangerous or otherwise undesirable regions. The following table [2] shows the distance in nautical miles between New York and various cities, as measured along a rhumb line and along a great circle.

| | Rhumb Line | Great Circle | Difference |
|---|---|---|---|
| New York to Boston, Mass. | 166 | 165 | 1 |
| New York to Chicago, Ill. | 622 | 619 | 3 |
| New York to Paris, France | 3290 | 3149 | 141 |
| New York to Tokyo, Japan | 6932 | 5856 | 1076 |

Tokyo is only about 5° nearer the equator than is New York, and the rhumb line is almost a parallel of latitude, bearing only a little south of west and passing over the middle of the United States and the Pacific Ocean; to follow the great circle an airplane would leave New York in a direction a little west of north, fly over Canada, Alaska, and Kamchatka, and arrive in Tokyo flying a little west of south; yet the great circle course is more than a thousand miles shorter.

It will be instructive to the student to find the great circle courses between various cities by stretching a string over the surface of a terrestrial

[1] A Mercator chart represents the meridians and parallels of latitude as straight lines, the parallels intersecting the meridians at right angles. Since the terrestrial meridians really converge to the poles, the map is badly distorted in the polar regions. The name Mercator is the Latinized form of the surname of Gerhard Kramer, a Flemish mathematician employed by Charles V of Spain to make maps for the use of his sailors.

[2] Condensed from Weems's *Air Navigation.*

globe; for example, between Valparaiso and Cape Town, Buenos Aires and Moscow, New York and Chungking.

**The Older Navigation.** In the methods of navigation practiced before Sumner and Marcq St. Hilaire—and still practiced by seamen of the old school—the line of position does not occur and the ship's latitude and longitude are determined separately. Latitude is determined from an observation of a body on or near the meridian, usually by a noon sight of the Sun, though any heavenly body whose declination is known may serve. In Figure 70, let the semicircle represent the visible half of the meridian, $O$ the observer, $S$ the Sun, $Z$ the zenith, $P$ the pole, $E$ the intersection of the meridian and the celestial equator, and $H$ and $N$ the south and north points of the horizon, respectively. The ship's latitude, being equal to the altitude of the pole, is represented by the arc $PN$, or by $EZ$, the declination of the zenith, since $PN$ and $EZ$ are equal, each being 90° minus the arc $ZP$. The zenith distance of the Sun is $SZ$ and is equal to 90° minus the altitude given by the noon sight; and the declination $ES$ of the Sun is given by the almanac. In the case pictured, $EZ = ES + SZ$, or the latitude of the ship equals the Sun's declination plus its zenith distance. If the Sun or the observer is south of the equator, the relation is changed only in regard to signs.

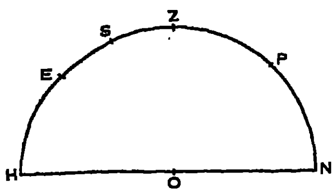


Fig. 70. *Latitude by Meridian Altitude.*

Polaris also is convenient for determining latitude because it is within a degree of the north pole and so its altitude is always within a degree of the observer's latitude. The correction to be applied to its altitude to obtain the latitude is given in terms of sidereal time in the almanacs.

Longitude is determined by the old method by means of a time sight made on a body when it is far from the meridian so that its altitude is changing rapidly; if the body is the Sun, the observation is made several hours before or after noon. The zenith distance given by the observation, the complement of the Sun's declination, and the colatitude carried by dead reckoning from the last preceding noon sight are the three sides of the $ZPS$ triangle which is solved for the angle $ZPS$, the hour angle of the Sun. This is identical with the local apparent solar time. Adding the equation of time taken from the almanac gives local mean time, and subtracting this from the Greenwich time furnished by the chronometer gives the longitude.

Until after the production of the first chronometer in 1713, no portable timepiece that could be depended on was available, and the accurate determination of longitude was impracticable. Had Columbus been able to determine his longitude, he would not have mistaken America for the East Indies but would have recognized it as a new land. Long after his time, a shipmaster sailing from Liverpool to Rio de Janeiro, for instance, would sail in a generally southwesterly direction until he reached the latitude of Rio, and then sail westward until he sighted land, having in the meantime no dependable knowledge of his distance from port.

Even chronometers, especially the early ones, could not be trusted implicitly throughout the long voyages of those days; therefore it was customary to verify their indication of Greenwich time by observing **lunar distances.** The Moon has an apparent eastward motion of about 30′ per hour and in its progress it passes certain bright stars near the ecliptic. The primary purpose of the founding of the Royal Observatory at Greenwich in 1675 was the prediction of the Moon's apparent place for purposes of navigation. For many years the nautical almanacs published predictions of the Moon's distance from certain stars in terms of Greenwich time throughout the year. By measuring these distances with the sextant and applying the proper corrections (the Moon's large parallax was especially troublesome), the navigator derived the Greenwich time for comparison with his chronometer. The method was laborious and inaccurate (an error of 1′ in the Moon's place produced an uncertainty of 2 minutes in the time) and now fortunately is obsolete.

**Books on Navigation.** This chapter can have done no more than convey a general idea of the navigator's problem and its relation to astronomy; and this is all that the author has intended. To go further, the student should study more technical works, do many exercises, and acquire practical experience. There are many books on the subject, some extensive and some brief, some old and many recent. The following are recommended:

*Pre-training Navigation*, by William T. Skilling and Robert S. Richardson. New York: Henry Holt and Company, 1942. Very concise and clear.

*The Way of a Ship*, by Lawrence C. Wroth. Portland, Maine: Southworth-Antoensen Press, 1937. A fascinating little book on the history of navigation.

*Primer of Navigation*, by G. W. Mixter. New York: D. Van Nostrand Company, 1944. A very readable book written for the beginner by a scholarly and experienced navigator. Contains numerous excellent exercises.

*Marine and Air Navigation*, by John Q. Stewart and Newton L. Pierce, experts in the teaching of navigation. Boston: Ginn and Company, 1944. Comprehensive, practical, and not too difficult.

*American Practical Navigator*, by Nathaniel Bowditch, outstanding navigator and mathematician of the old days of the clipper ships. Has been revised many times since his day. Washington: U. S. Hydrographic Office, Publication No. 9.

*Basic Marine Navigation*, by Bartolomaeus J. Bok and Frances W. Wright. Boston: Houghton Mifflin and Company, 1944. Elementary and practical.

# CHAPTER 5

# THE ORBITAL MOTION OF THE EARTH

**The Sun's Apparent Motion Explained by a Real Motion of the Earth.** In Chapter 1 (page 25) we described the apparent annual motion of the Sun among the stars, and remarked that this apparent motion is really due to an orbital motion of the Earth. This view was held as long ago as 280 B.C. by Aristarchus of Samos, but his successors, Apollonius, Hipparchus, and Ptolemy, rejected it in favor of the stationary Earth, and their ideas were held almost universally until the time of Copernicus (1473–1543), and pretty generally until at least a century later.

That it is possible that the apparent motion of the Sun is produced by a real motion of the Earth around the Sun is shown in Figure 71. Late in September, for example, the Earth is in such a position that the Sun appears in the direction of the stars of the constellation Virgo; Libra, on one side, and Leo, on the other, are too nearly in the Sun's direction to permit their stars to be easily seen; Scorpius is visible only in the early evening, being just east of the Sun, and Cancer may be seen just before dawn, west of the Sun; while at midnight, the stars of Pisces, near the vernal equinox, are on the observer's meridian. As the Earth moves in its orbit, with perfect silence and smoothness so that we are unaware of its motion except by observation of the heavenly bodies, the direction of the Sun from us changes, and by January the stars of Sagittarius are hidden, Scorpius has apparently emerged upon the west side of the Sun and can be seen before dawn, Aquarius is low in the west at twilight, and Gemini is on the meridian at midnight. The ecliptic, or Sun's apparent path, is thus explained as the projection of the Earth's orbit upon the celestial sphere as seen from the Sun, and, if we are to adopt the belief that the Earth moves, since the ecliptic is known to be a great circle, we must admit that the Earth moves always in the same plane and that this plane passes through the Sun.

The arc of the ecliptic, measured eastward from the vernal equinox to

the apparent position of the Sun as seen from the Earth, is, according to the definition in Chapter 1 (page 27), the **geocentric longitude** of the Sun. Since, for an observer on the Sun, the Earth would appear exactly at the opposite point of the celestial sphere, the **heliocentric longitude** of the Earth differs from the geocentric longitude of the Sun by just 180°. About March 21, for example, the Sun appears at the vernal equinox and its geocentric longitude is therefore 0°, while at the same time the Earth's heliocentric longitude is 180°.

**Proofs of the Earth's Revolution.** Studies of the planets by Kepler and Galileo had made it evident enough to reasonable persons before the middle of the seventeenth century that the Earth itself is a planet and revolves around the Sun. Now we have observational proofs that it does so. One of these is the annual parallactic displacement of stars; others depend on the finite velocity of light and include the effect of the Earth's changing position on the observed times of the eclipses of Algol-type stars, the annually periodic variation in the radial velocities of stars, and the aberration of light. The last-named was the earliest discovered, by the English astronomer Bradley in 1727. All require accurate and careful observation, and all have been revealed by observations made for various other purposes.

**The Annual Parallactic Displacement of the Stars.** It was remarked by Aristotle that if the Earth traveled in a great orbit as Aristarchus believed it to do, we should be brought into different regions of the stars at different times of year, and this would change the appearance of the constellations. Not being able to perceive any such changes, Aristotle concluded that the Earth was immovable. His fallacy lay in his failure to appreciate the enormous distances of the stars. The fact is that the Earth's motion does produce changes of apparent position, but the stars are so remote that these changes are too minute to be detected except by careful measurements with the telescope.

Suppose for a moment that a star were fixed at the point *A* in Figure 71 and that the other stars were all much more remote. If the Earth did not move, the star *A* would remain always in the same direction from us and would appear fixed among the constellations; but if the Earth revolves in the orbit shown, the star *A* must, in October, appear among those of the constellation Gemini and in April among those of Taurus. This apparent annual shift of a near star among its remote fellows is called its **parallactic displacement.**

The **heliocentric parallax** is the maximum value of the parallactic displacement, counted from the star's mean position, and is the angle

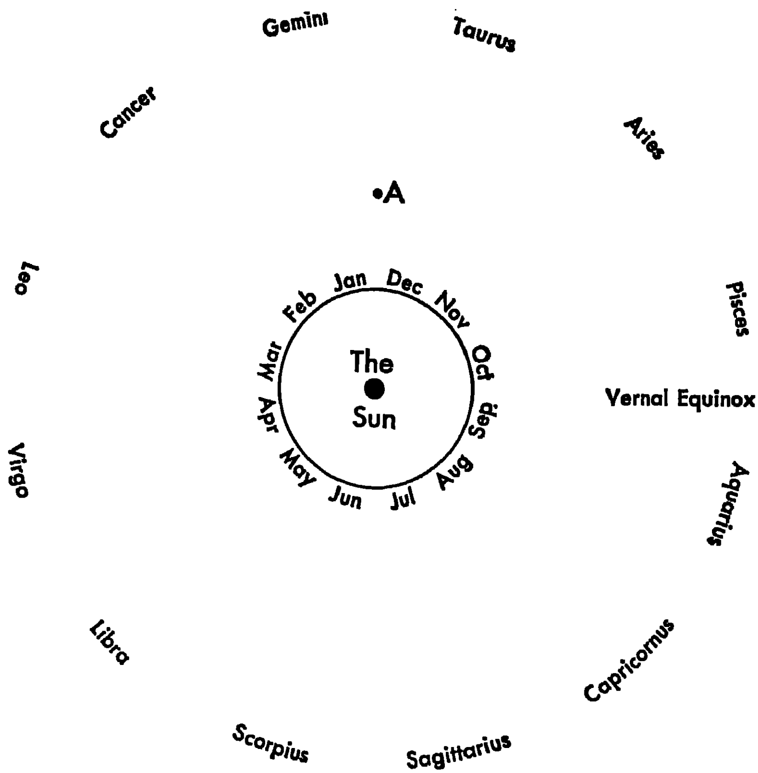


Fig. 71. *Annual Apparent Motion of the Sun.*

subtended at the star by the semidiameter of the Earth's orbit (Figure 72). The heliocentric parallax is inversely proportional to the distance of the star and its measurement affords the most direct means of determining that distance (page 331). Instead of being at the order of distance suggested in the figures, even the nearest star is so remote that its heliocentric

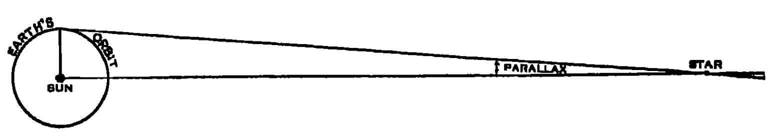


Fig. 72. *Heliocentric Parallax.*

parallax is very minute—only 0″.78—and to draw Figure 72 in correct proportion would necessitate drawing a triangle more than a mile long, the diameter of the circle representing the Earth's orbit being a half-inch.

All other known stars have still smaller parallaxes, and the vast majority of stellar parallaxes are too small for measurement even by the most modern methods. However, the parallaxes of more than 3000 stars have been measured (mostly since the beginning of the twentieth century), and so we have the testimony of more than 3000 witnesses that the Earth really does complete a circuit each year.

**Determination of the Velocity of Light.** That light does not travel instantaneously was demonstrated in 1675 by Olaus Römer in Denmark by a careful study of the times of the eclipses of Jupiter's satellites. Three of the bright satellites discovered by Galileo in 1610 are eclipsed (Figure 280) regularly at every revolution, and so by observing the interval between eclipses during a given season Römer could predict the times of future eclipses. He found that if the predictions were based upon observations made when the Earth was on the side of its orbit nearest Jupiter, the eclipses that occurred during the succeeding months came increasingly later than the predicted times until the Earth was on the opposite side of its orbit; then, as the Earth overtook Jupiter again, the error of the prediction diminished, and when the two planets were again at their shortest distance the eclipses were once more on time. Römer correctly attributed this alternate delay and hastening of the eclipses to the changing time required for the light reflected by the satellites to reach the Earth, and concluded from his observations that light requires about 600 seconds to travel the distance from the Sun to the Earth, a figure that has been changed by subsequent investigations to 500 seconds.

This study is complicated by the fact that the motions of the satellites are disturbed by their mutual attractions, and is also rendered inaccurate by the slowness with which the disappearance and reappearance of the satellites take place, which makes it impossible to observe the exact times of the eclipses. Moreover, to determine the velocity of light in this way one must know the distance from the Earth to the Sun in miles or kilometers, and this distance is not known so accurately as is the velocity of light as determined by modern experimental methods; and so the problem is now reversed and the times of the Jovian eclipses have been used for finding with greater accuracy the distance of the Sun.

The best-known experimental methods of determining the velocity of light are based upon two devices, known as the methods of Fizeau and of Foucault, having been first used by those French physicists in 1849 and 1850. The essential element in the former is a toothed wheel, and in the latter a rotating mirror. The principle of Fizeau's method is illustrated in Figure 73. Light from a bright lamp at $L$ is partly reflected and partly

transmitted by the thinly silvered glass plate *P*. The reflected portion proceeds between two teeth of the wheel *W* to a distant mirror *M*, is returned to *P*, where it is again divided, and the transmitted portion of the

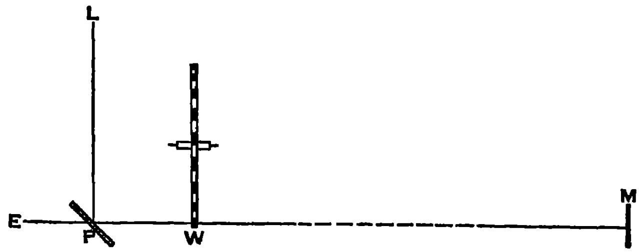


Fig. 73. *Fizeau's Method of Determining the Velocity of Light.*

returning ray is received by the observer's eye at *E*. When the wheel is rotated, the light is interrupted and proceeds in flashes which, however, unless the speed of the wheel is very low, are too brief to be perceived, and the eye has the impression of a continuous beam. This is seen until the speed of the wheel becomes so great that, while the light travels from *W* to *M* and back, a tooth has moved to the position of the space through which the light passed and so stops it on its return, in which case no light is seen. If the speed is made twice as great, the flash that escaped through one space returns through the next and the light reappears. Knowing the number of teeth on the wheel and also the speed when the light first disappears, the experimeter computes the time taken by a tooth in replacing a space, which is the same as the time taken by light in traveling the double distance between *W* and *M*.

In Foucault's method, light from *L* (Figure 74) falls upon a mirror *AOB*, which may be rotated about an axis parallel to its own plane, and is re-

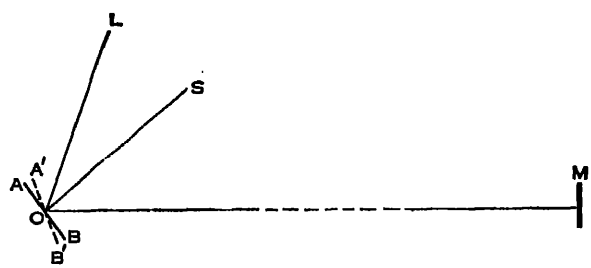


Fig. 74. *Foucault's Method of Determining the Velocity of Light.*

flected to the distant mirror *M* which returns it to *O*. Unless the rotating mirror occupies the position *AOB*, the light will be reflected by it not to *M* but in some other direction; if it does occupy that position, the return-

ing beam will be reflected back to $L$. Suppose the mirror to be rotated slowly. A flash will be sent to $M$ at each rotation, and will return to $O$ and be reflected very nearly to $L$. Suppose the speed to be increased so that, while the flash has traveled from $O$ to $M$ and back, the mirror's position has changed to $A'OB'$. The returning beam will then be reflected to $S$, and by measuring the angle $LOS$, the angle $AOA'$ through which the mirror has turned (which is, by the law of reflection, just half as great as the angle $LOS$) can be determined; and then, knowing the speed of the mirror in rotations per second, the experimenter can determine the time taken by it to turn through the angle $AOA'$ and hence the time taken by light to traverse twice the distance $OM$.

In the best determinations the reflected beam is several miles long, and is intensified by means of condensing and collimating lenses or mirrors. The most reliable value of the velocity of light is believed to be that given by Michelson, who, using the rotating-mirror method, worked at the problem with various associates and at different places and times during a period of more than fifty years. In 1921–1926 he used a light-path of 45 miles between Mount Wilson and Mount San Antonio in southern California. Later, to avoid the bad seeing and the uncertainties of refraction in a long path through air, he arranged to have the whole apparatus enclosed in a three-foot, mile-long, airtight pipe which was exhausted of air to a high degree of vacuity. The rotating mirror had the form of a 32-sided polygon, light was supplied by a powerful arc lamp, and the optical path within the pipe was extended by repeated reflections to ten miles. The observations, completed after Michelson's death by Pease and Pearson in 1933, give for the velocity of light *in vacuo*

$$c = 299{,}774 \text{ kilometers per second.}$$

Earlier values, by Michelson and by others, were somewhat higher, some being over 300,000 km./sec. Sufficiently accurate for many purposes is the round number, 300,000 km./sec., $3 \times 10^{10}$ cm./sec., or 186,000 miles per second.

**Evidence of the Earth's Revolution Derived from Eclipsing Stars.** Römer's discovery of the apparent delay and hastening of the eclipses of Jupiter's satellites would itself be proof of the revolution of the Earth to anyone who accepted the fact that Jupiter revolves around the Sun in a nearly circular orbit; but it was once generally believed (page 214) that Jupiter moved in a looped curve around a stationary Earth, approaching and receding from us annually; this view also would agree with Römer's discovery. There are, however, several hundred stars such as Algol, $\beta$ Persei (page 363), each of which is eclipsed at regular intervals by an unseen companion, and the observable times of these eclipses are affected in the same way as those of the Jovian eclipses. Since no one imagines

that each of these eclipsing stars alternately approaches and recedes from the Earth in a period of exactly a year, this effect constitutes proof of the Earth's revolution.

**The Annual Variation in the Radial Velocities of Stars.** By the **radial velocity** of a star or other object is meant the rate at which the distance between the object and the observer is changing. If this distance is increasing, the radial velocity is said to be positive; if diminishing, negative. In such remote objects as the stars, the change of distance produced even in many centuries by any known velocity is not sufficient to make any perceptible change of brightness or appearance; but the radial velocity can be readily detected and evaluated by measuring the displacement of lines in the stars' spectra. As will be shown when we discuss the Doppler-Fizeau principle (page 168), the measured displacement of any spectral line due to this effect is proportional to the ratio $V/c$ of the radial velocity to the velocity of light. In this way, the radial velocities of many stars and nebulae are determined to within a small fraction of a kilometer per second.

From a consideration of Figure 71 it may be seen that if the Earth revolves around the Sun, it will at some time of the year be traveling directly toward any given star on the ecliptic, while six months later it will be moving directly away from the same star. In October, for example, the orbital motion must carry us toward the stars of Gemini and away from those of Sagittarius, while in April the case must be reversed. Observation shows that, although the stars have motions of their own in various directions, the radial velocity of those in Gemini is about sixty km./sec. less, and of those in Sagittarius about sixty km./sec. greater (having due regard to sign) in October than in April and that similarly, in other parts of the zodiac, there is a variation of radial velocity in harmony with the idea that the Earth revolves yearly in a nearly circular orbit and with a velocity of about thirty kilometers (eighteen miles) a second, or about one ten-thousandth of the velocity of light.

This annual variation of radial velocity is of course less for stars that are not on the ecliptic, since they are not directly in the plane of the Earth's motion. It is proportional to the cosine of the star's latitude, and vanishes at the ecliptic poles. Among the stars are many spectroscopic binaries (page 357) which have orbital motions of their own around companion stars and so periodically recede from and approach the Earth; but in every such case the radial velocity due to this motion of the star is superposed upon that due to the orbital motion of the Earth, and the two effects can readily be disentangled.

**The Aberration of Light.** The effect considered in the preceding section is greatest when the Earth is at the points of its orbit where it is moving most nearly in the line of sight, toward or from the star. The effect that we shall next consider is least at those points and is greatest when the Earth is moving at right angles to the line of sight. To make this effect clear, let us first consider an analogous case. Suppose that rain is falling vertically and that it is desired to catch a raindrop in a narrow, straight tube in such a manner that the drop will enter the center of the top and fall upon the center of the bottom without touching the side. If the tube is stationary, this can be accomplished by simply holding it in an upright position, as at $A$ in Figure 75. But suppose the tube to be held by a person who is walking toward our right. It is now necessary to incline the tube forward, as at $B$, at an angle $\alpha$, such that, while the raindrop is falling through the height $V$ of the tube, the bottom moves a distance $u$ to the point vertically below the position that was occupied by the top of the tube at the moment the raindrop entered it. The tube must thus form the diagonal of a rectangle of which the vertical and horizontal sides $V$ and $u$ are proportional to the velocities of the raindrop and tube respectively. The relation connecting the three quantities is

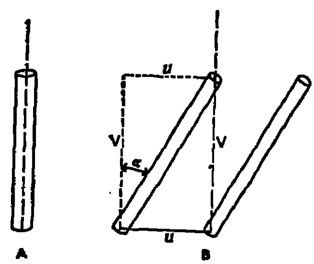

Fig. 75. *Aberration.*

$$\tan \alpha = \frac{u}{V}$$

or, if the person is walking very slowly so that $u$ is very small compared with $V$,

$$\alpha = \frac{u}{V} \times 206265''.$$

A similar effect, called **aberration,** is produced by the orbital motion of the Earth upon the apparent direction of the light that comes to us from a star. The tube in the above illustration is supplanted by a telescope in which it is desired to catch the light waves from a star in order that an image of the star may be formed upon the cross-wires or seen in the eye-piece. It is, in fact, necessary to direct the telescope *ahead* of the star's true position by an angle $\alpha$ which is equal to $u/c \times 206265''$, where $u$ is the velocity of the Earth in its orbit and $c$ is the velocity of light. If the Earth were fixed among the stars, this would not be the case, and if it

moved forever uniformly in a straight line we should be unaware of the existence of aberration, since the displacement would be constant and we should be ignorant of the star's true position. If, on the other hand,

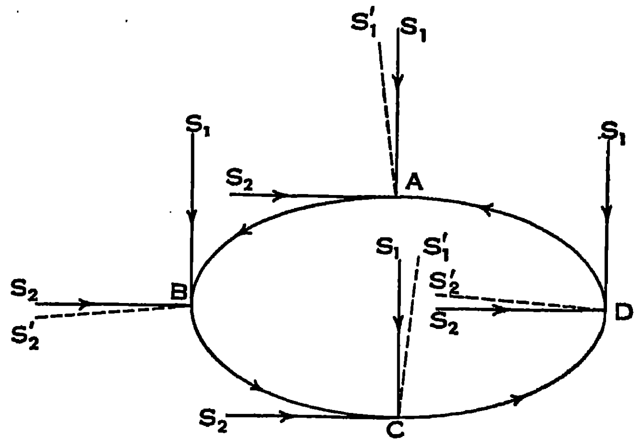


Fig. 76. *Aberration in Different Parts of the Sky.*

the Earth revolves in a closed orbit, the amount and direction of the effect of aberration on any given star must be different at different times of the year, and observation shows this actually to be the case.

The effect of aberration upon the apparent positions of stars in different parts of the sky may be studied in Figure 76, which represents the Earth's orbit in perspective as if seen from a point outside the plane of the ecliptic, with the Earth in four positions, *A*, *B*, *C*, and *D*. Rays of light are represented as coming from two stars: $S_1$, at the pole of the ecliptic, and $S_2$, on the ecliptic itself. When the Earth is at *A*, $S_1$ is displaced toward the left to $S_1'$, but $S_2$ is not displaced at all because, as the Earth is going straight to meet its light, the effect is the same as if the tube, in the raindrop analogy, were carried endwise upward. At *B*, the Earth is moving at right angles to both light rays, and both stars are displaced, by the same amount, in the direction of the imaginary onlooker. At *C*, the conditions are the reverse of those at *A*: $S_1$ is displaced toward the right, and $S_2$ not at all. At *D*, the conditions at *B* are reversed. A little thought will show that a star on the ecliptic, as $S_2$, seems to oscillate in a straight line through its true position, and that one at the pole of the ecliptic, as $S_1$, is always displaced, but in an ever-changing direction, so that, in the course of a year, it appears to describe a small, closed orbit on the surface of the celestial sphere. Stars between the ecliptic and its poles describe closed aberrational orbits which vary in form with the celestial latitude, as shown in Figure 77.

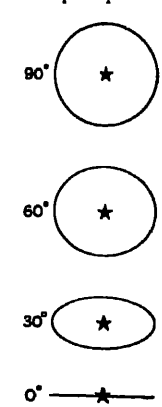


Fig. 77. *Aberrational Ellipses.*

The apparent displacement of a star, produced by aberration when the Earth, traveling at its mean speed, is moving at right angles to the star's light, is called the **constant of aberration.** Observation shows that its value is very nearly 20″.5. The mean orbital velocity of the Earth is therefore

$$u = \frac{20.5 \times c}{206265}$$

or slightly less than one ten-thousandth of the velocity of light, in agreement with the value noted in the preceding section.

**The Distance and Dimensions of the Sun.** If we multiply the Earth's mean orbital speed in miles by the length of the year in seconds ($60 \times 60 \times 24 \times 365.25$), we shall get the circumference of the Earth's orbit in miles; and if we then divide this by $2\pi$ we shall get the radius of the orbit (considered circular), or the Earth's mean distance from the Sun. This is a number of great importance in astronomy as it reveals the scale of the Solar System and is used as the astronomical unit of distance. The result of the arithmetical calculation proposed above will be in good agreement with the results of several other methods, some of which are believed to be even more reliable. The latest determination of this fundamental number, announced in 1944 by Spencer Jones, the English Astronomer Royal, is

**93,005,000 miles.**

It is believed to be accurate within one part in 10,000, i.e., within about 10,000 miles. The round number, 93 million miles or 150 million kilometers, is worth remembering.

Sound travels in air at a speed of a fifth of a mile a second; but if interplanetary space were filled with air and an explosion occurred on the Sun that was loud enough for the sound to reach the Earth, it would be fifteen years before we could hear it. Light, which has the greatest of known velocities, travels the distance in a little over eight minutes.

The mean apparent diameter of the Sun being half a degree, or, more exactly, 1920″, its real diameter is $\frac{1920 \times 93{,}005{,}000}{206265}$ or 866,000 miles. This is 109.3 times the diameter of the Earth, and since the volumes of spheres are proportional to the cubes of their diameters, the volume of the Sun is $109.3^3$, or about 1,300,000, times that of the Earth. Thus, great though the Earth is, it fills less than a millionth of the space occupied by the Sun.

**The Form of the Earth's Orbit.** As long ago as 120 B.C. it was noticed by Hipparchus that the time occupied by the Sun in its apparent motion from the vernal to the autumnal equinox was 186 days, whereas to go from the autumnal to the vernal required only 179; and as it was contrary to his sense of the fitness of things that a *heavenly* body should move otherwise than with uniform speed in a *perfect* curve (i.e., a circle) he inferred that the Earth was placed eccentrically within the orbit of the Sun. The unequal division of the circle by the straight line passing through the Earth and the equinoxes explained satisfactorily the observed difference of seven days, as may be seen in Figure 78.

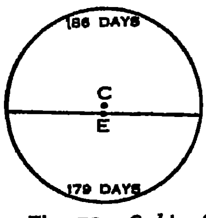

Fig. 78. *Orbit of the Sun According to Hipparchus.*

The investigations of Kepler, seventeen centuries after Hipparchus, showed that the orbit of each of the planets was an ellipse instead of a circle, that the Sun occupied the focus of the ellipse, and that the motion of the planet was not uniform; and Newton proved that these facts were necessary consequences of the attraction of the Sun according to the Law of Gravitation. Observational evidence of the form of the Earth's orbit may be obtained by measuring the apparent diameter of the Sun in different longitudes, for, there being no reason to suppose that the Sun's real diameter changes with the position of the Earth, any apparent change in its diameter must be attributed to a variation of the distance of the Earth.

The form of the Earth's orbit is shown in Figure 79, which was constructed as follows: The Sun's center is represented by the black dot *S*,

Table 3

| Date | Sun's Longitude, $\lambda$ | Apparent Semidiameter, $s$ | $\frac{10{,}000}{s}$ |
|---|---|---|---|
| January 1 . . . . . . . . . . . . | 280° | 978″ | 10.20 |
| February 1 . . . . . . . . . . . . | 311 | 975 | 10.25 |
| March 1 . . . . . . . . . . . . | 340 | 970 | 10.30 |
| April 1 . . . . . . . . . . . . | 11 | 962 | 10.38 |
| May 1 . . . . . . . . . . . . | 40 | 954 | 10.47 |
| June 1 . . . . . . . . . . . . | 70 | 948 | 10.54 |
| July 1 . . . . . . . . . . . . | 99 | 945 | 10.58 |
| August 1 . . . . . . . . . . . . | 128 | 947 | 10.55 |
| September 1 . . . . . . . . . . . . | 158 | 953 | 10.48 |
| October 1 . . . . . . . . . . . . | 187 | 960 | 10.41 |
| November 1 . . . . . . . . . . . . | 218 | 969 | 10.31 |
| December 1 . . . . . . . . . . . . | 248 | 975 | 10.25 |

and the line $S\Upsilon$ is taken as the direction of the vernal equinox. From $S$ are drawn lines which make with the line $S\Upsilon$ angles equal to the heliocentric longitude of the Earth at the first of each month, which is just 180° different from the geocentric longitude of the Sun given in Column 2 of Table 3.

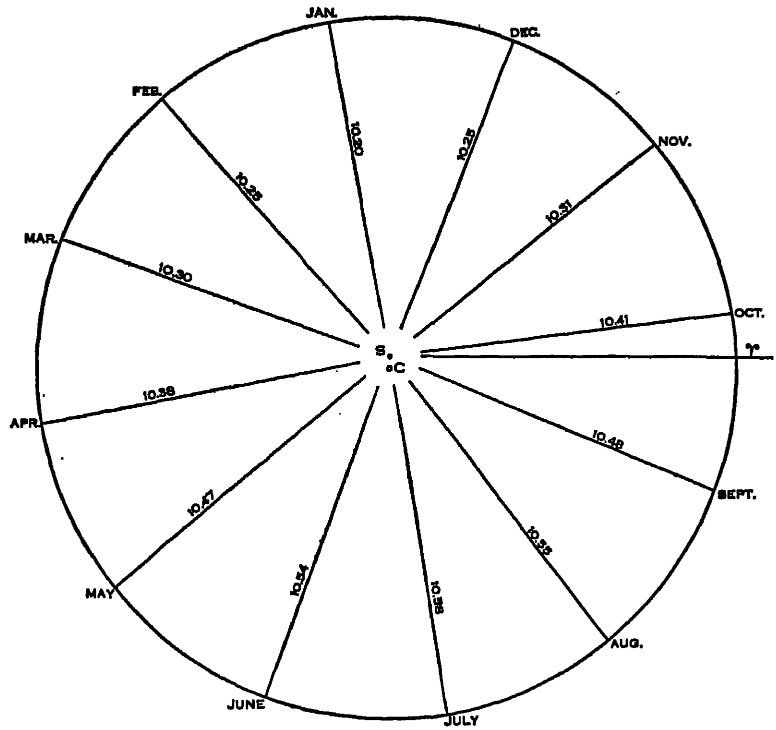


Fig. 79. *True Form of the Earth's Orbit.*

Column 3 of the table gives the apparent semidiameter of the Sun for the same date, and Column 4 gives the result of dividing 10,000, an arbitrarily chosen number, by the numbers in Column 3. Since the apparent diameter increases as the distance diminishes, the numbers in Column 4 are therefore proportional to the distance from the Sun to the Earth. The radiating lines in Figure 79 are cut off to lengths proportional to the numbers in Column 4, and the curve drawn through their extremities therefore has the form of the Earth's orbit. It is so nearly circular that the eye cannot distinguish it from a circle, but its center, which is at the point $C$, is very obviously not at the Sun. The appropriate mathematical treatment of the relation between the Earth's distance and its heliocentric longitude shows that the orbit is an ellipse having the Sun at one focus.

**Definitions Relating to the Earth's Orbit.** An ellipse may be defined as a curve, every point of which is so situated that the sum of its distances from two points within, called the **foci**, is a constant. This definition affords a simple means of drawing an ellipse, which is illustrated in Figure 80. Two pins are thrust through the drawing paper into a board, a loop of thread is tied loosely around them, and the thread is then kept stretched tight by a pencil while the pencil is drawn around the pins. The sum of the distances of the pencil from the pins is thus kept constant, being equal to the whole length of the loop minus the distance between the pins, and so the pencil traces an ellipse which has the pins as foci.

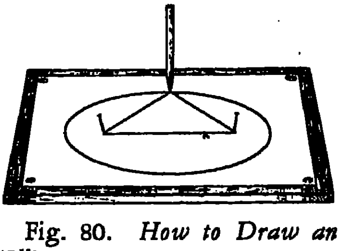

Fig. 80. *How to Draw an Ellipse.*

In Figure 81, the curve represents an ellipse of which the foci are at *F* and *S*. The point *C*, midway between them, is called the **center**. The longest diameter, *AP*, which is the one that passes through the foci, is the **major axis**, and the shortest diameter, *BD*, which is at right angles to the major axis, is the **minor axis**. The semimajor axis, *CP*, is often denoted by the letter $a$, the semiminor axis by $b$, and the distance, *CS*, from center to focus, by $c$. The **eccentricity** of an ellipse is defined as the ratio

$$e = \frac{c}{a}$$

or the ratio of the distance between the foci to the major axis. The eccentricity of an ellipse is never greater than 1, and the less the eccentricity the more nearly circular is the ellipse. A circle is an ellipse of eccentricity zero. The eccentricity of the Earth's orbit is 0.016.

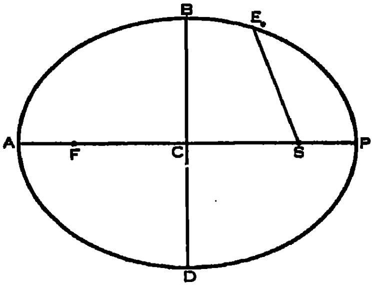


Fig. 81. *An Ellipse.*

Definitions relating to the Earth's orbit may perhaps be better illustrated in Figure 81, in which the eccentricity is exaggerated, than in the more accurate Figure 79. Since the Sun is at one focus, let it be represented by *S*. The point of the orbit nearest the Sun, which is the end *P* of the major axis, is called the **perihelion**; the most distant point, *A*, the **aphelion**. As may be seen in Figure 79, the Earth is at its perihelion about January 1 and at its aphelion about July 1.

Either the perihelion or the aphelion is sometimes referred to as an **apse**, and the line of indefinite length that passes through them and through the foci, of which the major axis is a segment, is known as the **line of apsides.**

The line *SE*, connecting the Sun with the Earth at any point of its orbit, is called the Earth's **radius vector**, and the angle *PSE*, made by the radius vector with the line of apsides, is its **true anomaly.**

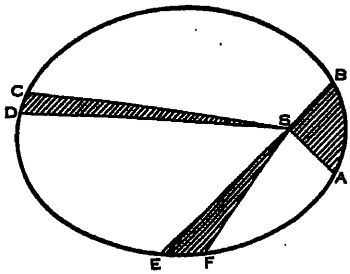


Fig. 82. *The Law of Areas.*

**Variation of the Earth's Speed. The Law of Areas.** The orbital speed of the Earth is not uniform, but varies in accordance with a famous principle which was discovered by Kepler and which is known as the **law of areas.** This law states: *The radius vector of the Earth passes over equal areas in equal intervals of time.* If the Earth moves from *A* to *B* (Figure 82) in a given length of time, say a week, then the distances *CD* and *EF*, over which it passes in a week when in other parts of its orbit, are such that the areas *ASB*, *CSD*, and *ESF* are all equal. Thus, the orbital speed is greatest at perihelion and least at aphelion.

The **angular velocity** of the Earth, which is the rate of increase of its heliocentric longitude or of its true anomaly, is related in a simple manner to the length of the radius vector. The area described by the radius vector in a short interval of time is nearly equal to that of the triangle bounded by the two limiting radii vectores and their chord. Let $\theta$ be the angle *CSD* in the slender triangle formed by the radii vectores *CS* and *DS*. If *DS* is taken as the base of this triangle, its altitude is $CS \sin \theta$ and its area is $\frac{1}{2}\, DS \times CS \sin \theta$. If the interval is very short, say one second (the Earth will have traveled only eighteen miles in that time), the sine of $\theta$ will be practically equal to the value of $\theta$ in radians, and the two radii vectores will be practically equal in length. Call their length $r$. The area passed over in a second is then equal to $\frac{1}{2} r^2\theta$. Since, according to the law of areas, this quantity is constant for all parts of the orbit, we have the general principle, *The product of the square of the radius vector and the angular velocity is a constant.*

The law of areas makes it possible to compute the length of the radius vector and the value of the true anomaly when the time that has elapsed since perihelion passage is known; but the rigorous solution of this problem, which is known as **Kepler's problem,** involves mathematical principles of some difficulty.

**The Seasons and the Climatic Zones.** With the exception of the internal heat of the Earth, which has but slight effect at the surface, our sole appreciable source of heat and light is the Sun. If this supply were permanently cut off from the whole Earth, as on the rare occasions of total

eclipses of the Sun it is cut off for a few minutes from a small region, the temperature would fall very rapidly and all activity and life would cease within a few days. The question is sometimes asked, Why do we not have the warmest weather in January, when the Earth is nearest the Sun? The fact is that the Earth as a whole does receive the most heat in January, but its orbit is so nearly circular that the difference between the perihelion and aphelion distances is too small—about 3,000,000 miles—compared with the mean distance of 93,000,000 miles, to make any obvious difference in the temperature.

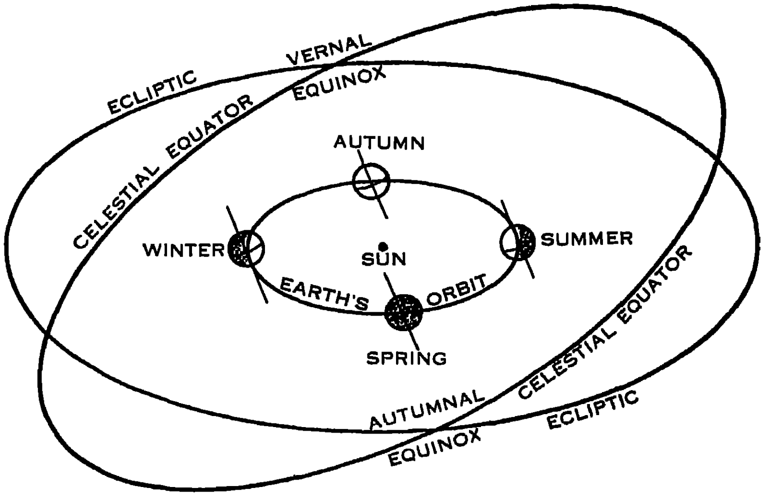


Fig. 83. *The Seasons.*

The changes of season which the Earth experiences are due mainly to the fact that the axis about which the planet rotates is not placed at right angles to the plane in which it revolves. It will be remembered that the ecliptic and celestial equator are the traces upon the celestial sphere of the planes of the Earth's orbit and equator, respectively, and that the angle between these circles is 23°5; therefore, since the Earth's axis makes a right angle with the plane of the equator, it must make an angle of 66°5 with the plane of the orbit. Since the equinoxes are, throughout the year, practically fixed among the stars, the axis must remain practically parallel to the position that it had at the beginning of the year. These relations are shown in Figure 83.

The position of the Earth in relation to the rays of the Sun in June is shown in Figure 84, and in December in Figure 85. In the northern hemisphere the weather is warmer in June than in December for two main

reasons. These may be understood by considering the point *O*, which may represent a locality somewhere in the United States. The first reason is that in June the rotation of the Earth keeps *O* a longer time in the sunlight than in the shadow, while in December these conditions are reversed.

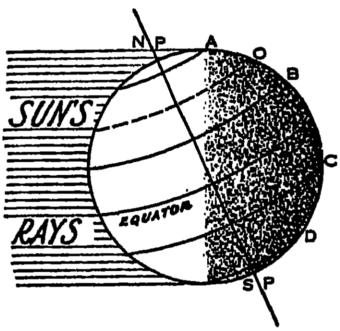


Fig. 84. *The Earth in June.*

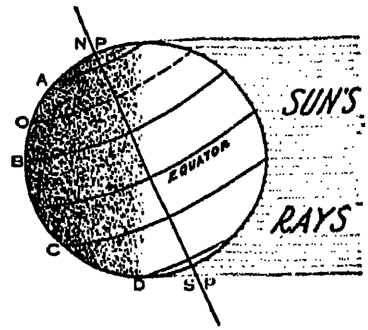


Fig. 85. *The Earth in December.*

In summer, therefore, more heat is received from the Sun every twenty-four hours than can be radiated away, and so the temperature continues to rise until some weeks after the summer solstice, when the loss of heat equals the inflow. The second reason is that, in June, the rays of sunlight fall more nearly normally upon the surface and a beam of sunlight of a given cross section is spread over a smaller area of the soil than the same beam can cover in December, when it falls more obliquely. This may readily be seen by noting the arc of the Earth's surface near *O* that is included between two of the parallel sun rays in the two figures. A point in the southern hemisphere, at the same distance from the equator as *O*, has similar changes of season, but at opposite times of year, the warm weather occurring in January and the cold in July.

The existence of climatic zones is due to the same cause as the change of seasons. The circles *A* and *D*, which are the parallels of latitude 66°.5 from the equator, are known respectively as the **arctic** and **antarctic circles**, and enclose the **frigid zones**, at every point of which, on at least one day of the year, the Sun does not set and at the opposite time of year does not rise. The parallels *B* and *C*, 23°.5 from the equator, are the **tropics of Cancer** and **Capricorn**, between which lies the **torrid zone**, where, at some time of the year, the Sun may be seen at the zenith. Between the torrid and the frigid zones are the **temperate zones.**

**The Calendar.** The solar day as a unit of time has already been discussed; it was seen that this unit was divided artificially into hours,

minutes, and seconds, subdivisions that are commensurable with the day. For the designation of long periods of time, however, the day is inconveniently short, and for a longer unit we naturally adopt the interval between successive recurrences of a given season. Since the seasons are determined by the position of the Earth's axis with reference to the Sun, which also determines the position of the equinoxes, the natural unit thus forced upon us is the tropical year, defined as the interval between successive arrivals of the Sun at the vernal equinox. Its length is very nearly 365.24219 mean solar days, but the fraction is not exactly known, and probably the exact value could not be expressed by a finite number of digits following the decimal point. This circumstance, the incommensurability of the day and the year, makes the measurement of time more complicated than other kinds of measurement, such as that of length, weight, or value, in which the larger units are defined arbitrarily so as to contain an exact number of the smaller, a foot, for example, being exactly twelve inches, and a dollar exactly one hundred cents.

The calendar, or system of keeping account of time over long intervals, is in most nations involved with the celebration of religious festivals. In Christian countries, the calendar now used is that known as the Gregorian, having been made the official calendar of the Roman Catholic Church by Pope Gregory XIII in 1582. This calendar superseded the Julian calendar, instituted in Rome by Julius Caesar. The ancient Romans had used a system of time-reckoning in which the principal unit was the lunar month, and since the year does not contain an integral number of months, they were obliged frequently to insert an "intercalary" month to keep the seasons in their places. By the advice of the astronomer Sosigenes, Caesar decreed that the Roman year should consist ordinarily of 365 days, but that every fourth year should contain 366, an arrangement which would work perfectly if the tropical year contained exactly 365.25 days, because the quarter of a day would accumulate to exactly one day in four years. The tropical year, however, is really nearly 0.008 of a day shorter than this, and so in the course of a thousand years the Julian calendar falls into error by nearly eight days. In 1582 the accumulated error was about thirteen days; but instead of shifting back to the time of Julius Caesar (45 B.C.), Gregory made the date of the vernal equinox the same as in the year A.D. 325, the year of the celebrated Council of Nicaea which decided upon the method of reckoning Easter as well as many other abstruse matters. He accordingly dropped only ten days from the Julian reckoning,

and decreed that the rule of adding an extra day every fourth year should be followed *except* in the case of those century years whose number is *not* divisible by 400. Thus, A.D. 2000 will be a leap year, but the years 1700, 1800, and 1900 were not. In four hundred years, the Julian calendar had one hundred leap years, but the Gregorian has only ninety-seven. The error of the Gregorian calendar amounts to only about one day in 3000 years.

The change to the Gregorian calendar was not immediately accepted in countries where the Pope's authority was not recognized. In England and her colonies it was made in 1752. We celebrate George Washington's birthday on February 22, but he was born on February 11, 1731 (Old Style). Russia, Greece, and other countries in which the Eastern orthodox church held sway adhered to the Julian calendar until 1923, when they adopted a leap-year rule which is even more accurate than the Gregorian, namely, that century years shall be leap years only in case their numbers when divided by 9 give a remainder of 2 or 6. The average length of the year thus determined is within three seconds of the true length of the tropical year, while the average Gregorian year is about twenty-four seconds in error. The Eastern and Gregorian calendars will agree, however, until A.D. 2800, when the difference will be one day, the Gregorian year being a leap year and the Eastern an ordinary one.

Non-Christian peoples have calendars of their own, some of which, like the Jewish and Mohammedan calendars, are based on the lunar month. The years are numbered from some historical or legendary event. The Mohammedan era began with the Hegira, or flight of Mohammed, in A.D. 622; the Jews reckon from the traditional date of the creation of the world, which they put in the year 3761 B.C.; and the ancient Romans numbered their years from the legendary date of the founding of Rome (A.U.C.). The Christian era began with the date of the birth of Jesus as inferred in the sixth century by Dionysius Exiguus, but subsequent investigations, based partly on the computed date of an eclipse that occurred at the time of King Herod's death, show that this may be in error, and that, according to our adopted reckoning, Jesus was probably born in the year 4 B.C.

The **Julian period**, which is often used in astronomical calculations, is a period of 7980 years, the least common multiple of three cycles that were much used in Roman chronology and are called the Roman Indiction, the Solar Cycle, and the Lunar Cycle. The current Julian period began January 1, 4713 B.C., on which date all three cycles began together. The name of the period has nothing to do with Julius Caesar, but was applied in 1582 by its inventor, Joseph Scaliger, in honor of his father. In using this device, it is customary to indicate dates by the total number of days that have elapsed since the beginning of the period, no reference being made to years at all. The **Julian Day** (abbreviated by J.D.) thus determined is understood by every astronomer, regardless of his nationality or religion, and the system is especially useful for obtaining the interval in days between two observations, such as those of the brightness of a variable star.

**The Months and the Days of the Week.** Two serious faults of both the Julian and the Gregorian calendar are the irregularity of the lengths of the months and their absurd nomenclature. Both may be traced directly to the changes made by the Caesars. Julius Caesar, in his reform of the Roman calendar, changed the

beginning of the year from March to January, and established the simple system by which the odd months should each have thirty-one days and the even ones thirty, except the second, which was to have thirty in leap years only and twenty-nine in common years. However, he introduced confusion in the names of the months by appropriating the month Quintilis to himself, for which reason we call it July, and retaining for the months to which they no longer applied, the numerical names Sextilis, September, etc., so that the name September, for instance, which means seventh, has for two thousand years been applied to the ninth month. After Julius came Augustus, who, in correcting an error that had been made by the pontiffs in carrying out the Julian decree, named the month Sextilis for himself and, to make it as long as that of Julius, stole another day from February and added it to August.

The origin of the week is not known with certainty, but the names of the days are almost certainly of astronomical origin. The ancient Chaldeans named the hours of the day for the seven heavenly bodies which they noticed moving in the zodiac, in what was then supposed to be the order of their distance from the Earth, viz., Saturn, Jupiter, Mars, the Sun, Venus, Mercury, the Moon. The day was then named identically with its first hour. Since in naming the twenty-four hours the seven names were used three times with three names over, the order of the days became that of Saturn, the Sun, the Moon, Mars, Mercury, Jupiter, Venus. Modern names for the days are derived from the corresponding words in Latin or Saxon.

**Proposed Calendar Reform.** Certain international organizations are hopefully endeavoring to overcome the inertia of ancient custom and to produce a more orderly arrangement of the calendar year. They proposed to place one day of each year and the additional day of leap year outside any month or week, to designate these days by special names such as Year Day and Leap Day, and to divide the remaining 364 days into equal months or quarters. In the proposed **International Fixed Calendar** of 13 identical months, each month would contain exactly four weeks, and any given day of the month would fall on the same day of the week in all months. The supporters of the 12-month **World Calendar** would divide the 364 days into quarters, identical in regard to distribution of weekdays, and each containing three months of 31, 30, and 30 days, respectively. Of the two propositions, the 13-month calendar has the advantage of uniformity of months, but the 12-month year may more easily be divided into halves and quarters and is less upsetting to established habits.

**Precession.** While it is true that, during a year, the Earth's axis remains very nearly parallel to its initial position, thus causing the seasons, its direction does change slowly and in the course of many years the change becomes conspicuous. The motion of the Earth is somewhat like that of a boy's top, which, set spinning with its axis inclined to the vertical, moves so that the axis slowly describes a vertical cone. The Earth's axis gyrates in a cone while keeping its inclination to the ecliptic practically unchanged, but the gyration is so slow that one complete circuit occupies 25,800 years. This motion of the Earth is called **precession.**

Since the celestial poles are the points where the Earth's axis meets the celestial sphere, this conical motion of the axis results in a circular motion

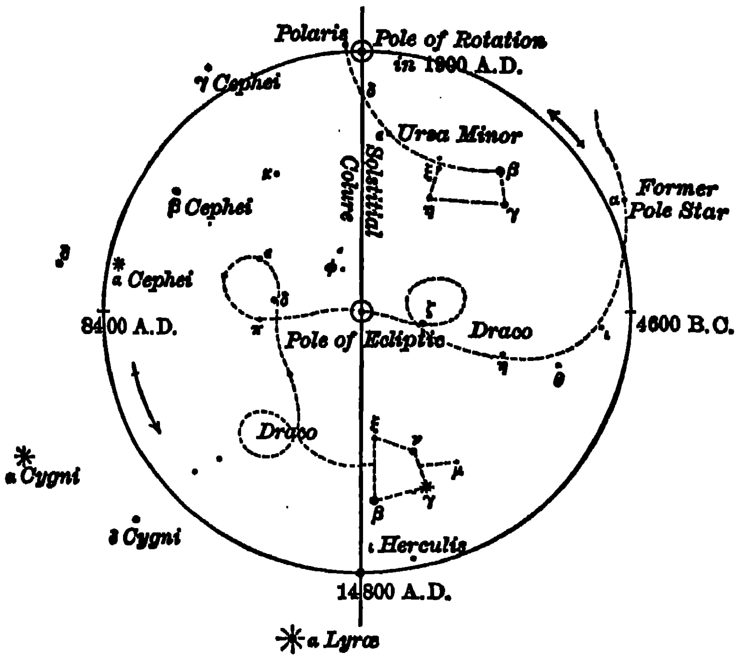


Fig. 86. *Precessional Path of the Celestial Pole.* (*From Young's* Manual of Astronomy.)

Fig. 87. *The North Pole Approaches Polaris. Star trails photographed at the Lick Observatory by J. C. Duncan, 1907, and J. F. Chappell, 1941. Photographs superposed so that the beginnings of the two trails of each star coincide.*

of the poles of rotation around the poles of the ecliptic, which for the north pole is illustrated in Figures 86 and 87. The radius of the circle in Figure 86 is equal to the obliquity of the ecliptic, 23°5. At present, the north "pole star" is $\alpha$ Ursae Minoris, but some 4000 years ago it was $\alpha$ Draconis, and in about 5000 years more it will be $\alpha$ Cephei. Because the circles of perpetual apparition and occultation are at a distance from the poles equal to the observer's latitude, precession makes a marked difference in the constellations that are visible at a given place. For example, in the year 3000 B.C. the Southern Cross, which is now invisible throughout the United States, except in extreme southern Florida and Texas, could be seen north of the site of Quebec.

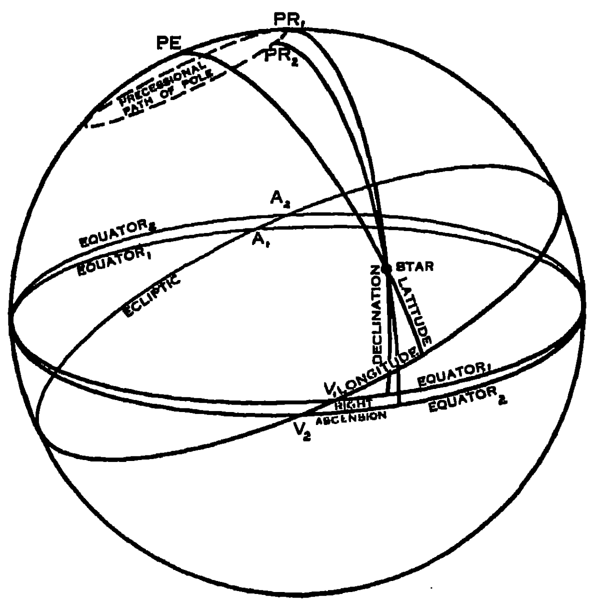


Fig. 88. *Precession.*

Since the celestial equator and the other circles of the equator system of coördinates are fixed with respect to the poles of rotation, precession causes them all to move with respect to the stars. As the equator is a great circle, the equinoxes, or points where it cuts the ecliptic, must remain opposite one another, and any motion of the equator necessitates a motion of the equinoxes. They move westward along the ecliptic at a nearly uniform rate of 50″.2 a year. Right ascension and longitude being reckoned from the vernal equinox and declination from the equator, these three coördinates of a star are all affected by precession; but latitude, which is

counted from the ecliptic, is not so affected. The longitudes continually increase, while the right ascensions and declin affected differently in different parts of the sky. The results of are illustrated in Figure 88. As the pole of rotation moves th short distance from $PR_1$ to $PR_2$, the equator tilts about the poin between the equinoxes so that the vernal equinox moves from and the autumnal from $A_1$ to $A_2$. The right ascension, declin longitude of the star shown are thus increased. The declinatio at the opposite point of the sphere would be diminished, but two coördinates would still be increased.

Precession was discovered in 125 B.C. by Hipparchus by comparing of the year determined by the dates of the heliacal risings of certain the stars could first be seen in the dawn after the Sun had passed them motion) with its length determined by the dates when the shadow of a or gnomon was at its average length. The former is the sidereal year between successive arrivals of the Sun at a given place among the stars length, as we now know, of 365.25636 mean solar days; the latter is year, already defined (page 113), with a length of 365.24219 days. shortness of the tropical year is due to the motion of the equinoxes meet the Sun. The origin of the word precession, which is perhaps a little is found in the fact that the position of the equinox at any epoch *precedes* of earlier epochs as the celestial sphere is carried westward by the diur

Since the time when the signs of the zodiac (page 27) were named fo zodiacal constellations, the equinoxes have moved more than the length and so the vernal equinox, which is even yet called the "first point some writers, lies actually in the constellation of Pisces.

The intersection of the celestial equator with the galactic circle, which as the origin of galactic longitudes (page 27), is of course also displ. precessional motion of the equator. The reasonable suggestion has bee Stroobant that the point of the galactic circle where the equator crosses in future taken as the origin and that its position be defined by means of longitude of the star Deneb, which is situated near the galactic circle an a very small proper motion.

**Nutation.** The cause of precession, as will be explained in (page 247), lies in the attraction of the Sun and Moon for the material at the Earth's equator. The Moon's share of this, if act would cause a conical motion of the Earth's axis around the Moon's orbit instead of the pole of the ecliptic; but, because of sion of the Moon's nodes (page 124), the pole of its orbit its around the pole of the ecliptic in a small circle with a period years. The result is that, superposed upon the precession there is a small oscillation of the pole of rotation to a distanc

9″ from the position that it would occupy if precession acted alone This motion is called **nutation** (nodding), since it causes the pole of rotation to "nod" toward and from the pole of the ecliptic.

**Reduction of Star Places.** Star catalogues have been published which give the right ascensions and declinations of thousands of stars. For the sake of convenience, these coördinates are always given as reckoned for some fixed and designated **epoch**, for example, the beginning of the year 1900. As the observations could not all have been made at that time, it was necessary for the authors of the catalogue to correct the observed

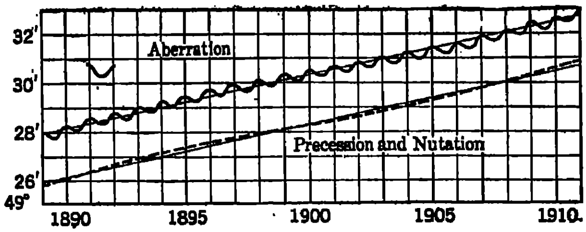


Fig. 89. *Change of Declination of α Persei.* (*Drawing by L. S. Flint.*)

apparent place of each star for the changes due to precession, nutation, and aberration that took place between the time of observation and the epoch of the catalogue. Moreover, the astronomer who uses the catalogue must make similar corrections in order to obtain accurately the place of the star at any subsequent date.

Figure 89 shows graphically the change in the declination of the star $\alpha$ Persei during a period of twenty years. Precession alone would cause the declination to increase at a nearly uniform rate; but superposed upon the straight line that designates this change are the nineteen-year sinuosity of nutation and the annual wave of aberration. For the nearest stars, a slight further complication is added by parallactic motion, and most stars have a small **proper motion** (page 345) of their own besides.

**Cause of the Change of the Equation of Time.** The equation of time has been defined (page 74) as the difference between mean and apparent solar time, the latter being the hour angle of the true Sun and the former the hour angle of the mean sun, which moves with uniform angular speed in the celestial equator. It may equally well be defined as the difference of the right ascensions of the mean and the true suns, for

$$\text{mean time} = \text{sidereal time} - \alpha_m$$
$$\text{app. time} = \text{sidereal time} - \alpha_t$$

where $\alpha_m$ and $\alpha_t$ signify the right ascension of the mean and the true Sun, respectively. Subtracting the second equation from the first gives

$$\text{equation of time} = \alpha_t - \alpha_m.$$

The equation of time varies throughout the year from the combined effect of two principal causes: the varying speed of the Earth due to the eccentricity of its orbit, and the obliquity of the ecliptic. To make the action of the first cause clear, let us define an **intermediary sun** as an imaginary body which moves in the *ecliptic* with the mean speed of the true Sun and which coincides with the true Sun when the Earth is at perihelion. The apparent motion of the true Sun is as if it moved in an ellipse and obeyed the law of areas with respect to the Earth situated at the focus $E$ (Figure 90). Let its position at different times of year be denoted by $S_1$, $S_2$, $S_3$, · · ·. Since, near perihelion, its angular velocity is greater than the mean, the true Sun forges ahead of the intermediary, and when the true Sun is at $S_1$ the intermediary is at $I_1$, the right ascension of the former is the greater, and the equation of time is positive. The angular distance between the two "suns" increases until the speed of the true Sun has fallen to its mean value, which occurs near the end of the minor axis of the orbit, when the equation of time is a maximum; then the true Sun slows up, and at aphelion $A$, where half the circumference and also half the area of the ellipse have been described, the two are together. From aphelion to perihelion the true Sun lags behind the intermediary. The portion of the equation of time which is due to eccentricity is thus zero at perihelion and aphelion, is positive during the first half of the year, and is negative during the second half. Its maximum value is about seven minutes, and its changes are as shown in the dotted curve of Figure 92.

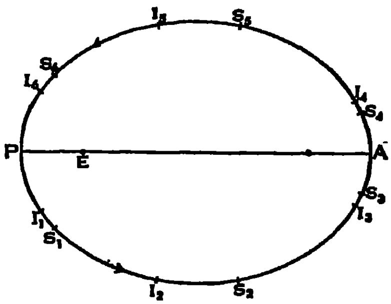

Fig. 90. *Equation of Time Due to Eccentricity.*

To explain the action of the second cause, let the intermediary sun, moving uniformly in the ecliptic, coincide with the mean sun, moving with equal angular speed in the celestial equator, when they are at the vernal equinox (Figure 91). After a brief interval, the intermediary sun will have moved to $I_1$ and the mean sun an equal distance to $M_1$; the longi-

tude of the former will equal the right ascension of the latter; but the right ascension of the intermediary sun, being the projection of its longitude upon the equator, will be less than that of the mean sun, and so the portion of the equation of time that is due to this cause will be negative. At the summer solstice each will have traveled 90° and their right ascensions will again be equal because the sixth hour circle passes through the solstice. Between the summer solstice and the autumnal equinox the intermediary sun will have the greater right ascension because of the convergence of the hour circles as they recede from the equator, but the two will coincide again at the autumnal equinox, each having traveled 180°, though by different routes; and the changes will be repeated in the second half year. (A celestial globe will be of great assistance in visualizing these relations.) The equation of time due to obliquity thus passes through zero four times a year—at the equinoxes and solstices. Its maximum numerical value, which is twice positive and twice negative during the year, is about ten minutes, and its changes are as exhibited in the broken curve in Figure 92.

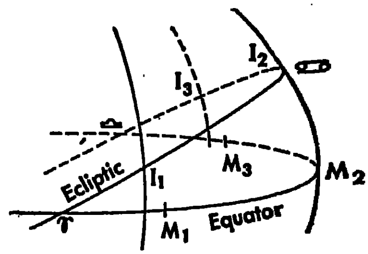


Fig. 91. *Equation of Time Due to Obliquity.*

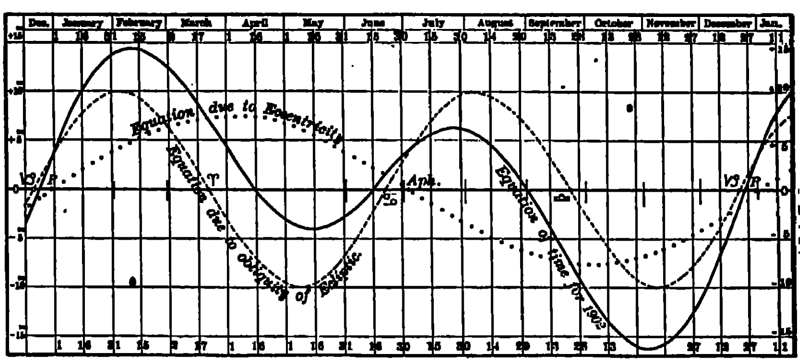


Fig. 92. *The Equation-of-Time Curve. (From Young's* Manual of Astronomy.)

To obtain the combined effect of the two causes for any date, it is necessary simply to add together the ordinates of the two curves. The actual value of the equation of time, obtained in this way, is shown in the continuous curve in Figure 92. It is zero about April 16, June 15, September 1,

and December 25, and reaches a maximum positive value of $+14^m\ 28^s$ on February 11 and a maximum negative value of $-16^m\ 21^s$ on November 3.

Reckoned by apparent time, the length of the morning—i.e., from sunrise to noon—is (the slight daily change of the Sun's declination being neglected) equal to that of the afternoon; but reckoned by mean time, noon does not generally thus occur midway between sunrise and sunset, and the change of the equation of time renders the resulting inequality of morning and afternoon different at different times of year. To make this clear, let $2L$ be the length of the day (from sunrise to sunset); then the apparent time of sunset will be $L$ (P.M.) and that of sunrise, $12^h - L$ (A.M.). Let $r$ and $s$ be the *mean* times of sunrise and sunset, respectively, and $E$ the equation of time; then,

$$r = 12^h - L + E,$$
$$s = L + E.$$

The effect is especially noticeable from about December 10 to January 5, for then $E$ is increasing rapidly while $L$ is changing but little; and so both $r$ and $s$ increase, the afternoon lengthening at the expense of the morning.

NOTE. Authorities differ as to the sign used for the equation of time. The *American Ephemeris* defines it as apparent time minus mean time, thus giving it the opposite sign to that used in this chapter.

## EXERCISES

1. At what times of year is the parallactic displacement of a star at the vernal equinox a maximum? When is its aberrational displacement a maximum?

2. To draw an ellipse with major axis 4 inches and eccentricity 0.5, what must be the distance between the pins? The length of the loop?

*Ans.* $2c = 2$ inches; $a + c = 3$ inches

3. Draw ellipses of eccentricity 0.2, 0.5, 0.9.

4. What is the eccentricity of an ellipse in which the minor axis is three-fifths as long as the major axis?

*Ans.* 4/5

5. What effect on the seasons would be produced if the obliquity of the ecliptic were reduced to zero? If the Earth's axis were made to lie in the plane of the ecliptic?

6. Is precession now increasing or diminishing the declination of the Pleiades? Of Arcturus? Of Antares? Of Deneb? (See star charts.)

7. Is the right ascension of any star being diminished by precession? If so, where is the star?

8. The star charts in this book (Maps 1–8) have the epoch 1950 A.D. What displacement of the equinoxes on the page would be necessary to change to epoch 2000 A.D.?

9. Toward what point of the celestial sphere is the Earth moving on June 22?

*Ans.* The vernal equinox

10. Toward what point of the celestial sphere is the Earth moving on September 22?

# CHAPTER 6

## THE MOON

**General Remarks.** The Moon is the satellite of the Earth and accompanies the latter on its journey around the Sun, at the same time revolving around the planet as a gull may circle about a moving ship. Because it is our nearest neighbor, the Moon is given perhaps an undue amount of attention by mankind, for as seen from any other body than the Earth it would seem insignificant. It resembles the Earth in its spherical shape and solid body and in having its surface diversified by mountains, valleys, and plains, but its diameter is only about a fourth that of the Earth and it has little, if any, atmosphere or water. Some of the important facts about the Moon are:

| | |
|---|---|
| Mean distance from Earth, | 60 Earth-radii, or 239,000 miles |
| Greatest distance, | 253,000 miles |
| Least distance, | 222,000 miles |
| Diameter, | 2163 miles |
| Mass, | 1/81.5 of Earth's mass |
| Mean density, | 3.39 × that of water |
| Surface gravity, | 1/6 that of Earth |
| Mean albedo, | 0.07 |
| Period of revolution (sidereal), | $27\frac{1}{3}$ days |
| Period of rotation, | Exactly the same as that of revolution |

**Apparent Motion of the Moon.** Even the most inattentive observer of the Moon must notice that it rises and sets a little later each day than on the preceding day. The average delay is about fifty-one minutes. This means that it does not keep up with the general diurnal motion of the celestial sphere, but, relative to the stars, has a motion of its own of about half a degree an hour (13° a day) toward the east. By careful noting of

the Moon's position among the stars on successive nights, its apparent path may be determined, and it is found to be very nearly a great circle that lies near the ecliptic, intersecting the latter at an angle of about 5°. This angle is called the **inclination** of the Moon's orbit, and the points of intersection are known as the Moon's **nodes,** the one where the Moon crosses from the south side of the ecliptic to the north being called the **ascending** node and the other the **descending** node.

Observations of the Moon's path among the stars in different years show that it changes markedly, the nodes moving westward along the ecliptic, while the inclination remains about the same. This change, known as the **regression of the nodes,** carries each node completely around the sky in about nineteen years. It is similar to the precessional change of the celestial equator but takes place much more rapidly.

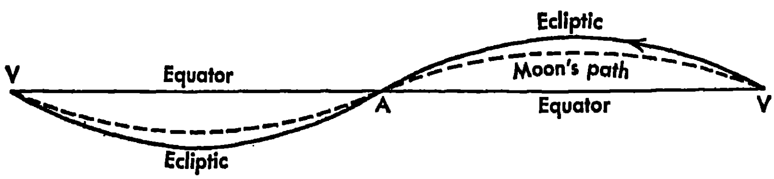


Fig. 93. *Moon's Descending Node at Vernal Equinox.*

The regression of the nodes produces a very noticeable change in the *diurnal* path of the Moon in different parts of the nineteen-year cycle; for when the descending node coincides with the vernal equinox (Figure 93), the Moon's path lies entirely between the ecliptic and the equator, and the maximum declination is only 23.°5 − 5° = 18.°5; but nine years later the Moon's orbit is shifted halfway around so that the nodes are interchanged (Figure 94), the Moon's path lies entirely outside the space bounded by the ecliptic and the equator, and the declination reaches 23.°5 + 5° = 28.°5. The total range of declination, and therefore of meridian altitude, is in the one case from −18.°5 to +18.°5, or 37°, and in the other, from −28.°5 to +28.°5, or 57°.

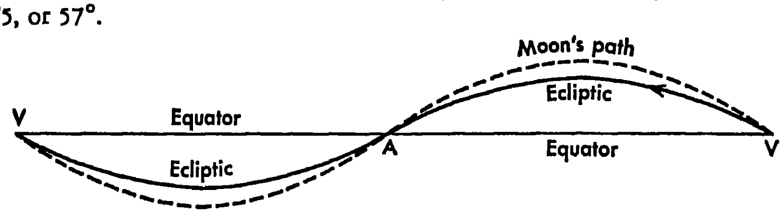


Fig. 94. *Moon's Ascending Node at Vernal Equinox.*

The difference of longitude between the Sun and the Moon is called the Moon's **elongation.** Since the Moon moves among the stars about 13° a day whereas the Sun moves only 1°, the Moon overtakes the Sun about thirteen times a year. When this occurs, the elongation is zero and

the Moon is said to be in **conjunction**; when the elongation is 90° east or west, the Moon is said to be in **quadrature**; when 180°, in **opposition**. Either conjunction or opposition is sometimes referred to as **syzygy**. At syzygy, the Sun, Moon, and Earth are nearly in the same straight line and would be exactly so if the inclination of the Moon's orbit were zero. Actually, the three bodies are in the same straight line only on the rare occasions when syzygy occurs at one of the nodes.

**The Month.** The word *month* is etymologically related to the word moon, and the month as a unit of time had its origin in the use of the Moon as a timekeeper. The months of the calendar, although arbitrarily made unequal in length, are approximately the time of a revolution of the Moon. Astronomers recognize a number of different kinds of month. The **sidereal month** is the interval between two successive arrivals of the Moon at a given apparent place among the stars; it averages 27.32166 mean solar days, but varies about three hours because of the perturbations of the Moon's motion (page 248). The **synodic month** is the time of a revolution with respect to the apparent place of the Sun—that is, from conjunction to conjunction. Suppose the Sun, Moon, and a star at a given moment to be in the same direction from the center of the Earth. After a sidereal month the Moon will have arrived again at the star, but the Sun will in the meantime have moved forward about 27°, and the Moon must travel two days longer to overtake it. The average length of the synodic month is, in fact, 29.53059 days.

If we let $M$ be the number of days in the sidereal month, then $360/M$ will be the average number of degrees passed over by the Moon (relatively to the stars) in a day—its **mean daily motion**. Similarly if $E$ is the number of days in a year, $360°/E$ will be the mean daily motion of the Earth, or, apparently, of the Sun; and if $S$ is the number of days in the synodic month, $360°/S$ will be the mean daily gain of the Moon upon the Sun. Hence,

$$\frac{360}{M} - \frac{360}{E} = \frac{360}{S}$$

or, more simply,

$$\frac{1}{M} - \frac{1}{E} = \frac{1}{S}$$

a relation which expresses the length of the synodic month in terms of the sidereal, or vice versa. The synodic month is the more natural as a unit of time, for in this period the Moon passes through its cycle of phases (page 129).

In computations relating to eclipses of the Sun and Moon, the **nodical month** is useful. It is the time of the Moon's revolution with respect to either node, and averages 27.2122 days in length.

**The Form of the Moon's Geocentric Orbit.** The Moon's apparent diameter varies from 1764″ when it is farthest from the Earth to 2013″ when it is nearest.

These numbers represent the geocentric diameter—that is, the diameter the Moon would appear to have if viewed from the distance of the center of the Earth. From the surface of the Earth, the distance is a little less, the observer being brought nearer the Moon as the Moon approaches the zenith. The resulting increase of the Moon's apparent diameter, called the **augmentation,** may easily be computed and allowed for. When the Moon is rising or setting, the distance of the observer (*MR* or *MS*, Figure 95) is nearly the same as that of the Earth's center *C*; but when the Moon is at the zenith, the observer *O* is nearer it by one Earth-radius, or $\frac{1}{60}$ of the whole distance, than is the Earth's center. The maximum augmentation therefore amounts to $\frac{1}{60}$ of the whole apparent diameter, or about 30″.

The augmentation may seem incredible to most casual observers, for they get just the opposite impression, the Moon seeming to them much larger when rising than when high in the sky. This is an optical illusion which familiarity usually dispels and which it is the province of the psychologist rather than the astronomer to explain.

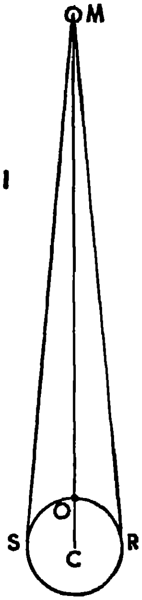

Fig. 95. *Augmentation of the Moon's Diameter.*

By following the method that has already been described (page 107) for determining the form of the Earth's orbit around the Sun, and drawing a "spider," the legs of which have lengths proportional to the reciprocal of the measured diameter and make angles with a fixed line equal to the Moon's longitude, the form of the Moon's geocentric orbit may be determined. Like the Earth's orbit, it turns out to be a nearly circular ellipse, but of greater eccentricity —averaging about 0.056. The point of the lunar orbit that is nearest the Earth is called the **perigee,** and the point most remote the **apogee.** Other terms relating to the orbit have the same significance as in the case of the Earth's orbit.

**The Moon's Distance and Dimensions.** The Moon's distance is determined by its parallax, or by a triangulation from two points of observation. Suppose that two observers *A* and *B* (Figure 96), situated on the same terrestrial meridian but widely separated in latitude, observe the zenith distance of the Moon with meridian circles. The oblateness of the Earth being neglected for simplicity (it can be accurately allowed for), these observations give the angles *ZAM* and *Z′BM*. The angle *ACB* is

the algebraic difference of the latitudes of the observers. In the triangle *ABC*, the sides *CA* and *CB* are radii of the Earth, and from these and the angle *ACB* may be computed trigonometrically the angles *CAB* and *CBA* and the distance *AB*. In the triangle *ABM* the base *AB* is now known, and the angles *BAM* and *ABM* may be found by subtracting *ZAM* + *CAB* and *Z'BM* + *CBA* from 180°; hence *AM* and *BM* may be computed. Finally, in the triangle *CAM*, the sides *CA* and *AM* are known, and the angle *CAM* is the supplement of *ZAM*; hence *CM*, the geocentric distance, may be computed in terms of the radius *CA* or *CB*. The Moon's mean distance is thus found to be 60 Earth-radii, or about 239,000 miles. Its distance at perigee is 222,000 miles, and at apogee 253,000 miles.

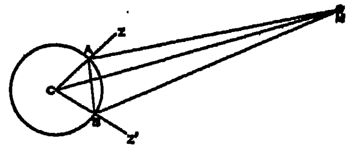

Fig. 96. *Determining the Moon's Distance.*

From the Moon's distance and its apparent diameter its real diameter may be found at once (page 11). It is 2163 miles, a little over a quarter of the Earth's.

**Form of the Moon's Heliocentric Orbit.** Since the Earth moves continually in its orbit around the Sun while the Moon circles around the Earth, the Moon's path around the Sun is a sinuous line, as suggested in Figure 97, where the continuous circle represents the path of the Earth and the dotted line that of the Moon. The proportions of the figure, however, are far from correct; it would not be practicable to represent them correctly for, the Moon's distance being only about 1/400 of the Sun's, if we represent the Earth's orbit by a circle having a radius as great as two feet, the departures of the Moon from that circle would be only about 1/16 of an inch. These departures are so small that the Moon's orbit is everywhere concave to the Sun.

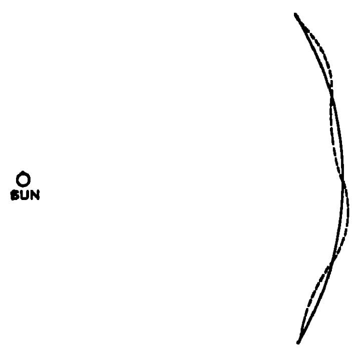


Fig. 97. *The Moon's Heliocentric Path.*

**Mass, Density, and Surface Gravity of the Moon.** As the Moon revolves, its gravitational attraction sways the Earth slightly from the position it would otherwise occupy, the two bodies revolving around their common center of mass (page 240). This monthly motion of the Earth causes a slight but observable parallactic displacement of the nearer

planets, from which the distance of the center of the Earth from the center of mass may be computed.

The center of mass of two homogeneous spheres is a point on the line joining their centers such that the product obtained by multiplying its distance from the center of either sphere by the mass of that sphere is the same as the corresponding product relating to the other sphere. For example, if $m$ and $M$ are the masses of the two spheres $A$ and $B$ (Figure 98), then their center of mass, $C$, is such a point that $m \times AC = M \times BC$. The center of mass of two bodies near the surface of the Earth may be determined as the point about which they balance when joined by a light, straight rod.

Fig. 98. *Center of Mass of Two Spheres.*

The distance of the Earth's center from the center of mass is found to be 2880 miles; hence the latter is about a thousand miles within the surface of the Earth. This distance is approximately 1/82.5 of the distance from the Earth to the Moon, and therefore the Earth's mass must be about 81.5 times the Moon's.

Since the volumes of spheres are as the cubes of their diameters, the Earth's volume is $(\frac{7920}{2163})^3$, or about 49, times the Moon's. The mean density of a body is its mass divided by its volume, and so the Moon's density is 49/81.5 times the Earth's or, since the Earth's density as compared to water is 5.53, that of the Moon is about 3.4.

By the **surface gravity** of a heavenly body is meant its attraction for bodies at its surface. It can easily be computed when the body's mass and radius are known (page 239); in the case of the Moon it proves to be about one-sixth that of the Earth. This means that a body that weighs six pounds on the Earth would weigh but one on the Moon. A given upward force would shoot a projectile six times as high there as here. A ballet dancer who weighs 120 pounds on the Earth would weigh only 20 on the Moon and could leap six times as high.

**The Phases of the Moon.** The most obvious phenomenon shown by the Moon, and one which must have excited the admiration of mankind from the earliest times, is its apparent change of shape from a narrow crescent to a full circle and back to the crescent form. This change of shape, or of **phase**, is due to two circumstances: first, the Moon shines only by reflected sunlight, and second, as it revolves around the Earth, different portions of its sunlit side are presented to our view. This may be understood by a study of Figure 99, or better by holding a tennis ball at arm's length in the light of a single lamp and turning slowly on one's

heel. When the Moon is at conjunction, its unilluminated side is turned toward us and for this reason and also because of the dazzling light of the Sun, the Moon is invisible. The phase is then that of new **moon.** Two or three days later it has moved several degrees east of the Sun and is visible in the western sky soon after sunset, but still only a little of the

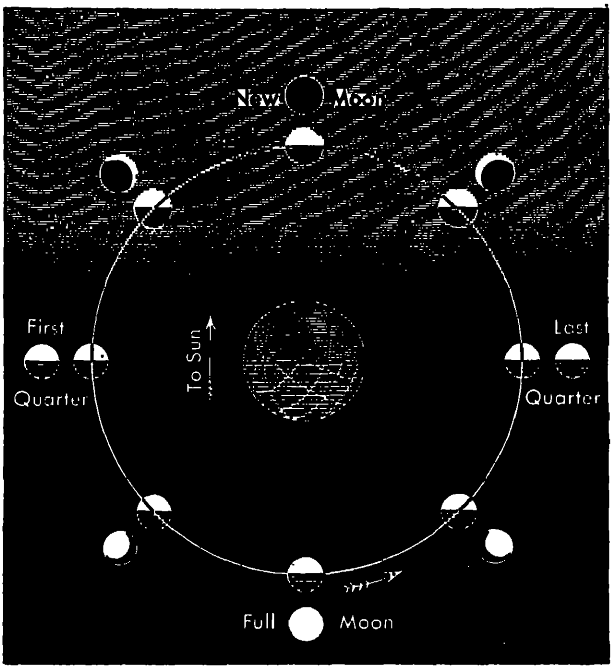


Fig. 99. *Explanation of the Moon's Phases.* (*After Young.*)

illuminated side is presented to us and it appears as a thin crescent. The crescent grows wider on succeeding nights until east quadrature is reached,[1] when we see half of the illuminated half, the Moon appears as a semicircle, and the phase is called **first quarter.** From east quadrature to west quadrature more than a quarter of the surface is visible, and the Moon is said to be **gibbous,** except at opposition, when the illuminated half is turned full upon the Earth, and we have **full moon.** After full moon the succession of phases is reversed, passing through **last quarter** at west quadrature to new moon again at the next conjunction.

The line which separates the dark half of the Moon from the sunlit half—the sunrise or sunset line—is called the **terminator** and always has

[1] More precisely, until the angle Sun-Moon-Earth is just 90°; but this occurs only a few minutes before quadrature.

the form of a semi-ellipse because it is a circle seen edgewise. At its extremities, the cusps of the Moon, it is tangent to the Moon's circular outline, or limb.

Fig. 100. *The Young Moon.* (*After M. A. Orr.*)

From Figure 99 it may be easily seen that the cusps or "horns" of the crescent Moon are always turned away from the Sun, and that therefore they are never pointed downward when the Sun is below the horizon—a fact sometimes disregarded by artists. In northern latitudes, moreover, the visible half of the ecliptic lies south of the zenith, and so the horns of the young Moon, when it is visible after sunset, are always turned upward and toward the left, as in Figure 100.

**Earthshine.** When the Moon is near conjunction and so appears to us as a crescent, an observer on the Moon would see the Earth as a great bright body, about two degrees in diameter, in the gibbous phase. The Earth at that time thus lights up the side of the Moon that is turned away from the Sun; it is often easy to see this earthlit lunar landscape, which is fancifully called the "old Moon in the new Moon's arms."

**Rotation and Librations of the Moon.** Observations of the markings on the Moon's surface show that the Moon rotates on an axis which is

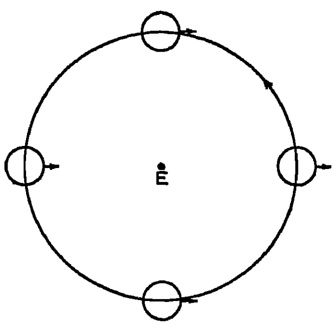


Fig. 101. *Revolution Without Rotation.*

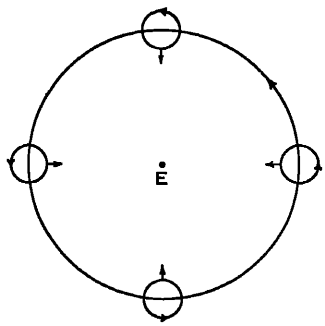


Fig. 102. *Revolution and Rotation in the Same Period.*

so placed that the plane of its equator makes an angle of 6°.5 with the plane of its orbit. The intersection of the two planes always coincides with the line of nodes, and the plane of the ecliptic lies between them, so that the Moon's equator makes an angle of only 1°.5 (6°.5 − 5°) with the plane of the ecliptic. Very remarkably, the rotation and revolution take place in the same direction, from west to east, and *in precisely the same*

*period*, so that one side of the Moon is always turned toward the Earth, and the opposite side, although nearer us than are any of the planets, is an undiscovered country wholly inaccessible to observation.

At first thought this may not seem to be a rotation at all, but a consideration of Figures 101 and 102 will show that it is. Suppose the orbit is circular and that an arrow is erected perpendicularly upon the Moon's surface. If there were no rotation, the arrow would be directed always to the same point among the stars, as in Figure 101; whereas, if it points always to the center of the orbit, as in Figure 102, its direction must sweep over a whole circle and coincide again with its initial direction at the end of one revolution, thus completing a rotation and a revolution in the same time.

The motion of the Moon is not, however, as if it were attached to the Earth by a rigid bar, for it has apparent balancing movements, or **librations**, which render about 18 per cent of its surface alternately visible and invisible. The principal librations are three in number and are known as the librations in latitude and in longitude, and the diurnal libration.

The cause of the libration in latitude is the inclination of the Moon's equator to the plane of its orbit, and is exactly analogous to the cause of the Earth's change of season. Just as the Sun shines alternately over the Earth's north and south poles in the course of a year, so we on the Earth may look over the Moon's poles alternately in the course of a month.

The libration in longitude is due to the fact that, whereas the Moon's rate of rotation is uniform, that of its revolution is variable, obeying the law of areas. Suppose an arrow is erected perpendicularly on the Moon on the line that joins the centers of the Earth and Moon when the latter is at perigee (Figure 103). From the direction of the Earth's center, it will be seen at the center of the Moon's disk. After a quarter of a sidereal month, the Moon's rotation will have changed the direction of the arrow just 90°, but the Moon's direction from the Earth will have changed 96°, so that the arrow will appear a little toward the east of the center of the disk, and we may see a little of the western side of that portion of the Moon which, at perigee, was concealed. After half a month, the radius vector has described half the area of the orbit, the Moon's direction has changed just 180°, and as the arrow has been turned through an equal angle by the uniform rotation, it again points to the

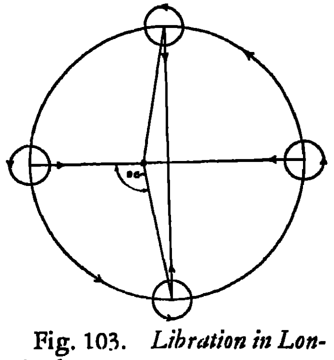

Fig. 103. *Libration in Longitude.*

center of the Earth. At the end of the third quarter, the rotation has proceeded 270° and the revolution only 264°, and we then see a little of the eastern rim of the back of the Moon. At the end of the month, the initial conditions are restored.

The diurnal libration is due to the change of the point of view of the observer as he is carried around by the Earth's rotation. When the Moon is on the horizon, the observer's line of sight makes an angle of about 1° with the line of centers, and this enables him to see about 1° beyond what would be the limb if the Moon were in the zenith.

Because of the small inclination of the Moon's equator to the ecliptic, it can have no perceptible change of season, and over the whole surface of the Moon the day is always nearly equal to the night, each being about two weeks long.

**Physical Libration.** The Moon is not quite spherical; the diameter which lies nearly in line with the Earth is longest, the equatorial diameter at right angles to this is about a third of a mile shorter, and the polar diameter is about a mile shorter still. There is thus a slight protuberance extending toward the Earth, and the librations in longitude and latitude cause the direction of the Earth's gravitational attraction for this protuberance to change slightly, producing a real but very slight irregularity in the Moon's rotation which is known as the **physical libration.** It amounts to only about a mile on the Moon's surface.

**Absence of an Atmosphere.** It is certain that the Moon does not possess an atmosphere which is at all comparable to that of the Earth. No clouds float over its surface, and the mountains and craters show no erosion by weather. More conclusive still, when the Moon passes between us and a star, or **occults** it, the star disappears and reappears with extreme suddenness and there is no evidence of any refraction of light at the Moon's limb. The refraction of the Earth's atmosphere deviates a horizontal ray by more than half a degree (page 66), and if the Moon were surrounded by gases of equal density a ray of starlight which passed the surface tangentially would be deviated by twice this amount, being refracted both on entering and on leaving the lunar atmosphere. Even a rare atmosphere should bend starlight slightly, delaying the disappearance and hastening the reappearance of the

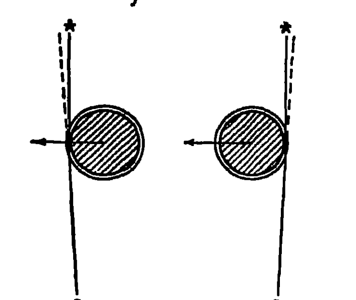


Fig. 104. *Effect of a Lunar Atmosphere.*

star as illustrated in Figure 104; but the observed duration of an occultation is always equal within a fraction of a second to that which may be computed from the known diameter of the Moon and its rate of motion. From this fact it is certain that the Moon's atmosphere, if any exists, has a refractive effect less than 1/2000 that of air. Since gases consist of molecules in rapid motion, it is doubtful that the Moon, with its low superficial gravity, could retain an atmosphere if it ever had one.

If there is no atmosphere on the Moon, no liquid water can be there, for it would evaporate in the sunshine of the long lunar day and from an atmosphere of water vapor. Water might, perhaps, exist there in the form of ice or snow in crevices where sunshine enters but little.

**Lunar Topography.** To the unaided eye there appear upon the bright surface of the Moon only certain vague, dark areas which suggest to the fancy such objects as the round face of a man, the profile of a lady, or a very long-eared donkey. In a telescope of low magnifying power these dark areas appear as smooth gray plains, so uniform that Galileo and his contemporaries thought that they might be seas and accordingly called them **maria,** a name still used by selenographers. Other parts of the lunar surface, even in a telescope such as Galileo's with its power of only thirty diameters, appear to be very rugged, with numerous mountains and other formations which, under favorable illumination by the Sun, cast long black shadows. With larger telescopes it is found that the smoothness of the maria is only relative, and that they too present a wealth of detail. The principal features of the lunar landscape are the maria, mountains, craters, rills, and rays.

The nomenclature used is based mainly on that of the Italian astronomer Riccioli, who mapped the Moon in 1651, naming the craters of the northern hemisphere for astronomers and philosophers of the ancient world and those of the southern hemisphere for his contemporaries, and giving the maria fanciful names—those of the western hemisphere mainly of pleasant connotation such as the Maria Tranquillitatis and Serenitatis; those of the eastern hemisphere more somber names such as Mare Nubium and Oceanus Procellarum. A few of the mountain ranges retain names given them earlier by Hevelius of Dantzig, who took them from terrestrial ranges. The most prominent are the Apennines, Caucasus, and Alps, which are situated north of the center of the visible disk.

The mountains are seen mostly in chains or groups, although on the surface of the Mare Imbrium are several isolated peaks of which a conspicuous one is Pico, about 8000 feet high. The Apennine range consists

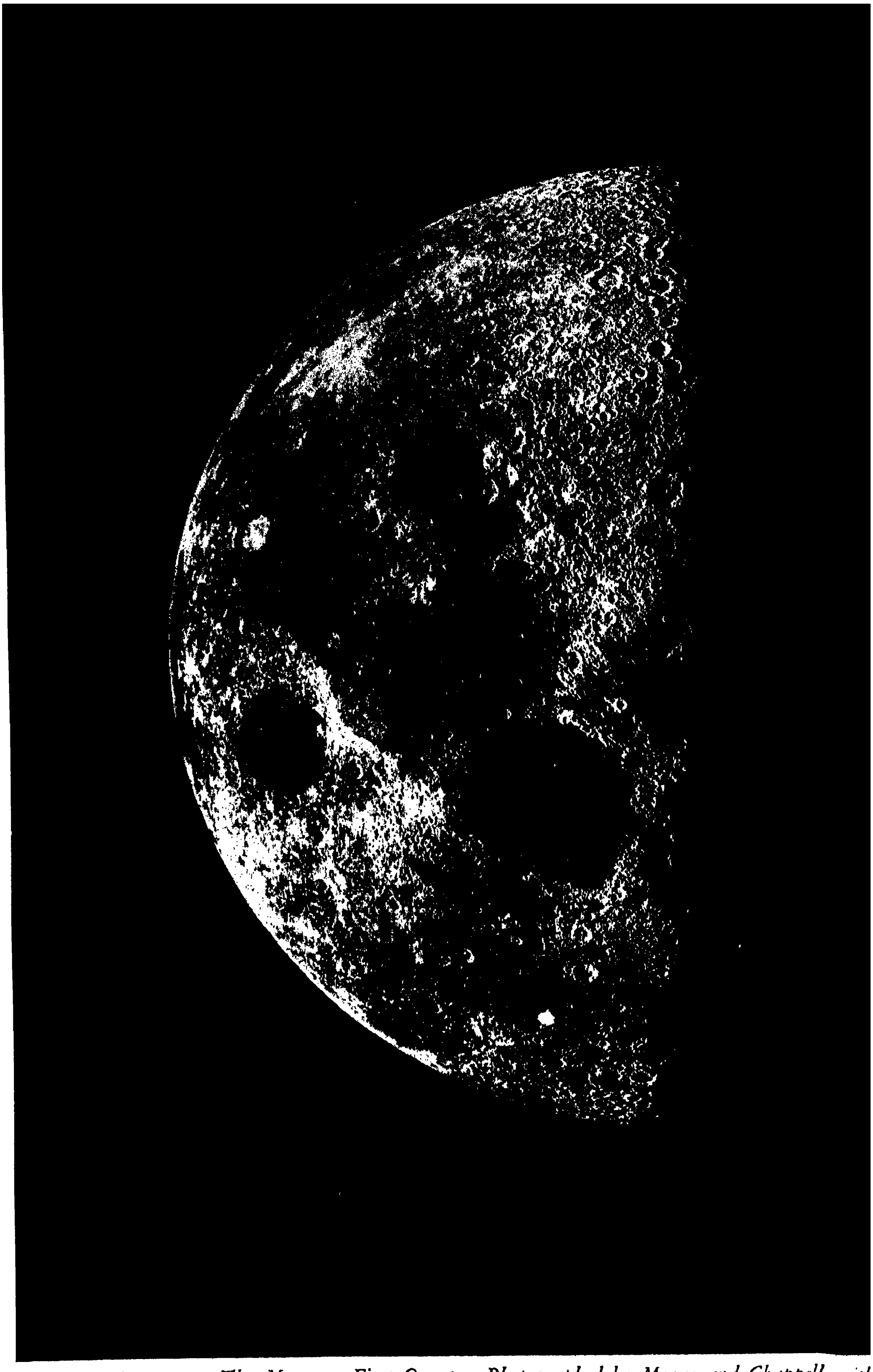

Fig. 105. *The Moon at First Quarter, Photographed by Moore and Chappell with 36-inch Lick Refractor, 1937.*

of about 3000 peaks and extends about 400 miles. Its loftiest peaks rise about 18,000 feet above the Mare Imbrium. The Alps are noteworthy for a great gorge 80 miles long and 4 to 6 miles wide. All these features are best seen near the time of the first or last quarter. The highest lunar mountains, which rise to 25,000 feet or more, are situated in ranges near the limit of the visible side of the Moon, where they are seen in profile when the conditions of libration are favorable.

The heights of lunar mountains are determined from the length of their shadows and a knowledge of the altitude of the Sun as it would appear from that point of the Moon, which can be obtained from the Moon's heliocentric position; also by a method used by Galileo, which depends upon the fact that the Sun rises earlier and sets later on a high peak than on lower mountains or the surrounding valleys. This may be understood by reference to Figure 107. Suppose the Moon were perfectly smooth except for a mountain of which the base is at *B* and peak at *P*, and suppose the Sun were just shining on the peak. To an observer on the Earth, the Moon's terminator would appear at *T*, where the sunlight is tangent to the lunar surface, while the illuminated peak would appear as a detached point of light on the dark side of the Moon—not an unusual appearance. The distance *TP* may be found with the micrometer in terms of the Moon's apparent diameter, which gives two sides, *TP* and the radius *TC*, of a right triangle, whence the hypotenuse, which is the sum of the Moon's radius and the height of the mountain, may be computed. Elevations on the Moon cannot be so definitely stated as those on the Earth, because the Moon has no sea level to which to refer them.

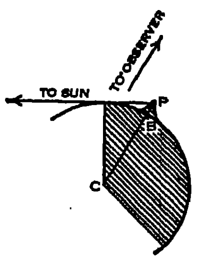


Fig. 107. *Measuring the Height of a Lunar Mountain.*

The **rills** (German *Rille,* a ditch or furrow) are narrow crevices from ten to over three hundred miles long and less than two miles wide. Their bottoms are generally invisible and their depth is consequently unknown. They are inconspicuous because of their narrowness, but over a thousand have been mapped. A remarkable example is the great Serpentine Cleft near the crater Herodotus.

The **rays** are narrow streaks which are lighter in color than their surroundings and which radiate from certain prominent craters, notably Tycho, Copernicus, and Kepler. They are inconspicuous or invisible under a low Sun, but stand out very prominently near the time of full Moon. They can be neither ridges nor furrows, for they cast no shadows. The rays that center upon Tycho extend many hundreds of miles across the surface of the Moon, regardless of craters, mountains, and seas.

The **craters** are by far the most numerous of the lunar formations.

Upward of 32,000 are shown on a map published in 1878 by Schmidt of Athens, and many others may be seen with present-day telescopes and on photographs made with them. The craters range in size from such "walled plains" as Clavius and Grimaldi, either of which is of greater area than the

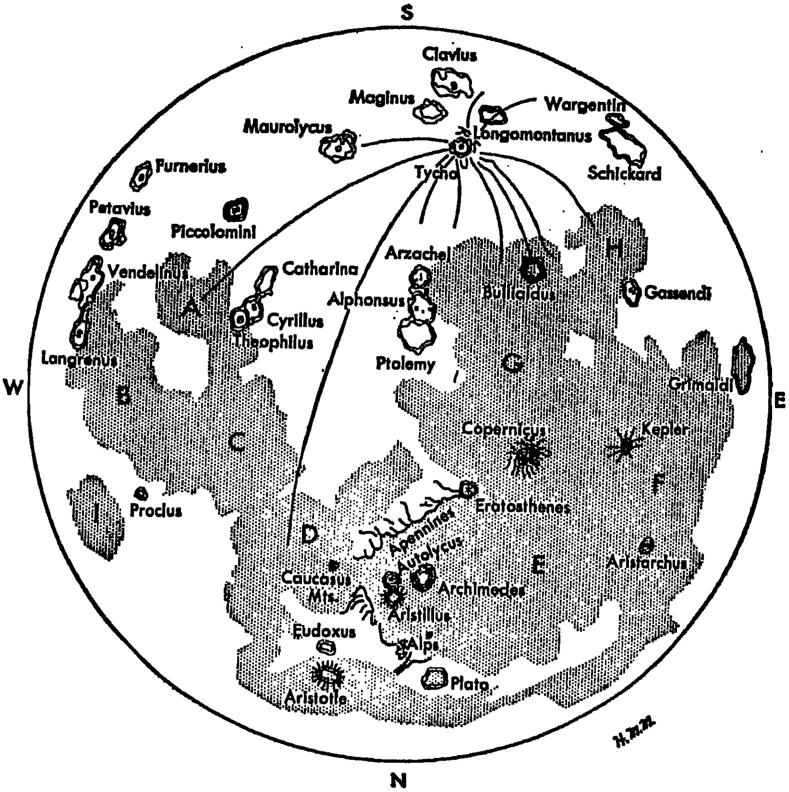


Fig. 108. *Map of the Moon, drawn by Helen M. Mitchell.* A, *Mare Nectaris; B, Mare Fecunditatis; C, Mare Tranquillitatis; D, Mare Serenitatis; E, Mare Imbrium; F, Oceanus Procellarum; G, Mare Nubium; H, Mare Humorum; I, Mare Crisium.*

State of Maryland, down to the smallest dot discernible in the greatest telescope, which is about a tenth of a mile across. Several are so large that, from the centers of their floors, their walls would be concealed by the spherical surface of the Moon. They are depressions, the floor being usually lower than the surrounding plain, and are roughly circular. Many contain lofty mountains which rise from their floors, and many large craters have numerous smaller ones superposed upon their floors or walls.

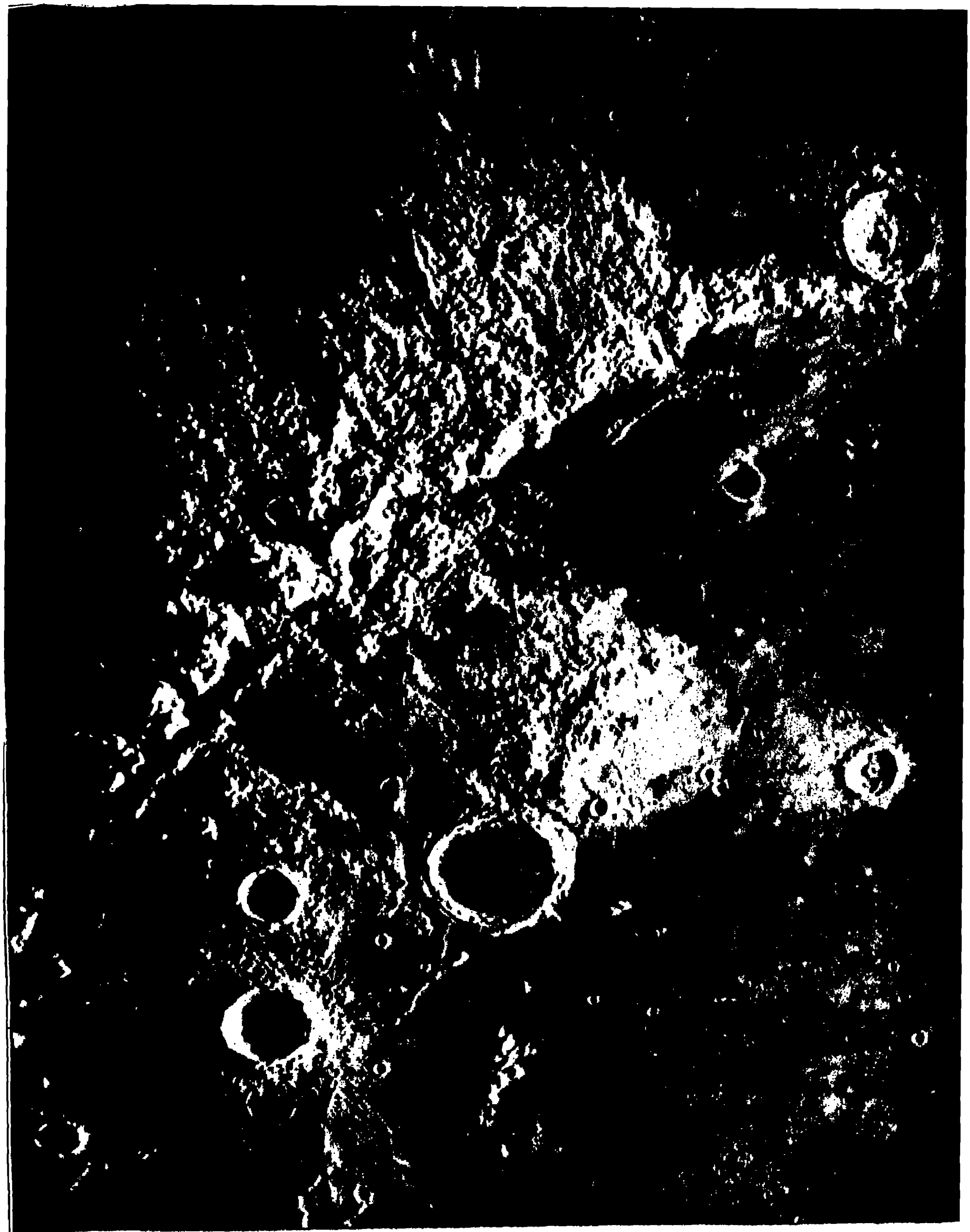

Fig. 109. *Sunset on Archimedes and the Lunar Apennines, from Photograph by Moore and Chappell with the 36-inch Lick Refractor, 1938.*

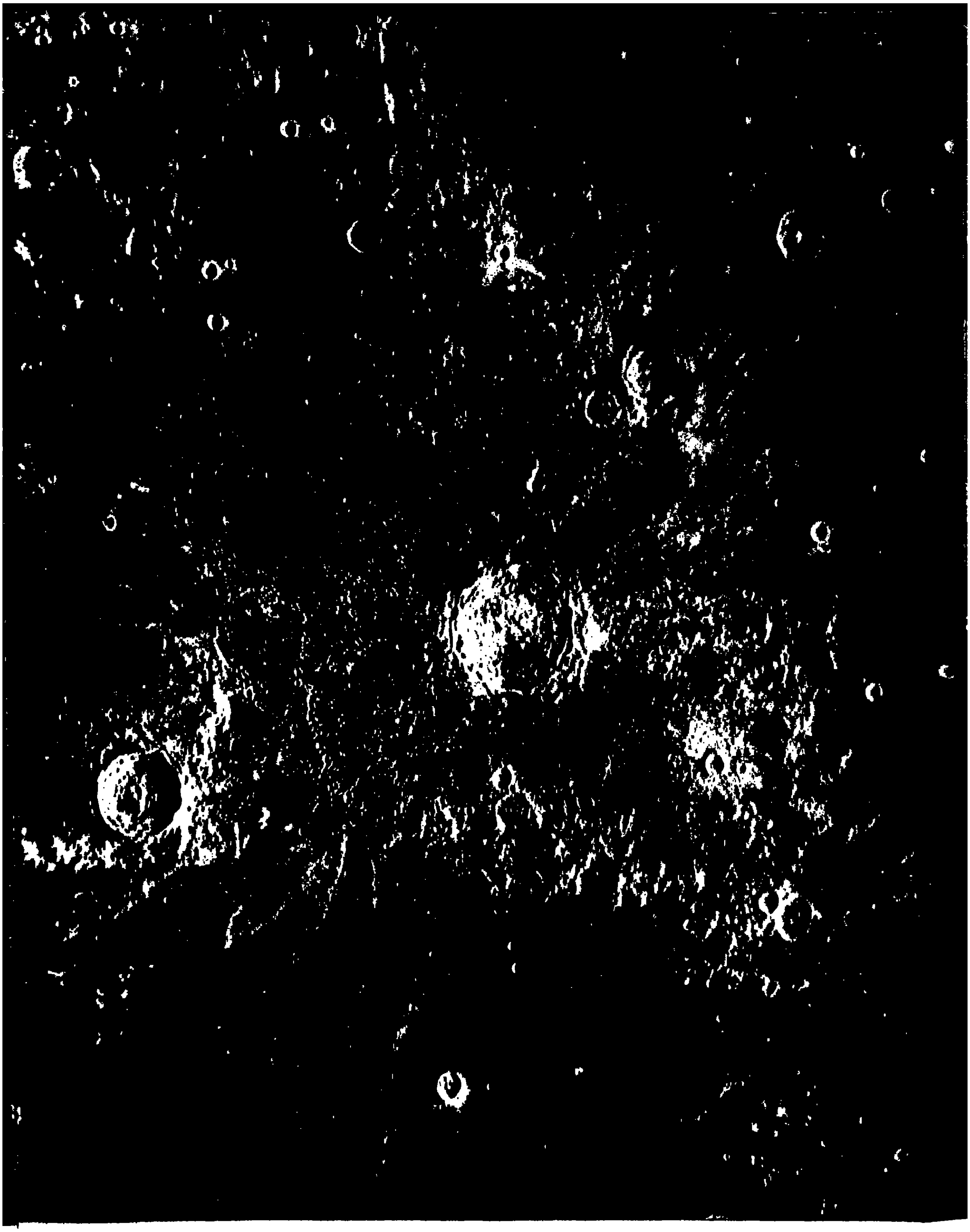

Fig. 110. *Afternoon on Copernicus and the Carpathians, from Photograph by Moore and Chappell with the 36-inch Lick Refractor, 1938.*

Some of the more important craters, with their diameters in miles, are listed below:

Proclus, 18; very bright, at edge of Mare Crisium
Linné, 6; has been suspected of change
Aristotle, 60
Eudoxus, 40
Eratosthenes, 38; at east end of Apennine range (Figures 109, 110)
Copernicus, 56; very conspicuous, surrounded by rays. Group of peaks in center (Figure 110)
Archimedes, 50 (Figure 109)
Piazzi Smyth, 6
Ptolemy, 115; near center of visible disk; Alphonsus, 83; Arzachel, 66; linked together
Tycho, 54; center of great ray system
Gassendi, 55
Schickard, 134
Wargentin, 54; floor level with brim
Grimaldi, 150; very dark
Langrenus, 90; Vendelinus, 50; Petavius, 100; Furnerius, 80—forming chain near west limb
Plato, 60; floor very dark in color
Kepler, 22; surrounded by rays
Aristarchus, 29; brightest object on the Moon
Catherina, 70; Cyrillus, 60; Theophilus, 64—forming three links of a chain

**The Best Time to Observe the Moon.** For observing the mountains, craters, and all other elevations or depressions on the Moon, the most favorable conditions are obtained when the object is near the terminator, for then it is brought into strong relief by the shadows. The maria and the ray systems are seen to best advantage near full Moon.

**Theories Regarding the Origin of Lunar Craters.** The craters on the Moon bear some resemblance to extinct volcanoes, and a theory which has been generally held attributes their origin to volcanic action. An objection is found in the enormous size of some of the craters. Owing to the small superficial gravity of the Moon, a given eruptive force would have much greater effect there than on the Earth, but this fact seems insufficient to explain so great a difference in size as exists between the largest terrestrial volcano and Grimaldi or Clavius. Moreover, there are no lava flows on the Moon, and most of the lunar craters differ from terrestrial volcanoes in their floors being lower than the surrounding plain.

An alternative theory supposes the craters to have been made by the impact of huge meteorites upon the Moon, possibly at an exceedingly remote time when the Moon's surface was plastic. To explain the lack of similar craters on the Earth, it is recalled that the Earth's atmosphere protects it from all but the largest meteorites (page 292), and greatly diminishes the speed of even these; and it is further argued that, during many millions of years, the surface of the Earth has been eroded by the weather and the ancient scars may thus have been obliterated, whereas no such action has taken place on the airless Moon. The meteoric explanation

derives some support from the appearance of Meteor Crater in Arizona (page 293). Airplane photographs of holes that were formed by the explosion of bombs dropped upon the ground from airplanes resemble lunar craters and lend some support to a view suggested by Ives, that the craters were formed by explosions due to the heat generated by the impact of meteors rather than to the splash made by the impact directly.

**The Light Received by the Earth from the Moon.** Comparisons of the brightness of the Moon with that of the Sun are difficult and uncertain. Probably the most reliable value is that deduced by Russell from the work of Herschel, Bond, Zöllner, and others, which makes the light given the Earth by the full Moon 1/465,000 that given by the Sun (stellar magnitude of full Moon, − 12.55). The brightness is much less at other phases, partly because of the smaller area of the visible illuminated surface, and partly on account of the presence of shadows; and it is less after full Moon than at the corresponding phase before full. Moonlight is somewhat yellower than sunlight; but, in defense of the artists who paint moonlight scenes in blue, it may be added that moonlight is much bluer than the ordinary lamplight with which it is likely to be compared.

The **albedo** (reflecting power, or ratio of light reflected to light received) of the Moon's surface ranges from about that of dark slate to that of white sand, and, according to Russell, has the low average value of 0.07. Studies of the polarization (page 166) of moonlight by F. E. Wright suggest that the reflecting surface is not bare rock but is more probably fine dust, perhaps of meteoric or volcanic origin.

**Temperature of the Moon.** Since the surface of the Moon is exposed to the rays of the Sun continuously for two weeks without the protection of an atmosphere, and then deprived of sunlight for an equal length of time, a great range of temperature is to be expected. Measurements of the heat which the Moon sends us indicate, for the sunlit side, a temperature well above 100° C. and for the dark side extreme cold—near the absolute zero of temperature (−273° C.). At the time of a lunar eclipse, the temperature falls very rapidly as the sunlight is withdrawn and rises with even greater rapidity soon after the surface emerges from shadow, showing that the Sun's heat does not penetrate deeply below the surface.

**The Question of Changes on the Moon.** If any changes have occurred on the Moon since it was first accurately mapped, they have been very slight. In the absence of any perceptible amount of air or water, no extensive erosion could take place, but the expansion and contraction of the rocks with the changing temperatures might indeed cause landslips

on some of the many steep slopes. Unless such a change affected an area of a square mile or more, it would be extremely difficult to detect, and he would be dogmatic indeed who would say that no such changes occur; but we have no certain evidence of them. The little crater Linné has been suspected of change, for its present appearance is distinctly different from that shown by drawings and descriptions made by Mädler early in the nineteenth century; but the evidence has failed to be universally convincing. A few observers, chief among whom was W. H. Pickering, have reported temporary greenish patches in Eratosthenes and some other craters which Pickering attributed to a low form of vegetation that passes through its life history in a synodic month. This would imply the presence in these craters of a slight amount of moisture and of some kind of atmosphere. Any observational evidence of changes on the Moon, however, must be accepted with caution because the appearance of certain parts changes strikingly with the direction of the Sun's rays and it is extremely difficult to secure two photographs or drawings under identical conditions of lighting.

If air and water are really totally lacking, life as we know it, either animal or vegetable, is of course impossible; and in the opinion of most astronomers the Moon is an arid, barren waste without life or sound or any change.

## EXERCISES

1. What are the greatest and least inclinations of the Moon's path to the horizon of an observer in latitude + 40°?

*Ans.* $78\frac{1}{2}°$; $21\frac{1}{2}°$

2. In what part of the sky must the Moon be in order that, for such an observer, its daily retardation in rising shall be the least possible?

*Ans.* Near the vernal equinox

3. At what time of year is the Moon in this part of the sky at full moon?

*Ans.* In September (this is the so-called Harvest Moon)

4. Assuming Russell's value, 0.45, of the albedo of the Earth, how does full earthlight on the Moon compare with full moonlight on the Earth?

*Ans.* Earthlight is about 90 times as bright as moonlight

5. Describe the diurnal motion of the stars, the Sun, and the Earth as seen from the Moon.

6. What interval of time must elapse between an occultation of Aldebaran and one of Antares? (See star maps.)

*Ans.* About nine years

7. What other bright stars may be occulted by the Moon?

8. Comment on Coleridge's lines:

> "The hornéd Moon, with one bright star
> Within the nether tip."

# CHAPTER 7

## ECLIPSES OF THE SUN AND MOON

**Shadows of the Earth and the Moon.** Since the Earth and the Moon are opaque and are illuminated by sunlight, each is accompanied on its orbital motion by a shadow which is ordinarily invisible and which extends into space in a direction opposite that of the Sun. Occasionally the Moon passes into the Earth's shadow and is darkened by a **lunar eclipse**; at certain other times its shadow falls upon the Earth, darkening the Sun for favorably situated observers, and so producing a **solar eclipse**. Evidently, a lunar eclipse can occur only at full Moon and a solar eclipse only at new Moon.

In Figure 111 the formation of the shadows of the Earth and the Moon is illustrated diagrammatically, but it is impracticable to present the figure in correct proportion. For an Earth of the size depicted, the Sun should be nearly two feet in diameter and about two hundred feet away, and the length of the shadows and the diameter of the Moon's orbit should be about six times as great as shown. The actual dimensions of the shadows may be found from geometric considerations as follows:

Let $L$ be the length of the Earth's shadow, $R$ the distance from the Earth to the Sun, $d$ and $D$ the respective diameters of the Earth and the Sun. The Earth's shadow is a cone, of which the cross section $HCK$ is bounded by the common tangents $BK$ and $AH$ drawn to the Earth and the Sun. On account of the Earth's great distance from the Sun as compared with the size of either body, these lines are nearly parallel (the angle between them is about half a degree, being slightly less than the apparent diameter of the Sun as seen from the Earth); hence the lines $AB$ and $HK$, which join the points of tangency, very nearly coincide with the diameters $D$ and $d$. In the similar triangles $ABC$ and $HKC$ we have $L/R + L = d/D$, whence $LD - d(R + L) = 0$, and $L = dR/D - d$. We have seen (page 106) that the Sun's diameter $D$ is about 109 times that

of the Earth, and so $d/(D - d) = 1/108$. Therefore, since $R$ is about 93,000,000 miles, the length $L$ of the Earth's shadow is $1/108 \times 93{,}000{,}000$, or 857,000 miles. This length varies by about 28,000 miles with the changing value of $R$, but always far exceeds the distance of the Moon.

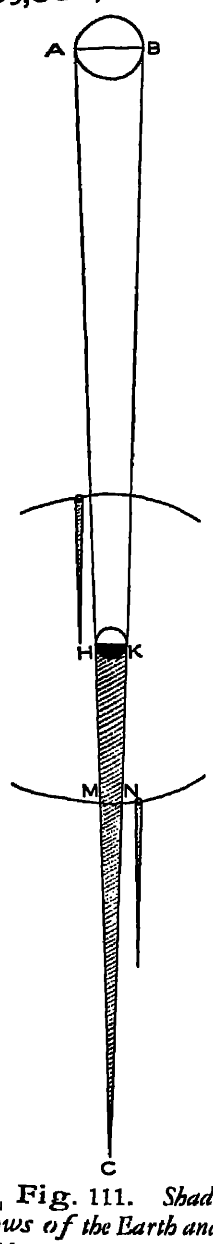

Fig. 111. *Shadows of the Earth and Moon.*

The length of the Moon's shadow, as found by a similar calculation, is about 232,000 miles on the average, and varies about 4000 miles either way. Since the Moon's distance ranges from 222,000 miles at perigee to 253,000 at apogee (page 127), its shadow is sometimes long enough to reach the Earth, but more often falls short. In the former case, an observer situated on the line of centers of the Moon and the Sun may see a total solar eclipse; in the latter, the apparent diameter of the Moon is less than that of the Sun, and an observer on the line of centers may see a bright ring, or *annulus*, of the Sun surrounding the black Moon; the eclipse is then said to be **annular.**

The diameter, $MN$, of the Earth's shadow at the place where the Moon crosses it may be computed from the similar triangles $HCK$ and $MCN$. Thus, if $\Delta$ represents the Moon's distance from the Earth when it crosses the shadow, $MN/L - \Delta = d/L$. The mean value of $\Delta$ being 239,000 miles, this gives for $MN$ about 5700 miles; but its actual value varies by about 400 miles with the varying distance of the Moon from the Earth and of the Earth from the Sun. At its greatest, $MN$ is about three times the diameter of the Moon, and, as the Moon travels a distance about equal to its own diameter in an hour, it may be wholly within the shadow (total lunar eclipse) about two hours, and the time from its first contact with the shadow until it finally leaves it may be as long as four hours.

The shadow cast by a luminous body of appreciable angular size, such as the Sun, consists of two parts: the shadow proper, or **umbra,** which is the part discussed above, and the **penumbra,** which is the region included between the prolongations of the tangents $AK$ and $BH$ (Figure 112). Within the umbra, the direct rays of the Sun are completely excluded, although

some sunlight, refracted by the air, is bent into the umbra of the Earth. Within the penumbra, there is direct illumination from a part of the Sun's face, but not the whole of it.

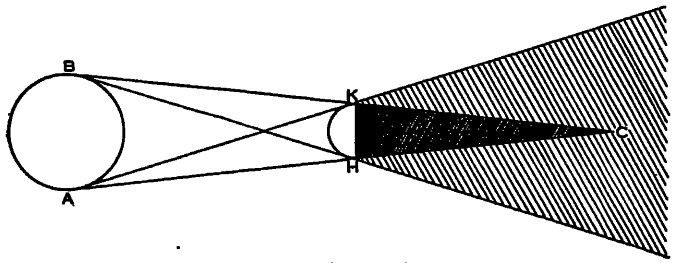


Fig. 112. *Shadow and Penumbra.*

**Effect of the Inclination of the Moon's Orbit.** If the Moon moved always in the plane of the ecliptic, there would be two eclipses, one of the Moon and one of the Sun, every month; but the plane of its orbit is inclined 5° to the ecliptic plane, and so the Moon usually passes either north or south of the line joining the Earth and Sun. It is only when the Earth's radius vector nearly coincides with the Moon's line of nodes that an eclipse can occur. Figure 113 represents the terrestrial orbit with the Earth and the lunar orbit in two positions, as seen from a point about 45°

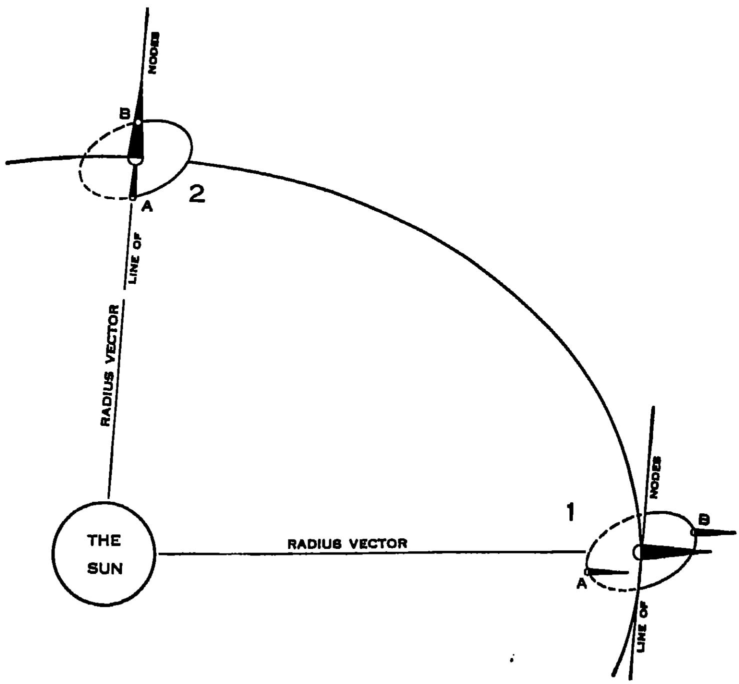


Fig. 113. *Why Eclipses Do not Occur Every Month.*

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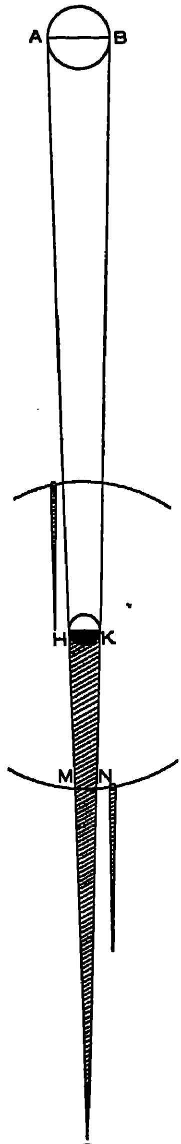

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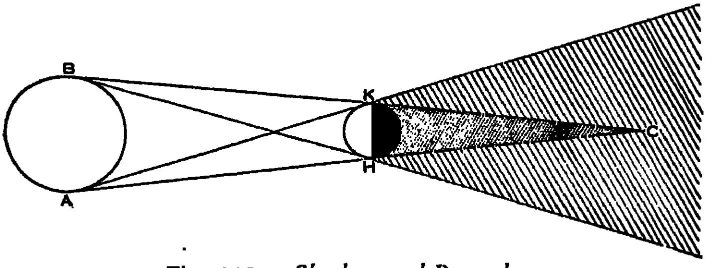



Fig. 112. *Shadow and Penumbra.*

**Effect of the Inclination of the Moon's Orbit.** If the Moon moved always in the plane of the ecliptic, there would be two eclipses, one of the Moon and one of the Sun, every month; but the plane of its orbit is inclined 5° to the ecliptic plane, and so the Moon usually passes either north or south of the line joining the Earth and Sun. It is only when the Earth's radius vector nearly coincides with the Moon's line of nodes that an eclipse can occur. Figure 113 represents the terrestrial orbit with the Earth and the lunar orbit in two positions, as seen from a point about 45°

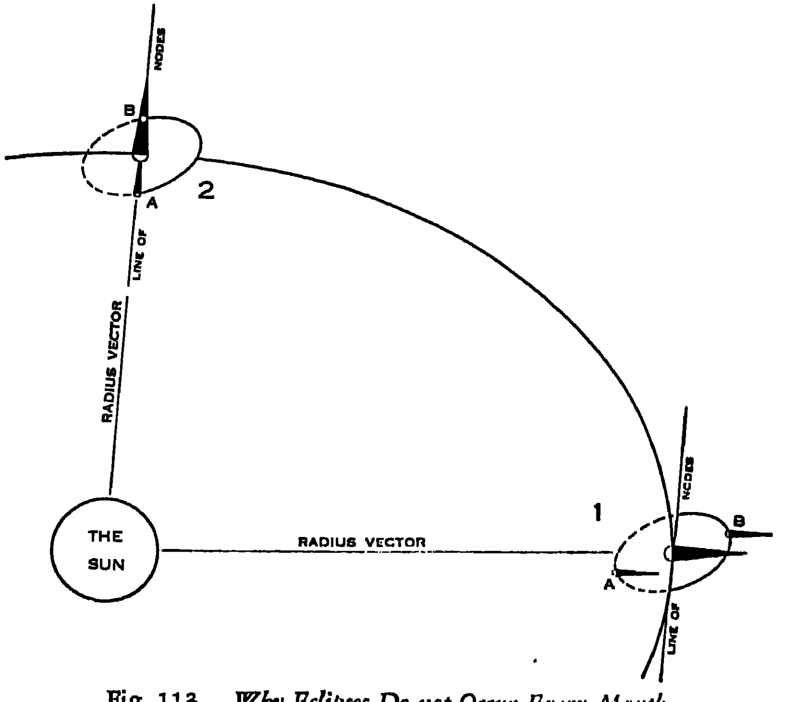



Fig. 113. *Why Eclipses Do not Occur Every Month.*

north of the plane of the ecliptic. The dotted portion of the Moon's orbit is south of the plane of the ecliptic, and must be thought of as lying behind the page on which the figure is drawn; the remainder is north of the ecliptic, or above the page. In Position 1, the Moon passes above the Earth's shadow at full Moon ($B$) and below the Earth's radius vector at new Moon ($A$), so no eclipse occurs. In Position 2, the radius vector coincides with the line of nodes, the points $A$ and $B$ lie in the plane of the ecliptic, and, if the Moon is in the proper phase, an eclipse is inevitable.

**Ecliptic Limits; Frequency of Eclipses.** For an eclipse to occur, the Earth's radius vector must be near the line of nodes, but the two lines need not coincide exactly. The greatest angular distance of the Sun (or of the Earth's shadow) from the nearer node which is compatible with an eclipse is called the **ecliptic limit**—lunar or solar, according to the kind of eclipse concerned. Figure 114 represents a portion of the ecliptic and

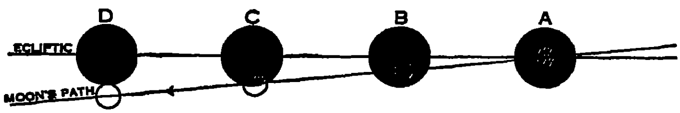



Fig. 114. *Lunar Ecliptic Limit.*

of the Moon's apparent path with the Earth's shadow and the full Moon in four positions. At $A$, both the Moon and the shadow are centered on the descending node. At $D$, the Moon just touches the shadow, and the distance $AD$, measured along the ecliptic, is the lunar ecliptic limit. Its value varies somewhat with the distance of the Moon, the dimensions of the shadow, and the inclination of the Moon's orbit. When the Moon crosses the shadow at the point where its diameter is three times that of the Moon ($1\overset{\circ}{.}5$), the distance between their centers is about $1°$, and, as the inclination is always near $5°$, or one-twelfth of a radian, the distance $AD$ is then about $12°$. The maximum value of the lunar ecliptic limit is in fact $12\overset{\circ}{.}3$, while the minimum is $9\overset{\circ}{.}5$.

There is thus, under even the most favorable conditions, an arc of the ecliptic less than $25°$ long, extending to the major limit on either side of the node, within which the Sun must be situated at the time of full Moon if a lunar eclipse is to occur. The Sun moves along the ecliptic about a degree a day, and so will cross this region in twenty-five days; but the synodic month is more than twenty-nine days long, and so it is possible for the Sun to pass the node without our having a lunar eclipse at all. If a lunar eclipse does occur, even if it is just within the limit so that it is

only a small partial one, before the Moon can be full again the Sun will have moved beyond the limit, and so only one eclipse of the Moon can occur at a given node passage. If the Sun passes one of the nodes early in January, it will pass the other in June and return to the first in December; for, on account of the regression of the nodes of about 19° a year, the interval between successive arrivals of the Sun at the same node, which is called the **eclipse year**, is only 346.62 days. The greatest possible number of lunar eclipses in a calendar year is therefore three, and a year may pass without any.

An eclipse of the Sun will be visible somewhere on the Earth if the Moon touches the boundary of the region *AKHB* (Figure 111 or 115) which encloses the Earth and the Sun. The geocentric distance of its center from the Earth's radius vector is then the angle *SEM* (Figure 115), which is the sum of the semidiameters *REM* and

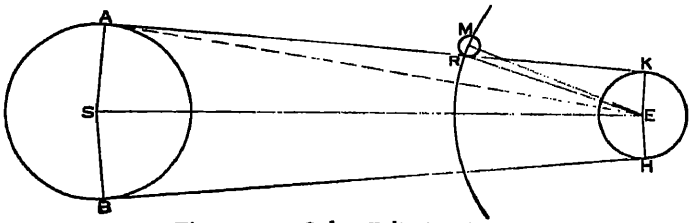



Fig. 115. *Solar Ecliptic Limit.*

*AES* of the Moon and Sun, respectively, and the angle *REA*. This last angle is equal to the difference between the Moon's geocentric parallax, *KRE*, and the Sun's parallax, *KAE*. The semidiameters of the Moon and Sun being each about 0°.25, while the maximum (horizontal) parallax of the Moon is about a degree and that of the Sun only 8″.8, t e angle *SEM* has a maximum value of 1°.5. The solar ecliptic limit is therefore about 50 per cent greater than the lunar.

The solar ecliptic limit ranges from 15°.4 to 18°.5. There is thus a region at least 31° long in which the Sun may be situated at the time of a solar eclipse, and since it requires longer than a synodic month to traverse this arc, one solar eclipse is inevitable and two are possible at a single node passage. If a (partial) solar eclipse occurs early in January near the solar ecliptic limit west of the node, a second will occur in the same month; two solar eclipses may likewise happen at the other node in the middle of the year, and a fifth may occur in December at the new Moon which follows the 346th day after the first; but a sixth could occur no less than 29 days later still, which would place it in January of the following year. The greatest possible number of solar eclipses in a year is therefore five, and the least is two.

When two solar eclipses occur at a given node passage, a lunar eclipse always takes place between them. The greatest possible number of eclipses in a single year is seven, either two of the Moon and five of the Sun or three of the Moon and four of the Sun. The least possible number is two, both of the Sun. In the 20th century, seven eclipses occur in each of the years 1917, 1935, and 1982.

**Recurrence of Eclipses; the Saros.** As the two conditions necessary for an eclipse are the appropriate phase of the Moon and proximity of the Sun to the Moon's node, an eclipse must repeat itself after an interval that contains without a remainder both the synodic month and the eclipse year. The smallest interval which even approximately does this is that which consists of 223 synodic months, being 6585.32 days (18 years, 11⅓ days—or 10⅓ days of the interval contains five leap years), which is only 0.46 day less than 19 eclipse years. The recurrence of eclipses in this interval was discovered in prehistoric times by the Chaldeans, who named the interval the **saros**, signifying repetition. During the course of a saros there occur about twenty-nine lunar and forty-one solar eclipses, ten of the latter being total, and these eclipses are repeated approximately in the next saros, but are then visible in longitude 120° farther west, the Earth having made in the meantime 6585 rotations and a third of a rotation over.


Fig. 116. *Paths of Total Solar Eclipses at Intervals of a Saros.*

Figure 116 shows the paths of three total solar eclipses occurring at intervals of a saros. After three saroses, or fifty-four years, one month, the eclipses return to nearly the same longitude, but, owing to the slight difference between the saros and an integral number of eclipse years, the

Sun has then a position a little farther west with respect to the node, and the eclipses of the new set occur a little farther north if at the descending node or farther south if at the ascending. Figure 117 shows the paths of a


Fig. 117. *Paths of Total Solar Eclipses at Intervals of Three Saroses.*

number of central solar eclipses occurring at intervals of three saroses. These eclipses take place at the ascending node, and so the series began at the north pole of the Earth and is sweeping slowly southward.

**Eclipses of the Moon.** The entrance of the Moon into the Earth's penumbra makes no perceptible change whatever in its brightness, but as it approaches the edge of the umbra a darkening becomes noticeable at its eastern limb. This darkening is so slight, however, that the edge of the umbra, when the Moon encroaches upon it, seems black by comparison. In a telescope, even the deepest part of the shadow may usually be seen to contain some light, and when the Moon is completely immersed so that the eclipse is total it ordinarily remains easily visible to the naked eye, for it shines dimly with a dull reddish light which is sunlight, refracted into the shadow by the Earth's atmosphere, as illustrated in Figure 118, and tinged with sunset colors by selective absorption.

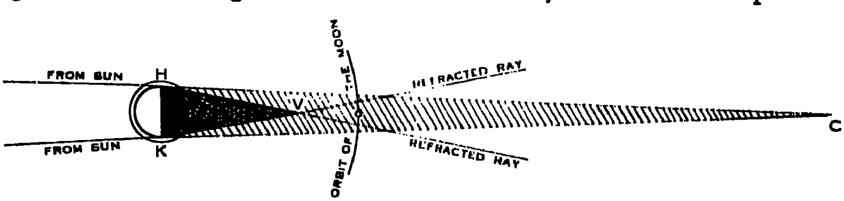



Fig. 118. *Refraction of Sunlight by the Earth's Atmosphere.*

The angles *CHV* and *CKV*, between the limits of the geometric shadow cone and the extreme refracted rays, are each somewhat over 1°, being twice the atmospheric refraction of sunlight at the horizon (page 66); hence the angle at *V*, which may easily be shown to be the sum of the angles *HCK*, *CHV*, and *CKV*, is about 2°.5, or five times the angle at *C*. The length of the cone *HVK*, from which sunlight is entirely excluded, is therefore about a fifth of the length of the geometric shadow or 170,000 miles—much less than the least distance of the Moon from the Earth. As seen from the Moon during a total lunar eclipse, the Earth would usually appear surrounded by a ring of brightly illuminated air; but if, as sometimes happens, the air throughout this ring contains dense clouds, most of this light is cut off. Under these circumstances, the Moon may, for a terrestrial observer, completely disappear.

**Eclipses of the Sun.** A lunar eclipse, whether total or partial, can be observed simultaneously from every place on the terrestrial hemisphere that is turned toward the Moon. It is quite otherwise with eclipses of the Sun. A total solar eclipse can be seen only within the umbra of the Moon's shadow, which, at the point where the Earth's surface cuts it, is at most only 168 miles in diameter; an annular eclipse can be seen within a region which may be 230 miles wide; and a solar eclipse is partial anywhere within the Moon's penumbra, which at the Earth's distance is some 4000 miles in diameter—sufficient to include about half of the exposed hemisphere. As the Moon moves in its orbit its shadow, like that of a vast bird, passes over the Earth's surface; and the solar eclipse is visible successively from all the points in its path.

The Moon and its shadow move eastward at a rate of about 2000 miles an hour. A point on the Earth's equator is carried by the Earth's rotation, also eastward, at about half that speed. Hence, the Moon's shadow passes an observer at the equator with a velocity of about 1000 miles an hour. In higher latitudes, where the observer's velocity is less, the shadow passes more rapidly; and if it falls obliquely, as it does when the eclipse occurs near sunrise or sunset, the relative velocity of the shadow and the observer may be as great as 5000 miles an hour. The greatest possible duration of a total solar eclipse for a single observer is $7^m\ 30^s$; of an annular eclipse, about $12^m$; and of the whole eclipse, from the first contact of the Moon's disk with the Sun until the last, a little over four hours. This maximum duration is achieved rarely indeed, and a total solar eclipse that lasts five minutes is unusually favorable.

Because of its brief duration and the very limited area within which it is visible, a total eclipse of the Sun is a phenomenon which many persons never see, although, taking the Earth as a whole, such eclipses are not rare.

According to Rigge, only two such events could be seen at London and only three at Rome in the twelve centuries between A.D. 600 and 1800.

Astronomers go to remote parts of the Earth to observe solar eclipses, often to be disappointed by cloudy weather after traveling thousands of miles. Probably no one has viewed the Sun's corona, which can be seen only while the eclipse is total, during an aggregate of thirty minutes.

**Total Solar Eclipses.** Many excellent writers and competent observers have described total solar eclipses, giving accounts which are fascinating and memorable; but anyone who has intelligently observed this most sublime of natural phenomena knows that all descriptions fail to convey an adequate impression of the reality.

A moment after "first contact" the eastern limb of the black Moon may, with the protection of a smoked glass or overexposed photographic film, be seen on the western limb of the Sun. As the Moon moves steadily eastward the black area grows, but for some time the only effect apparent on the Earth is in the images of the Sun formed by such small apertures as the interstices between the leaves of trees, which change from their usual circular shape to narrowing crescents—an effect which may of course be seen at any partial eclipse. About half an hour before totality, the landscape becomes perceptibly darkened and both the Earth and the sky assume an indescribably weird color because the light coming from near the Sun's limb is of a different quality from the stronger radiance of the center of the disk. As the light fades and its unusual color becomes relatively stronger, it is noticed not only by men but by other creatures; birds fly about and twitter excitedly, roosters crow, and dogs bark. Before the beginning of totality, fowls go to roost, dew or frost may form, and many flowers close their blossoms. Several minutes before "second contact" (the beginning of totality), ghostly **shadow bands** may be seen flitting along upon any exposed white surface. These are the shadows of waves in the Earth's atmosphere, rendered distinct by the narrowness of the remaining crescent of the Sun.

All this is but preliminary to the great spectacle of the total eclipse. Just before second contact a favorably situated observer may see the Moon's shadow in the air, looking like a vast thundercloud, but not so definitely limited, approaching from the west with a speed far exceeding that of any storm, being twenty miles a minute or greater. The thin crescent of the Sun's eastern limb breaks up into "Baily's beads," which are due to the Sun shining between irregularities at the limb of the Moon; while at

the western limb there appears already the glow of light from the inner corona, the wonderful pale envelope of the Sun which can be seen only when the main body is covered by the Moon. In the eclipse of 1925 one of the "beads" persisted after the corona was distinctly visible all around the Moon, giving an appearance that was likened by thousands of delighted spectators to a diamond ring. The majesty of the spectacle at this point is such that it seldom fails to silence the chattering crowds of spectators.

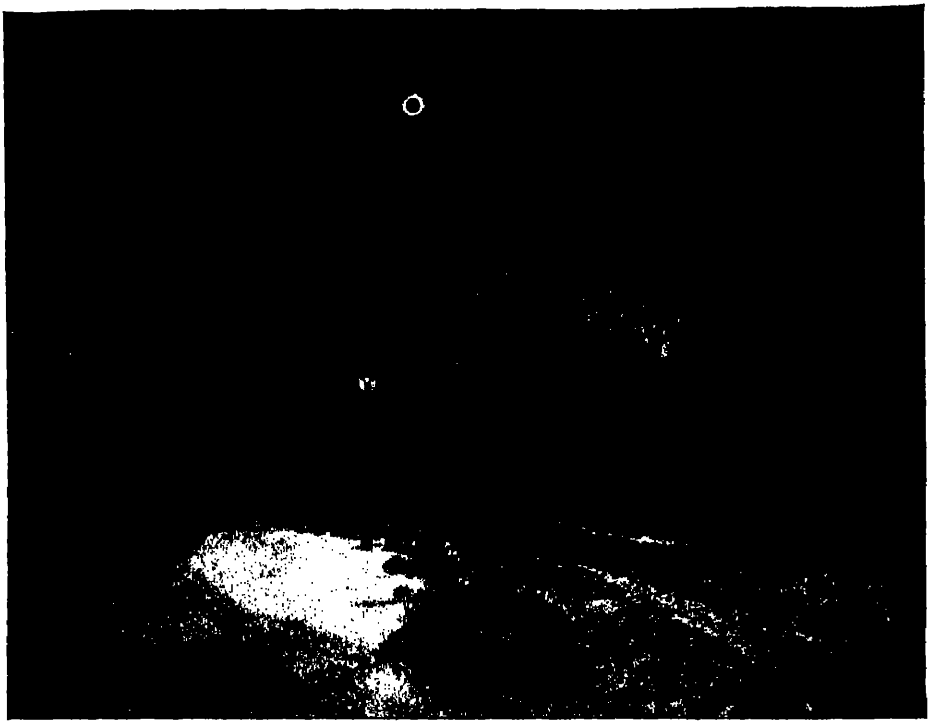


Fig. 119. *Total Solar Eclipse of 1925 January 24, Photographed in Bronx Park, New York City, by A. Fassbender.*

The "beads" last only an instant, and then the full glory of the corona presents itself. Its form is never the same at two different eclipses, but it always consists of a bright ring around the Sun (and Moon), with irregular streamers extending several solar diameters away. In color it is pearly white. At its inner edge it is very brilliant, contrasting sharply with the dead-black Moon; its outer streamers fade imperceptibly into the sky, which, if clear, is of a deeper blue than at other times, and on which are visible the brighter stars and planets. Often to the unaided eye and usually with a telescope, there may be seen rosy, flame-like prominences or

protuberances extending from the red chromosphere which is visible at the disappearing or the reappearing limb of the Sun. Strangely enough, no account of the prominences was ever given before the eclipse of 1842, although they have been seen or photographed at most total eclipses that have been observed since.

The only changes to be noted during totality are those due to the motion of the Moon as it covers the inner corona and prominences at the eastern limb of the Sun and uncovers those at the western limb. From third contact, the end of the total phase, to fourth contact, when the Moon finally leaves the Sun's disk, the events of the earlier part of the eclipse are repeated in reverse order.

The spectacular features of the eclipse are sometimes heightened by the presence of a few clouds, and even if the sky is completely overcast the sudden coming of darkness at midday is exceedingly impressive. The darkness of an eclipse is never very deep, for the corona gives about half as much light as the full Moon, and the air and clouds forty or fifty miles away, where the Sun is only partially eclipsed, are still brightly illuminated.

A total solar eclipse affords a unique opportunity for many important investigations, such as:

1. The study of the corona; visually, with long-focus cameras, and with the spectrograph, the photometer, and the polariscope.
2. Photography of the spectra of the chromosphere and prominences and the "flash spectrum" (page 186).
3. The search for new planets and other bodies near the Sun.
4. Determination of the exact relative position of the Moon and Sun by observation of the contact times.
5. Photography of the field of stars around the Sun to detect and measure the "Einstein displacement" due to the Sun's gravitational effect on light (page 252).

**Prediction of Eclipses.** From a knowledge of the elements of the orbits of the Earth and the Moon and of the changes of these elements such as the regression of the line of nodes, it is possible to predict far in advance the circumstances of any eclipse and the time, within a few seconds, when it will occur. The calculation of a lunar eclipse is simpler than that of a solar one since the circumstances of the former are the same for all points from which it is visible. In a great work named *Canon der Finsternisse*, Oppolzer of Vienna has given the approximate data concerning all the eclipses that have occurred since 1207 B.C. or that will occur up to

A.D. 2162, with maps showing, for central solar eclipses, the paths of the Moon's shadow on the surface of the Earth.

The latest total solar eclipse observed in the United States occurred 1932 August 31, and was visible in New England (Figure 120). The next to be favorably observed will be on 1963 July 20, also visible in New England.

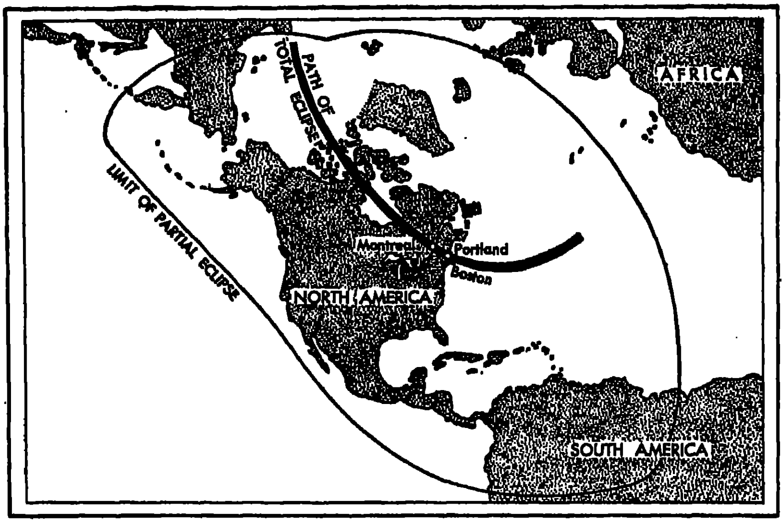



Fig. 120. *Region of Visibility of Solar Eclipse of 1932 August 31.*

Following are the dates, durations, and regions of visibility of some coming total solar eclipses (compiled by C. H. Smiley):

| | | |
|---|---|---|
| 1945 July 9 | 1.1 | Canada, Greenland, Russia |
| 1947 May 20 | 5.2 | Argentina, Paraguay, Africa |
| 1948 Nov. 1 | 1.9 | Africa |
| 1952 Feb. 25 | 3.0 | Africa, Persia, Russia |
| 1954 June 30 | 2.4 | Canada, Scandinavia, Russia |
| 1955 June 20 | 7.1 | India, Indo-China, Philippines |
| 1958 Oct. 12 | 5.1 | Chile, Argentina |
| 1959 Oct. 2 | 2.9 | New England (at sunrise), North Africa |
| 1961 Feb. 15 | 2.4 | Southern Europe, Russia |
| 1962 Feb. 5 | 4.0 | Borneo |
| 1963 July 20 | 1.5 | Alaska, Canada, New England |
| 1965 May 30 | 5.3 | Pacific Ocean |

EXERCISES

1. What is the latitude of an observer who sees the Moon at the zenith at the middle of a total lunar eclipse on December 22?

*Ans.* $23\frac{1}{2}°$ north

2. At the total solar eclipse of 1918 June 8, the Moon's shadow swept from a point south of Japan across the Pacific Ocean, and across the United States from the State of Washington to Florida. Where and when was the eclipse seen which followed at the end of a saros? At the end of two saroses? Where and when was the eclipse seen which occurred one saros earlier?

# CHAPTER 8

## SPECTROSCOPY

**The Analysis of White Light.** Although in ordinary speech the words *white* and *colorless* are often used as synonyms, it is a fact that white light, such as the light of the Sun, is in reality a mixture of light of all the different colors. This was first proved in 1666 by Sir Isaac Newton, who placed a triangular glass prism in the path of a beam of sunlight that he had admitted to a dark room through a hole in a window shutter, and found that the beam was not only deviated from its original path by the refraction of the prism, but was spread out into a band of light, red at the least refracted end and violet at the other. This band of light is called a **spectrum** and the separation of the colors by the prism is called **dispersion.**

To show that the colors were not bestowed upon the light by the prism, Newton isolated from the spectrum a ray of a single color by passing it through a hole in a second screen, and placed in its path a second prism, when he found that, although the ray was further deviated, its color was no further changed; and he also recombined the colors of the spectrum by reversing the second prism, whereupon the light again appeared white.

The branch of physical science that deals with the analysis of light, of which Newton's simple experiments were the beginning, is called **spectroscopy.** In the form into which it has been developed in the nineteenth and twentieth centuries, it has made possible the study of the chemical constitution, temperature, and motion of any source of light, whether it be a flame or electric spark in the laboratory or a distant body like the Sun or a star; and it has yielded much knowledge concerning the structure of matter and the nature and origin of light. It is to spectroscopy that we owe much of the advance of astronomy, and of physics and chemistry as well, since about 1860.

**The Rainbow.** The rainbow, one of the most beautiful of natural phenomena, is a spectrum produced by the dispersion of sunlight by

spherical raindrops. It appears as an arch of colored light having its center on the prolongation of the line from the Sun through the observer's eye, and therefore seems to move over the landscape if the observer moves. Most of the light of the rainbow is contained in the **primary** bow, which has a radius of about 42°; but there is also often seen a fainter **secondary** bow, of radius 51°, and sometimes certain very faint **supernumerary** bows near the inner edge of the primary and the outer edge of the secondary.

The complete explanation of these appearances cannot be entered upon here, but that of the primary bow is briefly as follows: A ray of sunlight, *SA* (Figure 121), falls upon a spherical drop of water at *A*, and is refracted and dispersed as at the surface of a prism. One ray, say the yellow, proceeds to *B*, where it is reflected at the inner surface of the drop, is refracted a second time at *C*, and emerges in the direction *CO*. It may be shown from the refractive index of water and the geometric relations involved that the direction *CO*, along which yellow light emerges most copiously, makes an angle of 42° with the incident ray *SA*. An observer at *O*, therefore, receives yellow light from all raindrops that are situated upon a cone having his eye for vertex, the prolongation of the line from the Sun through his eye as axis, and a semi-angle of 42°. In a shower of rain, as each drop falls, its place is taken by another and the observer has the impression of a continuous arch of light which would be a complete circle but that the supply of raindrops ends at the horizon. It is obviously impossible for a rainbow to be produced in the sky by the Sun when its altitude is more than 42°, and rainbows are most commonly seen in early morning or late afternoon. In both refractions at the surface of the raindrop, the red is deviated less than the yellow and the violet more; hence red light reaches the eye from drops outside the cone of yellow rays and violet from drops inside, and the colors of the spectrum are to be seen in the luminous arch, red at the outer edge and violet at the inner. The secondary bow is formed of light that enters the drop at the lower instead of the upper side, is twice reflected, and emerges from the upper side. In it, the order of the colors is reversed so that red is at the inner edge.

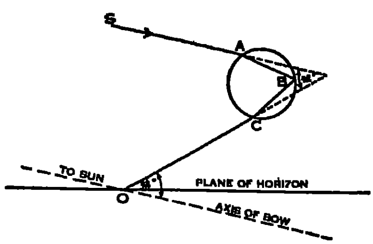



Fig. 121. *Optics of the Rainbow.*

**The Prism Spectroscope and Spectrograph.** When the source of light is a body of large angular size like the Sun, as in Newton's experiment and in the rainbow, the rays from different parts of the source strike the prism (or raindrop) at different angles and emerge at different angles, so that the colors in the resulting spectrum are mixed, the green light, for example, from one point of the source falling upon the red light from another. To obtain a *pure* spectrum with a prism from such a source it is necessary to pass the light first through a narrow slit placed parallel to the

refracting edge of the prism. Even then the colors will mix unless all the rays of a given color are made to pass through the prism parallel to one another and to meet at a focus after leaving it.

An instrument for studying the spectrum visually is called a **spectroscope**; if arranged for photographing the spectrum it is called a **spectrograph**. In the most common type of prism spectroscope the conditions necessary for producing a pure spectrum are met as follows: The light from the luminous body $F$ (Figure 122) first enters the **slit** $S$ (only a few thousandths of an inch wide), the rays from different parts of the source crossing at the slit and diverging beyond. At a distance from the slit equal to its own focal length is placed the **collimating lens** $C$, which renders the rays parallel. This broad beam of parallel rays then passes through the **prism** $P$, which disperses the light into its component colors. On emergence, all the rays of a given color are parallel to one another, but are not parallel to rays of other colors. The light then passes through the **objective** $O$, which focuses the red light at $R$ and the violet at $V$, the other colors being arranged between to form a spectrum; this may be photographed by placing a sensitive plate in the plane $RV$, or be studied visually with an eyepiece which, with $O$, would form the **view telescope**. For studying the spectra of the heavenly bodies, the spectrograph is ordinarily attached to an astronomical telescope, the slit being placed in the focal plane of the objective so that a bright image of the heavenly body is formed at the slit. The astronomical telescope thus becomes merely a collector of light. To secure high dispersion, in some spectrographs the light is passed successively through two or more prisms.

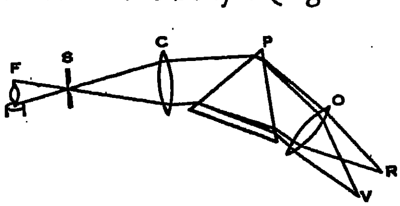



Fig. 122. *The Prism Spectroscope.*

For a general inspection of the spectrum, a convenient form of spectroscope is one provided with a **direct-vision prism**, which consists of a combination of prisms of flint and crown glass, usually arranged as in Figure 123. A wide-angle prism of flint glass producing considerable dispersion is placed between two inverted crown-glass prisms. Since crown glass has a smaller dispersive power than flint, these prisms rectify the deviation of the central ray of the spectrum without entirely destroying the dispersion. In small pocket spectroscopes intended for laboratory work, a direct-vision prism is used without a view telescope, because the emergent beam is already small enough to enter the pupil of the eye.

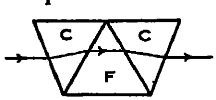



Fig. 123. *Direct-Vision or Amici Prism.*

Many other modifications of the form of the spectroscope are made for s]ʰ purposes.

**Development of Spectroscopy.** In 1802 Wollaston, in England, improve upon Newton's experiment by admitting the light to his prism through a narrow crevice (one-twentieth inch wide), thus obtaining a spectrum of greater purity, and noticed that the spectrum of sunlight was crossed in a number of places by dark lines which did not appear in the spectrum of a candle flame. A few years later, the great German optician Fraunhofer further improved the spectroscope by using a narrow slit and a collimating lens and viewing the spectrum through the small telescope of a theodolite. He thus observed about seven hundred of the dark lines, which have since been known as **Fraunhofer lines.** He found that the dark lines occupied fixed places in the spectrum, and although he was unable to explain their meaning, he realized their importance as points of reference and made a careful map of their positions, denoting the more prominent lines with the letters of the alphabet, beginning in the red with A.

In 1822, Sir John Herschel observed that a flame impregnated with certain metallic salts exhibited a spectrum consisting of isolated bright lines (each being an image of the slit), whose position was characteristic of the substance with which the flame was fed. Fraunhofer and others further noticed that the position of the single yellow line of which the spectrum of sodium seemed to consist was identical with that of the dark line in the spectrum of the Sun which Fraunhofer had lettered D. In 1859 Kirchhoff and Bunsen, by a brilliant series of investigations, showed that the spectra of different chemical substances which were brought to incandescence in a gaseous form were characteristic of the substances, no two being alike, and that dark lines were produced by the absorption of light by gases through which it passed. This work gave great impetus to the study of spectroscopy and formed the basis of much of its application to astronomy.

**Artificial Sources of Spectra.** Astronomical spectra are interpreted partly by comparing them with the spectra of sources of light artificially produced. The spectra of a few elements, among which are sodium, lithium, and strontium, may be obtained by placing small quantities of their salts in a flame such as that of the Bunsen burner; but for most substances the more powerful stimulus of electricity is needed. All the metals yield characteristic spectra when an electric arc or high-voltage spark is formed between terminals composed of the metal. In the electric furnace, controlled temperatures between those of the flame and the arc are produced. Permanent gases such as hydrogen, helium, and neon are made to glow in a vacuum tube which is not quite a vacuum but a tube of glass or quartz, exhausted of air and containing a little of the gas at very low pressure, and provided with sealed-in metallic terminals for leading in a high-voltage current. The tubes of the ubiquitous neon street sign are vacuum tubes and so, of a special type, are the tubes of fluorescent lamps.

**Kinds of Spectra.** Spectra are generally classified as **bright-line, continuous,** and **dark-line** spectra. In the first kind, all the light of the spectrum is contained in separate bright lines, each an image of the slit, the spaces between the lines being completely vacant. A continuous spectrum is one in which the light is spread out in a continuous band without interruptions. A dark-line spectrum consists of separate dark lines which, to be detected, must be seen against a background of continuous spectrum. Such a spectrum is the reverse of a bright-line spectrum. Examples of these different types of spectrum are shown in Figure 124.

**The Principles of Spectral Analysis.** The work of Kirchhoff and Bunsen established the following principles, which, although there are exceptions, form a general basis for the analysis of sources of light by means of their spectra.

1. *An incandescent solid, liquid, or compressed gas gives a* ***continuous spectrum.*** Examples are found in the spectrum of red-hot or molten iron and in that of the glowing filament of an electric lamp. (A rarefied gas also yields a continuous spectrum if it consists to a considerable extent of free electrons, as in the depths of some of the stars.)

2. *An incandescent gas under low pressure gives a* ***bright-line*** *spectrum, the positions of the lines being characteristic of the chemical nature of the gas.* Bright-line spectra are often called **emission spectra.** Every known chemical element has more than one line in its spectrum, although in many cases two lines, as the two visible lines of sodium, which are yellow, are so close together that they appear as one in small spectroscopes. The spectra of many of the metals contain hundreds or thousands of lines. No two elements are known to have spectra with even a single line in common. Lines of many elements occur in what seems to the eye to be the same *color* but they always differ, though perhaps only slightly, in their *position* in the spectrum. For example, the spectrum of helium contains a prominent yellow line, the color of which the eye can scarcely distinguish from that of the sodium line; but when the two spectra are confronted in the same spectroscope, it is found that the helium line lies farther toward the violet.

3. *When light from a source that gives a continuous spectrum shines through a gas whose temperature is lower than that of the source,* ***dark lines*** *appear in the spectrum in the positions characteristic of the bright lines belonging to the gas.* In other words, a gas absorbs from a beam of light just those rays that it emits, and no others. Dark-line spectra are often referred to as **absorption spectra.** The lines in such spectra are never totally *black,*

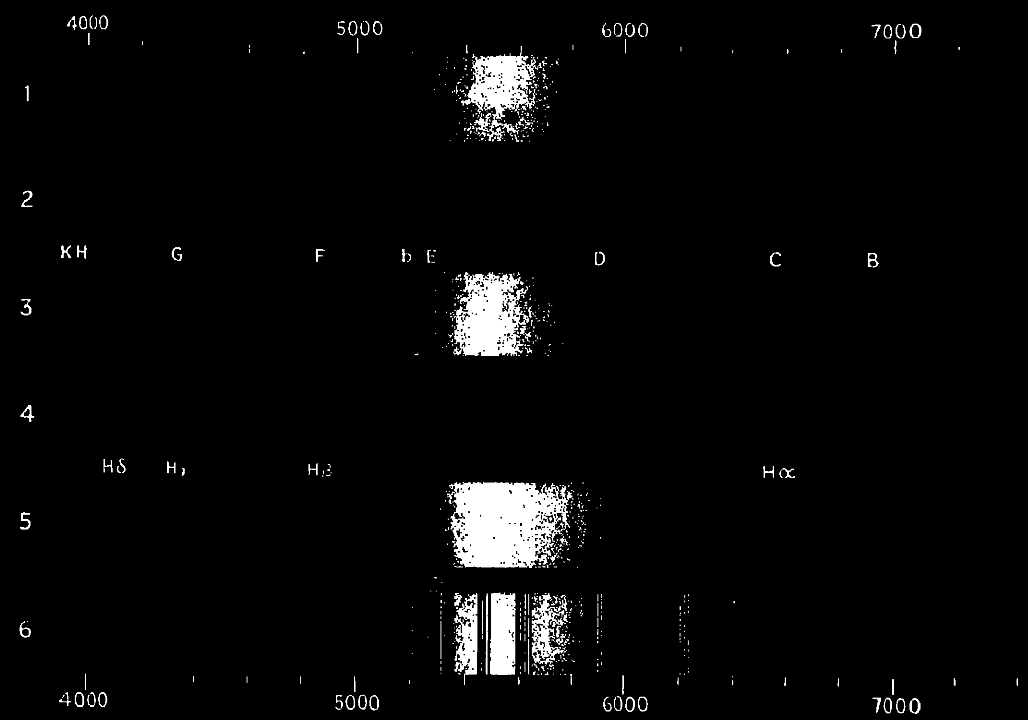



Fig. 124. Grating Spectra, Visible Region

(1) Continuous Spectrum; (2) Flame Spectrum of Sodium; (3) Solar Spectrum showing some Lettered Fraunhofer Lines; (4) Emission Spectrum of Hydrogen; (5) Spectrum of Sirius; (6) Spectrum of Betelgeuse. (Drawn by Agnes A. Abbot.)

but appear dark by contrast with the luminous background of continuous spectrum; the gas through which the light shines is emitting as much light as ever. In cases where the gas is hotter than the source of continuous spectrum, its lines may even appear bright against the less brilliant background.

The three principles may be illustrated as follows: Before the slit of a spectroscope is placed an incandescent electric lamp; the glowing filament is an incandescent solid, and a continuous spectrum is seen in the spectroscope, illustrating the first principle. The spectrum is brightest if an image of the filament is thrown upon the slit by means of a convex lens of which the ratio of aperture to focal length is about the same as that of the collimator, so that the collimator and prism are filled with light. Let an alcohol lamp be placed between the slit and the electric lamp, and let the latter be turned off. The blue alcohol flame is so nearly transparent that probably no light will be seen in the spectroscope, but if a little common salt (sodium chloride) is inserted on a wire at the base of the flame the sodium will be vaporized, the flame will be colored yellow by incandescent sodium gas, and in the spectroscope will be seen the two bright yellow lines of the sodium spectrum, exemplifying the second principle. Now let the electric lamp be turned on again, so that its light passes through the sodium flame; the continuous spectrum again appears, but this time it is crossed by a pair of *dark* lines in the yellow in the position that was occupied just before by the bright sodium lines. This illustrates the third principle.

The word *line* is not to be interpreted here in its strict mathematical sense; every spectral "line" has width, and both the width and the intensity of different lines may be different. Absorption lines are often described by their equivalent width, the width which a totally black gap in the spectrum would require in order to subtract the same amount of energy. The equivalent width of a line thus depends upon both its actual width and its intensity.

Lines in the emission or absorption spectra of gases which are composed of molecules instead of simple atoms are very numerous and are grouped closely together to form bands, which are usually sharply defined on one side and fade off gradually on the other. Common examples of such gases are the oxygen and nitrogen of the air and the carbon molecules which shine in the base of a blue gas flame. The spectrum of the latter consists of bands which are sharp on the redward side and is called, from its earliest investigator, the Swan spectrum.

Selective absorption is sometimes produced also by liquids and solids, for example, by the Crookes glass used for special kinds of spectacle lenses, and by a solution of chlorophyll, the coloring matter of green leaves. This kind of absorption is manifested by dark bands that are broad and hazy, and not made up of fine lines.

**The Solar Spectrum.** Part 3 of Figure 124 represents the spectrum of the Sun as it appears in a grating spectroscope of moderate dispersion. A few of the many dark lines are shown, and are marked with their Fraunhofer letters. According to the third principle of spectral analysis, since the solar spectrum is a dark-line one, we may infer at once that the Sun consists of a central core of highly heated material in such a state as to give a continuous spectrum, and that this core shines out through an envelope or atmosphere of less intensely heated gases. Moreover, by comparing the solar spectrum with the spectra of terrestrial substances, the presence of many familiar elements in this gaseous envelope may be established. Figure 125 shows

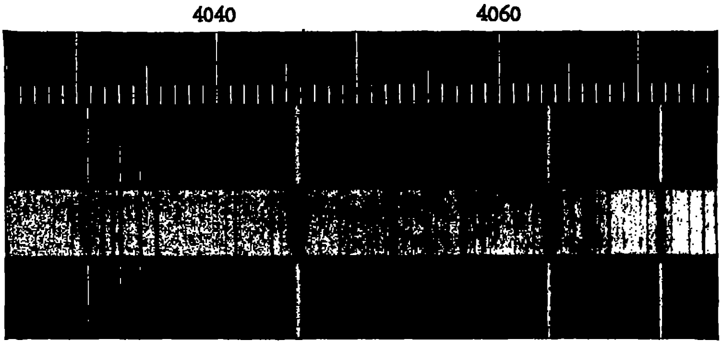



Fig. 125. *Part of the Solar. Spectrum with Iron for Comparison. (Photograph from Mount Wilson Observatory.)*

a portion of the violet end of the solar spectrum, confronted above and below by the corresponding part of the spectrum of an arc that was formed by passing an electric current between rods of iron. In the making of the photograph, the middle of the spectrograph slit was covered while the ends were illuminated by the arc; and the ends were covered while the middle was illuminated by sunlight. The spectra are thus made to correspond, and it is seen that each of the bright iron lines has a dark counterpart in the spectrum of the Sun. Some three thousand of these coincidences have been established, giving three-thousandfold evidence that the atmosphere of the Sun contains iron. Since, in order to produce spectral lines, the iron must be so hot as to be not only melted but vaporized, and since the core must be hotter still, this feature of the spectrum is alone sufficient to inform us that the Sun is intensely hot.

The principal Fraunhofer lines are as follows:

A. Band in the extreme red, sharply defined on the side toward the violet, due to oxygen, but belonging to the Earth's atmosphere instead of the Sun's, as shown by its intensification when the Sun's altitude is low.[1]
a. Rather narrow band in the red, due to terrestrial water vapor.
B. Band in the red, similar to A, due to terrestrial oxygen.
C. Line in the red due to solar hydrogen; identical with $H\alpha$.
D. Double line in orange-yellow, due to sodium in the Sun.
E. Close group of lines in green, due to iron and calcium.
b. Close group in green, due mainly to magnesium.
F. Hydrogen line in blue-green, identical with $H\beta$.
G. Hydrocarbon band in blue-violet.
H. Very broad calcium line in extreme violet.
K. Very broad calcium line, similar to H, in very extreme violet. K was not lettered by Fraunhofer, and can be seen only when very bright sunlight is used; but on photographs, H and K are the most prominent of all.

**The Spectra of Stars and Nebulae.** The great majority of stars, like the Sun, have dark-line spectra. Many star spectra, indeed, are practically identical with the solar spectrum, but among others there is great diversity in details such as the number, arrangement, and intensity of the lines. Some stellar spectra even contain some bright lines. It is certain that the stars are suns (or rather, that the Sun is one of the stars), and it is equally certain that among the stars there is great variety of physical conditions.

Many nebulae have spectra consisting of bright lines and are thus known to be clouds of shining gas. Others have spectra identical with the spectra of nearby stars, and are believed to shine by reflected starlight. Still others —the numerous extra-galactic nebulae or distant galaxies—have dark-line spectra such as would be expected from the mingled light of many stars.

The spectra of stars and nebulae and their interpretation will be discussed at greater length in later chapters of this book.

**Dispersion by a Grating.** A diffraction grating is made by ruling fine, equidistant grooves or "lines" with a diamond point upon a polished surface of glass or metal. When white light is transmitted or reflected by such a ruled surface it is dispersed, and the dispersion is the greater the closer the ruling. The dispersion of a grating having, for example, 15,000 lines to the inch is much greater than that of a glass prism. Unlike a prism, the grating causes red light to deviate more than violet, and instead of a single spectrum the grating produces a double series of spectra, the dis-

[1] Also by the absence of a Doppler-Fizeau displacement when the spectra of the receding and approaching limbs of the Sun are compared (page 168).

persion of the second being nearly twice that of the first, that of the third about three times that of the first, and so on. Fine gratings have been widely used in the study of the Sun, but not so much in investigations of other objects because they were wasteful of light. This deficiency has been mitigated in gratings made recently at Pasadena under the direction of Babcock, who uses a specially shaped diamond so that most of the light is thrown into a single spectrum.

**The Wave Nature of Light.** The action of the grating upon light shows that light consists of waves, the length of which is different in light of different colors, and it enables the experimenter to measure the length of these waves.

A complete discussion of the grating is far beyond the scope of the present book, and for this the reader is referred to advanced works on physical optics; but an elementary idea of the theory may be obtained from the following simplified description of the action of the transmission grating. For practical reasons, most gratings are ruled on metal surfaces and are reflection gratings; their theory is somewhat more difficult.

In a homogeneous medium, waves spread out from a point-source with equal velocity in all directions, so that the wave front is a sphere with the source at the center and the light is propagated in straight lines radially from the source. If the light encounters an obstacle, the waves bend slightly around the edge, and the limit of the shadow cast by the obstacle is not perfectly definite. This bending is called **diffraction.** The shorter the wave, the less the bending; light waves are so short that the diffraction is difficult to detect unless the source of light is very small and the edge of the obstacle very sharp. In the grating, the diamond lines are effective obstacles with very sharp edges, the light being transmitted or reflected by the narrow spaces of polished glass between them. Let $A$ and $B$, Figure 126, be the upper edges of two adjacent spaces of a grating, seen edgewise and highly magnified, and let the grating be illuminated from the left side with light of a single wave-length. Waves of light will be diffracted and will proceed in all directions from $A$ and from $B$. Let the light be focused upon a screen $CD$ by means of a convex lens. Waves that proceed from $A$ and $B$ in a direction normal to the grating will meet at $E$, after having traveled equal distances. Hence, the crest of a wave arriving from $A$ will meet the crest of one arriving from $B$, and the two will reinforce each other—there will be **constructive interference.** At a certain point $H$ above $E$, and at another $H'$ the same distance below $E$, the wave from one of the apertures will have traveled farther than the one from the other by a distance equal to half the wave-length; there a crest from $A$ will meet a trough from $B$, and the waves will neutralize each other—there will be destructive

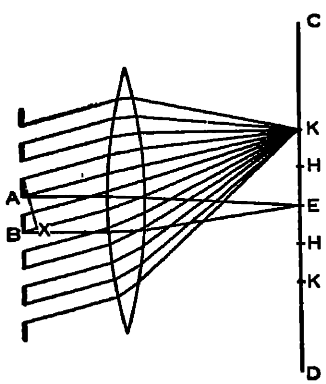


Fig. 126. *Action of the Diffraction Grating.*

interference. At $K$ and $K'$ the difference of path, $BX$, is one whole wave-length, constructive interference again occurs, and a bright point like that at $E$ is produced; and there will be a succession of these bright points where the difference of path from adjacent apertures is 0, $\lambda$, $2\lambda$, $3\lambda$, $\cdots$, where $\lambda$ is the wave-length of the light. If light of a different wave-length is used to illuminate the grating, its bright points fall between those that we have established, and so light of different wave-lengths is sorted out and a spectrum is formed. The greater the wave-length $BX$, the greater will be the distance $EK$, and from the distance $EK$ and the distance of the screen from the grating it is easy to calculate the wave-length.

The grating thus sorts out the light according to wave-length, producing a series of strips, in each of which the light of greatest wave-length is deviated most. What we actually see is a series of spectra in which the red is deviated most; hence, color must be a matter of wave-length, and red waves must be the longest of those that impress the eye. In grating spectra as ordinarily produced, the distance between any two lines is nearly proportional to their difference of wave-length, whereas in prism spectra the dispersion is much greater for short waves than for long waves.

**Wave-Length, Wave-Number, and Frequency.** It is customary to denote the wave-length of light by $\lambda$ and to express it in a unit called an **angstrom,** which is one ten-millionth of a millimeter, or $10^{-8}$ centimeters. Measurement with diffraction gratings shows the wave-length of light to be very small, that of the reddest visible light being about 7800 angstroms (0.000078 cm.) and that of the extreme violet about half as great.

Closely related to wave-length are two other numbers, called wave-number and frequency. The **wave-number** $n$ is the number of waves in a length of one centimeter measured along the line of advance of the waves. If the wave-length is expressed in centimeters, then $n = 1/\lambda$; if in angstroms, then $n = 10^8/\lambda$. The **frequency** $\nu$ is the number of waves that arrive at a given point in one second and is therefore $c/\lambda$ if $\lambda$ and $c$, the velocity of light, are expressed in the same units. Thus, when $\lambda$ is expressed in angstroms,

$$\nu = \frac{3 \cdot 10^{18}}{\lambda}.$$

The wave-length of radio waves (which also travel with velocity $c$) is often expressed in meters, and their frequency is expressed in kilocycles (one thousand waves per second) or megacycles (one million waves per second). In kilocycles, then, the frequency of radio waves = 300000 ÷ wave-length in meters; in megacycles, frequency = 300 ÷ wave-length in meters.

**Comparison of Light and Sound.** It is well known that sound consists of waves in the air and that the frequency of the waves determines the pitch of the

sound, a high note being due to a short wave-length. The wave-length of middle C of the scale is about four feet, enormously greater than that of visible light. Ascending the scale one octave divides the wave-length by 2, ascending two octaves divides it by 4, and so on. Since the wave-length of violet light is about half that of red, it is evident that the eye is sensitive to but a single octave of light, whereas the ear detects sound over a range of some ten octaves.

The velocity of sound is about 1090 *feet* per second, and that of light is, as we have seen, about 186,000 *miles* per second. The frequency of middle C is 256 waves per second. That of yellow light, having the wave-length of the D line of sodium, is about 500,000,000,000,000 ($5 \times 10^{14}$) per second.

Sound waves are *longitudinal*—that is, the motion of the particles of air is backward and forward along the line of advance of the wave. Waves on the surface of a pond are *transverse*, the water particles vibrating up and down at right angles to the line of advance. Light waves also are transverse, but the vibration is of an electromagnetic nature, not a motion of material particles. Unlike sound, light does not depend upon air or any other substance for its transmission from one point to another.[2] In ordinary (unpolarized) light the vibrations are in all directions within planes normal to the ray. In the special cases where they are ordered within those planes—along a straight line, in an ellipse, or in a circle—the light is said to be **plane polarized**, or elliptically or circularly polarized.

The ear is provided with a special organ, called the organ of Corti, by which we recognize sounds of different pitch, even when they are produced simultaneously, as by an orchestra or a choir; but the eye has no such mechanism, and its place may be said to be taken by the spectroscope.

**Invisible Radiation.** Although the eye is not sensitive to light of wave length much less than 3900 angstroms, a photographic plate placed beyond this point in the spectrum shows the existence of much shorter waves which constitute **ultra-violet light**. The spectra of the heavenly bodies have been studied down to about $\lambda 2900$, but to waves shorter than this the Earth's atmosphere is almost perfectly opaque, so that the extreme ultra-violet part of celestial spectra has never been observed. Using reflection gratings enclosed with their sources of light in vacua, Millikan has detected lines in the laboratory spectra of certain elements due to waves as short as 200 angstroms.

The wave-length of X-rays is of the order of one angstrom. Even the finest artificial gratings are much too coarse for use in the study of X-ray spectra, and so recourse is had to crystals, in which the natural orderly arrangement of molecules replaces that of the ruled grooves of the grating. The $\gamma$-rays emitted by radioactive substances are of smaller wave-length than X-rays, and the highly penetrating **cosmic rays** are believed by Millikan and others to be waves of still smaller length—of the order of 0.001 angstrom.

[2] In the eighteenth and nineteenth centuries, physicists speculated concerning the **luminiferous ether**, a hypothetical, highly elastic, perfectly incompressible medium supposed to pervade all space and yet to offer no resistance to the motions of planets and stars. With the rise of the theory of relativity, the idea of an ether has lost its prominence.

A delicate thermometer or thermopile, or a specially sensitized photographic plate, can show that beyond the red of the visible spectrum lies a great range of wave-lengths belonging to what is called **infra-red** light. There is, in fact, no break in the series between these waves and the long waves produced at radio broadcasting stations, which are in some cases several miles long. The longest known ether wave is thus some 100,000,000,000,000,000 times as long as the shortest, a range of about sixty octaves, to only one octave of which the human eye is sensitive. Ultraviolet, visible, and infra-red waves are all included in the physicist's term **radiation.**

**The Doppler-Fizeau Principle.** An observer who stands near a railroad track while a locomotive with a sounding bell passes may notice that the pitch of the sound drops suddenly at the moment when the engine is nearest. The reason for this may be seen from Figure 127. Let the engine move along the straight line from right to left, and let the positions of the bell at the moments of emitting successive wave crests be *A*, *B*, · · · *F*. The wave emitted at *A* spreads in the air in all directions with a velocity of 1090 feet a second, and when the engine has arrived at *F* this wave is represented by the largest circle, which is centered at *A*. The crest emitted at *B* spreads in the next circle, which is centered at *B*, and so on. Hence, when the bell has arrived at *F*, the successive waves have the positions shown by the five circles in the figure, and are crowded together in front of the bell and drawn apart behind it. That is, the sound received by an observer in front of the locomotive must be of shorter wave-length, and hence of higher frequency and pitch than the sound received in the rear. A similar effect may be noticed by passengers on a rapidly moving train in the pitch of the sound of warning bells at grade crossings. Here the bell is stationary and the observer moves. The wave length is not modified, but the frequency with which the waves are received by the observer is greater as he approaches the bell than as he recedes, and the effect on the pitch is the same as if the wave-length were changed.

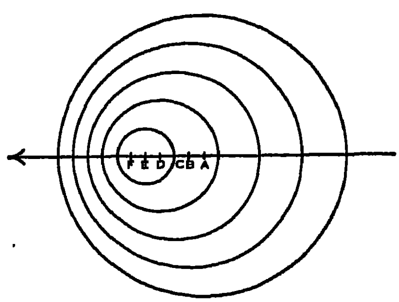



Fig. 127. *The Doppler-Fizeau Principle.*

The effect of motion on the pitch of sound was first explained in 1842 by Doppler of Prague, who realized that a similar effect would take place in waves of light, and

inferred that a rapidly approaching star would be blue and a receding one re‹ inference, however, is incorrect, for the star differs from the bell in emittin‹ having a great range in length; since the effect of recession is to lengthen waves, the whole spectrum is shifted, a little of the visible portion being the infra-red and a little of the ultra-violet being shifted over into the visible. over, the change of wave-length due to any velocity as yet detected in tl would not be sufficient to make a change in color that could be noticed by ı

The correct astronomical application of Doppler's principle was by Fizeau in a paper read before a learned society in Paris in 1848 l published until 1870. Although all wave-lengths are represented spectrum of a star, each *line* of the spectrum represents but a singl‹ length; hence the effect of motion along the line of sight is to mod wave-length of each line, and so to displace the line in the spectrun Doppler-Fizeau principle, which is of incalculable importance in omy, may be stated as follows: *When the distance between an observe source of light is increasing, the lines of the spectrum lie farther to the r their normal positions, and when the distance is diminishing they lie fa the violet, the displacement being proportional to the relative velocity of or approach.* The formula for the change of wave-length of a spectra

$$\Delta\lambda = \frac{v}{c}\lambda$$

where $v$ is the relative velocity of the source and the observer alc line of sight (called the **radial velocity** or **sight-line velocity**), $c$ velocity of light, and $\lambda$ is the wave-length of the line. The Doppler effect is illustrated in Figure 128, in which the upper and lower spe

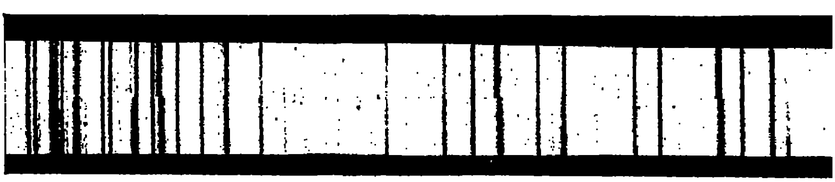


Fig. 128. *Spectra of Approaching and Receding Limbs of the Sun.*

of points near the Sun's western and eastern limbs respectively, tak‹ high dispersion at the 150-foot tower telescope on Mount Wils further enlarged. Most of the lines shown are caused by absorp the Earth's atmosphere, and the two spectra have been confronted these lines match; but there are also several lines due to iron in t and they are relatively displaced, those from the eastern limb tow violet, because the Sun is rotating, the eastern limb approaching

the western limb receding. The relative sight-line velocity is about 4 kilometers per second.

**The Laws of Radiation and the Continuous Spectrum.** All bodies, unless at the "absolute zero" of temperature (representing the complete absence of heat and assigned by physicists at $-273°$ Centigrade) emit some radiation, although it may not be perceptible to the eye. If the body is "black"—that is, capable of completely absorbing radiation of all wave-lengths[8]—its spectrum is continuous and the relations of its temperature to the quantity and quality of its radiation are expressed by certain laws which have been well established by the experimental and theoretical researches of physicists. Although the stars probably do not radiate exactly like black bodies, these laws afford valuable means of investigating their temperature. The **effective temperature** of a star is defined as the temperature at which a black body of equal area would emit the same amount and kind of radiation.

The **Stefan-Boltzmann law** states that *the total rate of radiation of a black body (all wave-lengths combined) varies as the fourth power of the absolute temperature;* that is,

$$E = \sigma T^4$$

where $E$ is the rate of emission of energy, $T$ the absolute temperature, and $\sigma$ a constant.

This law is by no means self-evident; for example, a doubling of the temperature might reasonably be expected merely to double the rate of emission of radiation, but in fact it would increase the rate of emission sixteenfold.

The intensity of the radiation emitted by a body is not the same at all wave-lengths, but is a maximum at a certain point in the spectrum whose position depends upon the temperature of the body. For cool bodies this point of maximum intensity of the continuous spectrum lies far in the infra-red, but with rising temperature it shifts to smaller wave-lengths and enters the visible portion when the temperature reaches about 4000° C. An obvious inference is that blue-white stars like Rigel are hotter than red stars like Antares. The precise relation is simply stated by **Wien's law:** *The wave-length of the point of maximum intensity in the continuous spectrum of a black body is inversely proportional to the body's absolute temperature;* or,

$$\lambda_{max} = \frac{C}{T}.$$

[8] This technical use of the word black does not agree with the ordinary use unless the body is so cool as not to emit visible light. If heated to incandescence, the technically "black" body would appear white-hot.

**Planck's Equation,** which states the rate of emission, $E_\lambda$, of radiation of any particular wave-length $\lambda$, is more complicated:

$$E_\lambda = \frac{c_1\lambda^{-5}}{e^{\frac{c_2}{\lambda T}} - 1}$$

where $e$ is the base of natural logarithms. Both the Stefan-Boltzmann law and the Wien law may be derived from Planck's equation.

If the rate of radiation is expressed in ergs per square centimeter per second, the wave-lengths in centimeters, and the temperatures in Centigrade degrees, the constants of the above laws have the following values:

$$\sigma = 5.72 \times 10^{-5}$$
$$C = 0.289$$
$$c_1 = 3.71 \times 10^{-5}$$
$$c_2 = 1.435$$

The laws of radiation may be better understood from a study of the curves in Figures 129 and 130, which were drawn from data calculated by Planck's formula. In Figure 129, the rate of emission of energy in ergs per square centimeter per second (ordinates) is plotted against wave-length in angstroms (abscissae) for values of the absolute temperature ranging from 3000° to 8000°. The ordinate of any point is Planck's function $E_\lambda$; the abscissa corresponding to the highest point of the curve is Wien's $\lambda_{max}$; and the area under the curve is the total emission $E$ as given by Stefan's law. The shift toward violet of the top of the curve and the rapid increase of area which accompany increasing temperature illustrate Wien's law and the Stefan-Boltzmann law respectively.

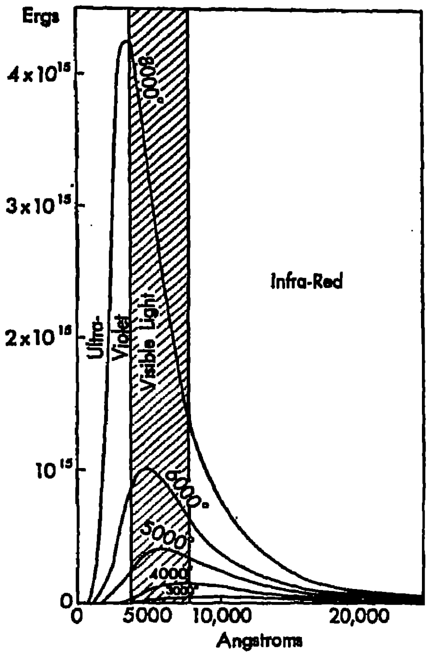



Fig. 129. *Distribution of Energy in the Spectrum of a Black Body.*

The height of the curve increases so fast with increasing temperature that it is not practicable to plot curves in the usual way for the great range of temperatures that is believed to exist among the stars. In Figure 130 use is made of a logarithmic scale, both the ordinates and abscissae increasing in a geometric instead of an arithmetic progression, and it is thus possible to represent both large and small numbers in the same drawing. The curves show that at either very high or very low temperatures, only a small fraction of the radiation of a star is sensible to the eye; and that at temperatures of millions of degrees, which are believed to exist in the interiors of stars, while the radiation of all wave-lengths is enormous the

extremely short waves (X-rays) vastly predominate. However, such short waves are unable to penetrate any great thickness of the star's atmosphere and so are imprisoned within the interior, to be transformed slowly into longer waves which as slowly filter out (page 378).

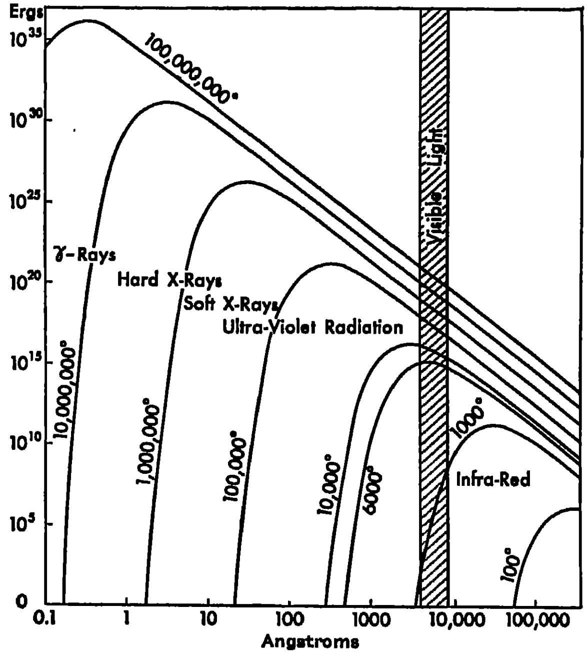



Fig. 130. *Energy Curves of a Black Body.*

**Fluorescence.** When radiation is absorbed by the atoms of matter, it may be reëmitted as radiation of the same wave-length (resonance) or of a different and generally greater wave-length (fluorescence). Certain minerals fluoresce visibly in beautiful tints under the action of ultra-violet light and attract much attention when so exhibited in museums. Such materials are used as lining in the tubes of fluorescent lamps, in which an electric current, passing through mercury vapor, generates within the tube a supply of ultra-violet radiation to which the glass is of course opaque but to which the fluorescent material responds brilliantly. The luminosity of comets is accounted for by fluorescence of their gases caused by the radiation of the Sun, and that of gaseous nebulae by fluorescence in the powerful ultra-violet radiation of neighboring hot stars.

**Absorption of Radiation by the Air; the Greenhouse Effect.** The Earth's atmosphere absorbs all radiation from the heavenly bodies of wave-length less than about 2900 angstroms, so that astronomers are

prohibited from investigating this region of solar and stellar spectra, which is doubtless rich in information. This absorption of the ultraviolet is due mainly to ozone high in the stratosphere. Fabry and Buisson have shown that a layer of ozone only three millimeters thick, if it were at sea-level density, would be enough to account for the obscuration.

The red and infra-red beyond λ7000, while not completely blocked, is heavily banded by atmospheric water vapor and to a less extent by atmospheric oxygen and carbon dioxide. The powerful radiation of the Sun between λ2900 and λ7000 is freely transmitted by the air and, being absorbed by the soil, warms the Earth's surface to a temperature near 300° absolute. The warm soil then emits radiation, but it is of the extreme infra-red kind only, and as this is partly absorbed by the air it does not escape rapidly into space. For this reason the top of a lofty mountain, where the air is attenuated, is cooler than the base. In the same way, the energy of the Sun is trapped by the glass in the roof of a greenhouse, the glass being largely opaque to infra-red radiation.

**The Pressure of Radiation.** It was shown theoretically by Clerk Maxwell in 1873, and since 1900 has been proved by experiment, that radiation exerts a pressure on any surface upon which it falls. The pressure of sunlight on the Earth's surface is so small—about two pounds per square mile—that it is difficult to detect; but in the immediate neighborhood of the Sun or other star, and especially in the interiors of stars, its effect on small bodies must be important.

The radiation pressure upon a body is proportional to the body's area or (for a sphere) to the square of its radius. The Sun's gravitational attraction for the body is proportional to its mass or, for a given density, to the *cube* of its radius. Both are inversely proportional to the square of its distance from the Sun. Therefore, whatever the distance, the ratio of radiation pressure to gravitational attraction is inversely proportional to the first power of the radius; hence, there must be a certain critical diameter for a body of given density, for which the light-pressure equals the attraction. Bodies having a diameter less than this critical value (which, for unit density, is about 0.0015 mm.) will be repelled instead of attracted by the Sun.

**The Zeeman and Stark Effects.** In 1896 Zeeman, in Holland, found that when a flame or other source of light is placed in a magnetic field, the lines of its spectrum are each doubled, trebled, or even further multiplied; and that, when the source is viewed from a direction at right angles to the lines of magnetic force, the light is plane-polarized, while if viewed along the lines of force—through a hole bored in the pole-piece of the magnet—the light is circularly polarized, the polarization conforming to a simple theory which was given a little later by Lorentz. In 1913 the German physicist Stark discovered that spectral lines may be split also by an electric field, the components of each line being plane-polarized. The Zeeman effect has been studied chiefly in the spectra of metals; the Stark effect, in the spectra

of hydrogen and helium. Both require strong fields and high dispersion for their demonstration in the laboratory; with moderate equipment the lines appear merely broadened or even unchanged. These phenomena, particularly the Zeeman effect, are of some importance in astronomy and have proved to be powerful tools for the physicist in the interpretation of spectral phenomena and of the relations of matter to energy.

**Series of Lines in Spectra.** In the spectra of many elements, the lines are so situated as to form series in which the wave lengths or frequencies fall into simple numerical formulae. Recognition of these series has proved a leading clue to the interpretation of spectra in terms of the structure of atoms; accordingly in modern physics and astrophysics it has outstanding significance.

The most obvious spectral series is one of hydrogen (Figures 124 and 131). A wide gap intervenes between the red line, Hα, and the green one, Hβ; a shorter gap between Hβ and the blue line, Hγ; and as we proceed into the ultra-violet the gaps become shorter (and the lines weaker)

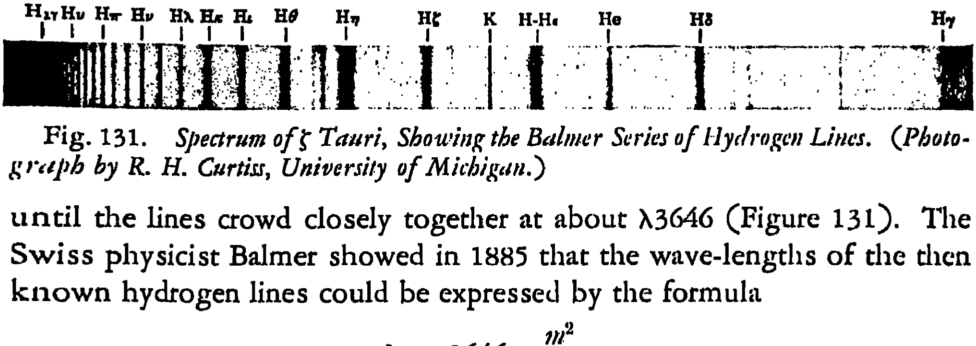



Fig. 131. *Spectrum of ζ Tauri, Showing the Balmer Series of Hydrogen Lines. (Photograph by R. H. Curtiss, University of Michigan.)*

until the lines crowd closely together at about λ3646 (Figure 131). The Swiss physicist Balmer showed in 1885 that the wave-lengths of the then known hydrogen lines could be expressed by the formula

$$\lambda = 3646 \cdot \frac{m^2}{m^2 - 4}$$

by giving to *m* the successive values 3, 4, 5, 6 · · ·, as in Table 4. More than thirty hydrogen lines obeying this formula have been observed in the spectra of the solar chromosphere and of some stars, and beyond the limit at λ3646 there extends a weak continuous spectrum, discovered by Wright in the stars and by Evershed in the chromosphere.

Table 4

THE BALMER SERIES

| Line | *m* | λ |
|---|---|---|
| Hα | 3 | 6563 |
| Hβ | 4 | 4861 |
| Hγ | 5 | 4340 |
| Hδ | 6 | 4102 |
| Hε | 7 | 3970 |
| . . . | . | . . . . |
| Limit | ∞ | 3646 |

Three other series of hydrogen lines are now known: the ultra-violet Lyman series extending from λ1216 (Lyman α) to λ512; the infra-red Paschen series converging to λ8204; and the far infra-red Brackett series with limit at λ14588. Only the Balmer and Paschen series have ever been observed in celestial spectra, the others being obstructed by the Earth's atmosphere. These series are seen to be mutually related if, in formulae of the Balmer type, we change from wave-length λ to wave-number $n$ by taking the reciprocal and multiplying by $10^8$:

THE HYDROGEN SPECTRUM

$$\text{Lyman series:} \quad n = R\left(\frac{1}{1^2} - \frac{1}{m^2}\right), \quad m = 2, 3, 4, \cdots$$

$$\text{Balmer series:} \quad n = R\left(\frac{1}{2^2} - \frac{1}{m^2}\right), \quad m = 3, 4, 5, \cdots$$

$$\text{Paschen series:} \quad n = R\left(\frac{1}{3^2} - \frac{1}{m^2}\right), \quad m = 4, 5, 6, \cdots$$

$$\text{Brackett series:} \quad n = R\left(\frac{1}{4^2} - \frac{1}{m^2}\right), \quad m = 5, 6, 7, \cdots$$

$R$ stands for the number 109678, which is known as Rydberg's constant. Other elements exhibit spectral series of a more complicated character.

**The Particles of Matter.** All matter, whether gaseous, liquid, or solid, is composed of **atoms**. Ninety-two kinds of atoms are recognized, and the atoms of each kind are particles of one of the fundamental substances or **chemical elements** which are listed in Table 5. On Earth the atoms of the 92 elements combine in many thousands of ways to form more or less complex **molecules**, the ultimate particles of chemical compounds. Few such combinations take place at the high temperatures of the stars, so that stellar matter is mostly atomic gas. The molecule is too small to be seen even with the highest-power microscope, and the still smaller atom was long believed to be the ultimate particle of matter. (The word was derived from Greek ἄτομος, which means indivisible.)

Twentieth-century physicists have revealed a subatomic world and have shown that many phenomena, including the phenomena of spectra, are best explained by supposing that an atom is an electrical structure composed of still smaller particles, that most atoms are highly complicated, and that natural laws operate within the atom which are not all in evidence in the world we perceive directly with our senses. Following Sir Ernest Rutherford (1911), we picture the atom as a **nucleus** which possesses a positive electrical charge and most of the atom's mass, accompanied by a retinue of **electrons**, or units of negative electricity that revolve around

the nucleus like planets around the Sun. The average electronic orbit, though only a few billionths of a millimeter in size, is thousands of times larger than the nucleus, so the atom is conceived to be mostly empty space.

Table 5

THE CHEMICAL ELEMENTS

| Atomic Number | Name | Symbol[a] | Atomic Number | Name | Symbol[a] |
|---|---|---|---|---|---|
| 1 | Hydrogen | H | 48 | Cadmium | Cd |
| 2 | Helium | He | 49 | Indium | In |
| 3 | Lithium | Li | 50 | Tin | Sn |
| 4 | Beryllium | Be | 51 | Antimony | Sb |
| 5 | Boron | B | 52 | Tellurium | Te |
| 6 | Carbon | C | 53 | Iodine | I |
| 7 | Nitrogen | N | 54 | Xenon | Xe |
| 8 | Oxygen | O | 55 | Caesium | Cs |
| 9 | Fluorine | F | 56 | Barium | Ba |
| 10 | Neon | Ne | 57 | Lanthanum | La |
| 11 | Sodium | Na | 58 | Cerium | Ce |
| 12 | Magnesium | Mg | 59 | Praseodymium | Pr |
| 13 | Aluminum | Al | 60 | Neodymium | Nd |
| 14 | Silicon | Si | 61 | Illinium | Il |
| 15 | Phosphorus | P | 62 | Samarium | Sm |
| 16 | Sulphur | S | 63 | Europium | Eu |
| 17 | Chlorine | Cl | 64 | Gadolinium | Gd |
| 18 | Argon | A | 65 | Terbium | Tb |
| 19 | Potassium | K | 66 | Dysprosium | Dy |
| 20 | Calcium | Ca | 67 | Holmium | Ho |
| 21 | Scandium | Sc | 68 | Erbium | Er |
| 22 | Titanium | Ti | 69 | Thulium | Tm |
| 23 | Vanadium | V | 70 | Ytterbium | Yb |
| 24 | Chromium | Cr | 71 | Lutecium | Lu |
| 25 | Manganese | Mn | 72 | Hafnium | Hf |
| 26 | Iron | Fe | 73 | Tantalum | Ta |
| 27 | Cobalt | Co | 74 | Tungsten | W |
| 28 | Nickel | Ni | 75 | Rhenium | Re |
| 29 | Copper | Cu | 76 | Osmium | Os |
| 30 | Zinc | Zn | 77 | Iridium | Ir |
| 31 | Gallium | Ga | 78 | Platinum | Pt |
| 32 | Germanium | Ge | 79 | Gold | Au |
| 33 | Arsenic | As | 80 | Mercury | Hg |
| 34 | Selenium | Se | 81 | Thallium | Tl |
| 35 | Bromine | Br | 82 | Lead | Pb |
| 36 | Krypton | Kr | 83 | Bismuth | Bi |
| 37 | Rubidium | Rb | 84 | Polonium | Po |
| 38 | Strontium | Sr | 85 | Virginium[b] | — |
| 39 | Yttrium | Y | 86 | Radon | Rn |
| 40 | Zirconium | Zr | 87 | Alabamine[b] | — |
| 41 | Columbium | Cb | 88 | Radium | Ra |
| 42 | Molybdenum | Mo | 89 | Actinium | Ac |
| 43 | Masurium | Ma | 90 | Thorium | Th |
| 44 | Ruthenium | Ru | 91 | Protoactinium | Pa |
| 45 | Rhodium | Rh | 92 | Uranium | U |
| 46 | Palladium | Pd | 93 | Neptunium[c] | Np |
| 47 | Silver | Ag | 94 | Plutonium[c] | Pu |

[a] Derived from the Latin form of the name. [b] Existence not universally accepted.
[c] "Transuranian" element; produced artificially.

The positive charge of the nucleus is held by particles called **protons,** each of which has a charge precisely equal to that of an electron, and a mass about 1835 times the electron's mass. The simplest atom, that of hydrogen, consists of a single proton forming the nucleus and a single revolving electron. In other atoms the nucleus contains both protons and **neutrons.** The neutron is a particle that has a slightly greater mass than the proton but no electrical charge.

The number of protons in the nucleus, which in the normal atom is equal to the number of attendant electrons, is called the **atomic number** of the element. The number of protons and neutrons together is called the **mass number** and is approximately equal to the atomic weight, which is measured by the chemist. It is the atomic number, not the mass number, that determines the chemical element to which the atom belongs. Atoms having the same atomic number but different mass numbers form different **isotopes** of the same element. Most of the chemical elements are mixtures of isotopes of which the chemical properties are identical. The dream of the early alchemists, of transmuting one element into another, is fulfilled spontaneously by radioactive elements such as uranium and radium, and artificially by giant cyclotrons and electrostatic generators, when the nuclei of atoms are broken up and their atomic numbers changed.

**The Emission and Absorption of Radiation.** An atom in its ordinary or normal state emits no energy; but it may absorb energy if it is subjected to a rise in temperature, to impact with another particle, to radiation arriving from without, or to electrical stimulation. An atom which has just absorbed energy is said to be **excited.** The absorbed energy is generally retained less than a millionth of a second; it is emitted by a transition to a state of lower excitation or a complete return to the normal state. Remarkably, the change of energy content of an unrestricted atom is never gradual but, in each transition, is a definite amount (**quantum**) which, if in the form of radiation, is proportional directly to the frequency.[4] Thus, from a shining body of gas, radiation is emitted in certain definite frequencies (and wave-lengths) and in no others; hence, when this radiation is analyzed by a spectrograph, it produces a spectrum consisting of isolated bright lines. When white light is sent through such a body of

[4] This fact is expressed by Planck's famous equation

$$E = h\nu$$

where $h$, known as Planck's constant, has the value $6.55 \times 10^{-27}$ ergs per second.

gas the atoms absorb from it the energy of just those frequencies which they are able to emit, and dark lines occur in exactly the same places as the bright lines of emission. Thus we have Kirchhoff's second and third principles of spectral analysis.

Bohr of Denmark explained the hydrogen spectrum in 1913 by supposing that the single hydrogen electron is capable of revolving only in any one of many orbits whose distances from the nucleus are proportional to the squares of the integral numbers—that is, to 1, 4, 9, 16, etc. The orbits are usually elliptic but may be circular; the four inner orbits are represented as circles in Figure 132. So long as the electron continues to revolve in any one of these orbits, the atom neither emits nor absorbs light; but, contrary to the laws of nature in the visible world, the electron is capable of falling or rising instantaneously from one orbit to another. If it falls from an outer to an inner orbit, the atom loses energy by one quantum and emits a wave of a length that depends on the distance fallen: if from the third to the second orbit, H$\alpha$ light is emitted; if from the fourth to the second, H$\beta$ light; if from the fifth to the second, H$\gamma$ light; and so on. If the electron falls from any outer orbit to the first, a line of the Lyman series is emitted; if to the third, a line of the Paschen series; if to the fourth, a line of the Brackett series. When energy is imparted to the atom it is received in an integral number of quanta, the electron jumps outward to a definite orbit, and an absorption line may be observed.

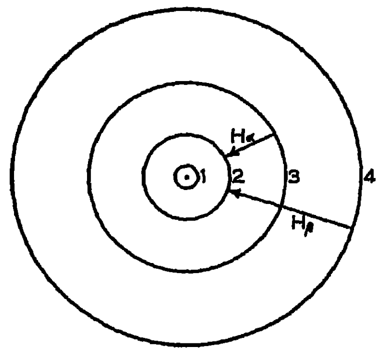



Fig. 132. *Orbits of the Hydrogen Electron.*

When an atom is acted upon with sufficient power, as by a very high temperature which increases the violence of its collisions with its neighbors, one or more electrons may not only be expelled to outer orbits, but be entirely lost to the atom. The atom is thus left with an excess positive electric charge and is not merely excited but **ionized.** Such an ionized atom, or positive **ion,** readily captures any free electron that may be near; and the fall of such electrons, at random speed, produces a continuous spectrum such as that observed beyond the limit of the Balmer series. The rate of recovery of a body of gas from ionization depends upon the density of the gas: if the density is high, the probability of encounters of atoms and free electrons is likewise high.

The spectrum of an ionized atom is quite different from that of a non-ionized atom, and so, as a body of gas is subjected to higher and higher temperatures and more and more atoms become ionized, the spectrum is modified by the emergence or strengthening of certain lines which are referred to as **enhanced lines.** The temperatures and pressures at which a given percentage of different elements is ionized are very different; hence the relative intensity of the enhanced and ordinary lines in the spectra of the stars gives a key to these conditions, a fact first pointed out by Saha of Calcutta in 1921.

The helium atom has two revolving electrons and a nucleus that consists of two protons and two neutrons. When one of the electrons is lost by ionization, the helium ion resembles the hydrogen atom in form but is about four times as massive and the charge on the nucleus is twice as great. The spectrum of ionized helium resembles the spectrum of hydrogen, and when one of its series of lines was discovered by E. C. Pickering in the hot star $\zeta$ Puppis in 1896, it was attributed to hydrogen and called the Pickering series.

There are certain transitions that can occur within the atom only when the time interval between collisions of atoms is exceptionally long. These transitions give rise to spectral lines which in ordinary sources are **forbidden** but which may be conspicuous in the spectra of very large bodies of highly rarefied gas such as the galactic nebulae.

The Rutherford-Bohr theory affords a picture of atomic processes which appeals to the imagination and has answered many questions, but it does not answer all the questions that have arisen in the recent progress in atomic and nuclear physics. These are treated by methods such as the wave mechanics of Heisenberg and Schrödinger, which are much too technical and mathematical for description here. It has appeared not only that light partakes of a corpuscular as well as an undulatory nature, but also that matter partakes of the nature of waves as well as of the nature of particles.

## EXERCISES

1. What is the *wave-number* of green light, the wave-length of which is 5000 angstroms? What is the *frequency* of such light?

*Ans.* 20,000; $6 \times 10^{14}$

2. Find the change of wave-length of $H\beta$, due to the orbital motion of the Earth, in the spectrum of a star toward which the Earth is directly moving at the moment of observation (take the Earth's orbital velocity as 30 km./sec. and the velocity of light as 300,000 km./sec.).

*Ans.* 0.49 angstrom

3. If a photograph of the spectrum of the star in Example 2 has a length of 25 millimeters between $H\beta$ and $H\gamma$, what is the linear displacement of $H\beta$ on the plate?

4. What is the wave-length of the brightest point in the spectrum of a black body whose temperature is $11000°$ K? What color would you expect a star of this effective temperature to have (e.g., Vega)?

5. Verify the wave-lengths of the hydrogen lines of the Balmer series given in Table 4.

6. Derive the formula for wave-number in the Balmer series from Balmer's $\lambda$ formula.

# CHAPTER 9

# THE SUN

**Importance of the Sun.** From three main points of view the Sun is to us inhabitants of the Earth, the most important of astronomical bodies. First, it is the ruler of the Solar System, controlling the motions of the Earth and all the other planets, the comets, and the streams of meteoric bodies, and influencing the motions of the satellites. This aspect of the Sun will appear in succeeding chapters.

Second, the Sun is a star, not by any means the largest or brightest or hottest star, but important to us because it is much the nearest, our next stellar neighbor being some 275,000 times as far away and the great majority of visible stars many times farther still. The Sun is the only star whose features we can study in detail.

Third, the radiation of the Sun is almost the sole source of power, warmth, activity, and life upon the surface of the Earth, the only exceptions worth mentioning being the tides and the activity of volcanoes and geysers which originates in the internal heat of the Earth.

Consider, for example, the manifestations of life on a city street, at night when it might seem that the Sun had no influence. The street is lighted and cars are being propelled upon it by the energy of electric current which is generated, perhaps, at a distant waterfall. The generators are run by turbines which are driven by the weight of falling water; but in order to fall, the water must have been raised to a higher level than that of the sea, and this was done by the radiation of the Sun which warmed the water of the ocean, causing it to evaporate and rise to form clouds which were wafted over the land by Sun-generated winds to fall as rain. If the Sun's radiation were cut off, the cataract would cease even before the existing supply of water was exhausted, for the temperature would speedily fall so low that the water would all be converted to immobile ice. Or perhaps the current is generated by the burning of coal. The energy then comes from the combination of atmospheric oxygen with carbon which was stored in plants ages ago under the mysterious action of the Sun's rays known as **photosynthesis,** and we are thus making use of "canned sunlight." The energy which drives the gasoline engines of automobiles may be traced to a similar ancient source; that of our own bodies is solar energy, stored not

so long ago by photosynthesis in the plants that form our food or the food of animals whose flesh we have eaten.

**General Description of the Sun.** The distance and diameter of the Sun have already been given (page 106). The distance is so great that, *to subtend an angle of 1″*—which is pretty small to observe with an ordinary telescope in the poor seeing that usually prevails in the daytime—*a marking on the Sun must be 450 miles in diameter*. The Sun is so large that, if the Earth were placed at its center, the Moon's orbit would lie only a little more than halfway out toward the surface.

The mass of the Sun (page 245) is about 333,000 times that of the Earth, and its surface gravity about 27.6 times the Earth's (a person who weighs one hundred pounds here would weigh nearly a ton and a half if transported to the Sun). Its form is that of a sphere. Its average density is about 1.4 times that of water. The temperature of its visible surface is about 6000° C.; that of the interior is certainly much higher, and is estimated at 20,000,000° C.

The Sun rotates in the same direction as the Earth upon an axis which is inclined 83° to the plane of the ecliptic and is directed to a point about halfway between Polaris and Vega, in $\alpha = 19^h 04^m$, $\delta = +64°$. The rotation is not the same all over the Sun, for a point at the equator turns faster than points in higher latitudes, the sidereal rotation period being 25 days at the equator, 27.5 days at latitude $\pm 45°$, and about 33 days at latitude $\pm 80°$. This shows, of course, that the surface of the Sun cannot be solid, for its parts move past one another; and its high temperature and other facts prove it to be gaseous throughout in spite of its high average density.

Upon the intensely brilliant visible surface of the Sun, which is called the **photosphere**, are often seen relatively dark spots called **sunspots**, some of which are many times larger than the Earth. Above the photosphere is the red **chromosphere** from which rise the vast flame-like **prominences**, and beyond all extends the tenuous **corona** which is conspicuous at the time of a total solar eclipse. The abundance and size of the spots and prominences, the form of the corona, and the magnetism of the Earth (which is thus shown to be connected with the Sun) all vary, for some unknown reason, in an irregular period which averages about 11 years.

As shown by the spectrum, the layer of the Sun which produces the Fraunhofer lines contains, in a state of vapor, some 65 of the chemical elements which are known upon the Earth. Those that have not been found

represented in the solar spectrum are rare on Earth or their strongest lines lie in the unobservable part of the spectrum. Light elements, especially hydrogen and helium, are proportionately much more abundant on the Sun than on Earth, and on the Sun there are some unfamiliar compounds, mostly oxides and hydrides. Otherwise, the chemical composition of the Sun appears to be not essentially different from that of the Earth.

**Methods of Observing the Features of the Sun.** To look directly at the Sun through a telescope would be disastrous, for the concentration of rays is so great that the observer would be quickly blinded. A piece of paper or other inflammable object placed in the usual position of the eye at a telescope directed to the Sun is at once set afire. For visual observations, one may make use of various devices known as helioscopes, which reflect away the greater part of the light after it has entered the telescope. (To reduce the aperture of the objective by a perforated cap would decrease the light but would also diminish the resolving power.) Another method is to project upon a white cardboard screen an enlarged image of the Sun formed by racking the eyepiece outward until the screen and the focal plane of the objective are at the conjugate foci of the ocular. The image may then be seen by a number of observers simultaneously.

For photographic observation, which for serious work has now almost entirely superseded the visual method, the intensity of the light is an advantage rather than a drawback, for it makes possible the use of slow plates, which are of finer grain than fast plates, and of very short exposures —one one-thousandth of a second or less. The photograph is usually made in the focal plane of a long-focus objective, the telescope often being mounted permanently in a horizontal or a vertical position and "fed" by a **coelostat** which reflects the light in a constant direction.

It was discovered in 1868 by Lockyer in England and Janssen in France that the solar prominences, which until then had been seen only during eclipses, could be observed in full sunlight by the aid of the spectroscope. Prominences cannot be seen under ordinary circumstances for the same reason that stars cannot be seen in daylight—the background of sky is too bright. The effect of the spectroscope is to spread the light of the sky (which is reflected sunlight) into a long spectrum and so to reduce its intensity; but the spectrum of the prominences is a bright-line one, and the spectroscope separates the bright lines without widening them and hence without weakening them, so they may be seen against the weakened background of the sky. To observe the prominences, therefore, a spectroscope of high dispersion —having a fine grating or a train of prisms—is attached to the tube of a telescope with its slit in the focal plane of the objective. The telescope is so directed that the slit is at one point nearly tangent to the image of the Sun but does not quite touch

it; if a prominence is situated at the corresponding point of the Sun's limb, the bright lines of its spectrum may be seen against the corresponding dark lines in the sky spectrum. Since an image of the prominence is formed on the slit-plate by the objective, the prominence-light entering the spectroscope comes from a narrow strip only of the prominence, and the bright lines of the spectrum are themselves images of the telescopic image of this strip, showing interruptions, for example, corresponding to any rifts that exist in that strip of the prominence. By widening the slit, a wider strip may be seen, and it is often possible thus to view the whole prominence if the sky is very clear. Widening the slit, however, admits more sky light without brightening the image of the prominence, and a slight haze, which has the effect of both dimming the prominence and brightening the sky, will make the observation of a whole prominence of any size impossible. The spectral line most frequently used for this kind of observation is the red (C or $H\alpha$) line of hydrogen, because it is the brightest in the prominence spectrum.

About 1890, Hale in America and Deslandres in France invented the **spectroheliograph**, by means of which the entire Sun is photographed in the light of a single spectral line. This instrument consists of a spectrograph of high dispersion which, in addition to the slit through which the light enters (and which we shall here call the first slit), is provided with a second slit placed in the focal plane of the camera objective and adjusted to the width of a line of the spectrum. The spectroheliograph is attached to a large telescope, the objective of which we shall call the *main objective*. The first slit is placed in the focal plane of the main objective, which is directed to the Sun. A narrow strip of the Sun's image is thus admitted by the first slit and a spectrum is formed, each line of which is an image of that narrow strip. The second slit is so placed that the light[1] of a single line passes through it, and behind the second slit is placed the photographic plate. Upon the plate there is thus formed a monochromatic image of the strip of the Sun's surface showing the distribution of the particular element, say hydrogen, that is responsible for the spectral line. A complete picture of the Sun, or rather of that portion of a certain layer which is composed of a single element, is then built up on the plate in one of two ways: either the main telescope is moved slowly so that the Sun's image travels at right angles to the first slit while at the same time the photographic plate is moved across the second slit, causing successive strips of the Sun to fall upon successive strips of the plate; or the plate and the main telescope are both kept stationary while the spectroheliograph with its two slits is moved slowly between them.

[1] It should be recalled that the Fraunhofer lines are *dark* only by contrast with the continuous spectrum which lies between them, and are not really without light.

In 1925 Hale perfected the **spectrohelioscope,** which makes perceptible to the eye the solar features which before had been detected only by the spectroheliograph. It is arranged precisely like the spectroheliograph except that the photographic plate is replaced by an eyepiece and the two slits are given a transverse motion so rapid that the eye receives the impression of a persistent image. The range of this motion is sufficient to admit a field of view covering a considerable portion of the Sun's surface.

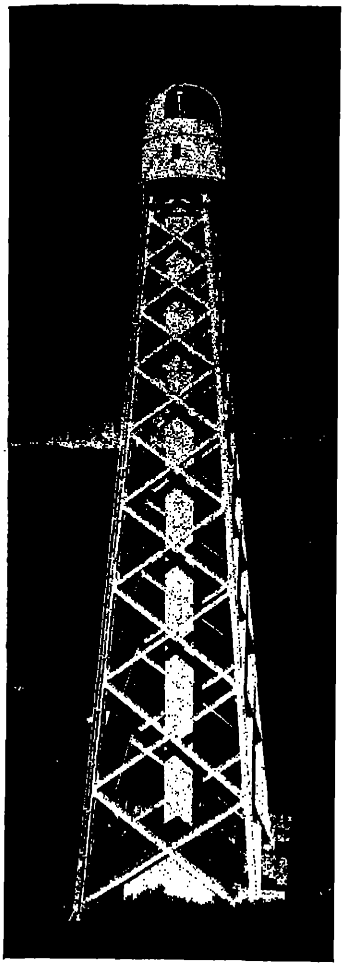


Fig. 133. *The 150-Foot Tower Telescope on Mount Wilson.*

About 1938 Öhman, followed by Evans and by Pettit, developed the **quartz-polaroid monochromator,** a device which acts as a light-filter and transmits light of a very narrow range of wavelengths. When attached to a telescope and adjusted to transmit the H$\alpha$ line, it so subdues the extraneous light as to make possible the observation, both visual and photographic, of remarkably fine solar detail. During the 1930's a modification of the spectroheliograph was perfected at the McMath-Hulbert Observatory with which vivid moving pictures are taken of the rapidly changing solar features.

In Figure 133 is shown the great tower telescope of the Mount Wilson Observatory. The tower is made double, each visible upright and crosspiece enclosing a similar member that belongs to the inner tower. The outer tower supports the dome and protects the inner tower from the wind; the inner one supports at the top a coelostat, an arrangement of mirrors driven by clockwork so as to reflect the Sun's light vertically downward, and, just below this, a twelve-inch lens of 150 feet focal length which forms a seventeen-inch image of the Sun at the foot of the tower. In a seventy-five-foot well beneath is placed a grating spectrograph whose slit is in the focal plane of the lens so that the spectrum of any part of the Sun may be photographed with enormous dispersion. It was with this spectrograph that the photograph of the solar spectrum reproduced in Figure 125 was made. The movements of the dome, coelostat, and spectrograph are, of course, controlled electrically.

**The Sun's Appearance.** As observed visually or on direct photographs, the Sun presents a clear-cut circular disk which is brighter at the center than at the limb, and which has a granular surface, the "granules" being some hundreds of miles in diameter. Photographs made in rapid

succession have shown that the granules are short-lived, the majority lasting but about half a minute. They perhaps represent something similar to the crests of waves on a storm-tossed sea. The darkening at the limb is more pronounced in violet light, and therefore on ordinary photographs, than it is in the light to which the eye is most sensitive. This effect is to be expected because the light from the depths of the Sun comes to us through a greater thickness of the outer layers at the limb than at the center. The outer layers absorb and scatter this light somewhat, the violet more

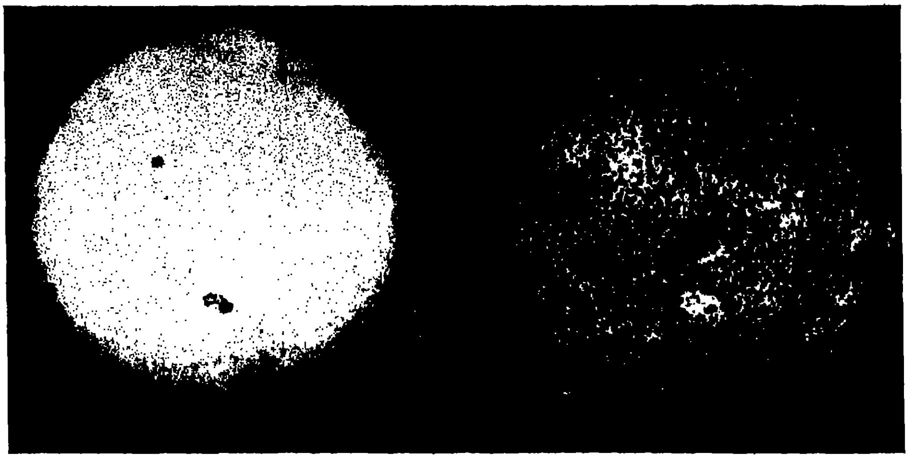


Fig. 134. *The Sun, 1906 July 30. Left, direct photograph; right, spectroheliogram in light of the H line of calcium. Small white dot shows comparative size of the Earth. (Mount Wilson Observatory.)*

than the longer waves, and the light they contribute is less strong and less blue than that from beneath because of their lower temperature.

Large, irregular, bright areas called **faculae** may usually be seen in various positions, but especially near the limb, where they appear more plainly against the less brilliant background. In their spectra the enhanced lines are stronger than in the spectrum of the surrounding photosphere, indicating a higher temperature or lower pressure, or both; and it is probable that the faculae are somewhat elevated above their surroundings. The **sunspots**, which, when present, are the most prominent feature of all, will be discussed in later sections.

On spectroheliograms the granules are not seen, but the whole surface of the Sun appears covered with a multitude of light and dark markings to which Hale has given the name of **flocculi**. Many of these directly overlie faculae and are similar to them in form. Spectroheliograms are

made usually in one of the strong lines (H or K) of ionized calcium or in the Hα line of hydrogen (the Hα line, being in the red, requires the use of specially sensitized plates). The calcium flocculi are bright over extensive areas, especially in the neighborhood of sunspots. The hydrogen flocculi, especially those which appear on Hα spectroheliograms, are usually more clearly defined than the calcium flocculi, and the largest ones are dark (Figures 134, 135).

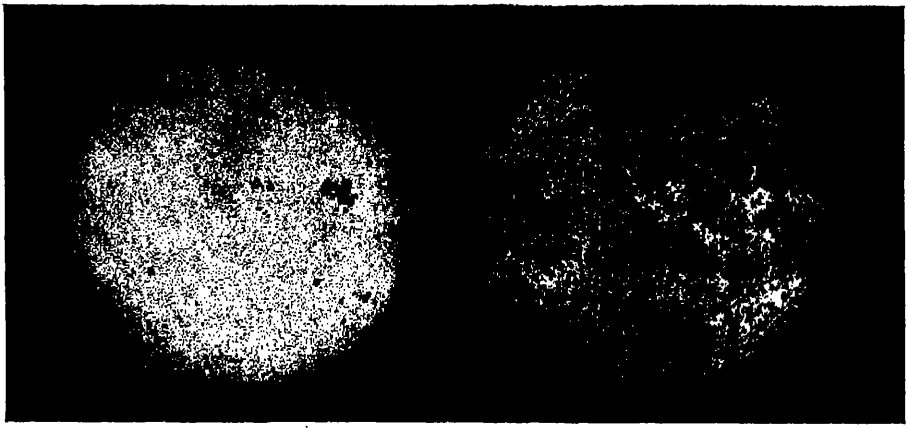


Fig. 135. *The Sun, 1917 August 12. Left, direct photograph; right, spectroheliogram in light of the Hα line of hydrogen. (Mount Wilson Observatory.)*

Although the flocculi have somewhat the appearance of clouds, they are not at all like the clouds in the Earth's atmosphere but are composed of highly heated gases which absorb light of certain wave lengths only, and are transparent to much the greater part of the Sun's light, so that they are invisible when the Sun is observed directly. The great dark hydrogen flocculi are usually prominences projected upon the photosphere, and may be seen as prominences when they have been carried by the Sun's rotation to the limb.

**The Flash Spectrum; Layers of Different Height.** Since, according to the third principle of spectral analysis, the Fraunhofer lines are due to the absorption of certain wave lengths from the light of the photosphere, it is possible, during a few seconds at the beginning or end of a total solar eclipse, to observe these lines as bright or emission lines; for at that time the bright photosphere is hidden by the Moon, while the chromosphere, which produces the lines, is still exposed. This phenomenon was looked for and discovered at the eclipse of 1870 by Young of Dartmouth College,

who named it the **flash spectrum.** It is vividly described in the discoverer's own words as follows:

> . . . At the moment when the advancing Moon has just covered the Sun's disk, the solar atmosphere of course projects somewhat at the point where the last ray of sunlight has disappeared. If the spectroscope be then adjusted with its slit tangent to the Sun's image at the point of contact, a most beautiful phenomenon is seen. As the Moon advances, making narrower and narrower the remaining sickle of the solar disk, the dark lines of the spectrum remain for the most part sensibly unchanged, though becoming somewhat more intense. A few, however, begin to fade out, and some even turn palely bright a minute or two before the totality begins. But the moment the Sun is hidden, through the whole length of the spectrum, in the red, the green, the violet, the bright lines flash out by hundreds and thousands, almost startlingly, as suddenly as stars from a bursting rockethead.[2]

A slender source of light, such as the crescent-shaped layer of gas here studied, may itself serve as a slit, and the slit and collimator of the spectroscope may be dispensed with. The flash spectrum may be thus seen or photographed by simply using a prism placed over the objective of a telescope. Such a combination is called an **objective prism** or sometimes, if used photographically, a **prismatic camera.** The lines of a spectrum thus produced are of course curved like their sickle-shaped source (Figure 136). The bright lines of the chromospheric spectrum have been

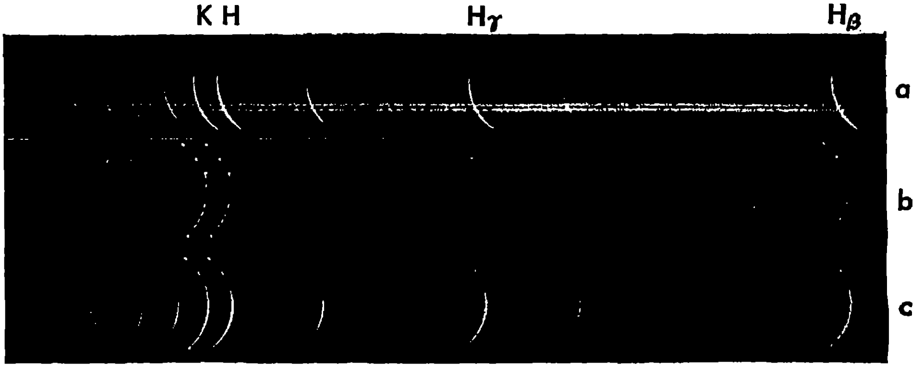



Fig. 136. *Flash Spectra Photographed by Anderson at Middletown, Connecticut, During the Total Solar Eclipse of 1925 January 24. (a) Beginning of totality; (b) middle of totality; (c) end of totality.*

photographed by Adams at Mount Wilson without an eclipse by means of the 150-foot tower telescope, the scale of the image of the Sun being so great as to permit the slit of the great spectrograph to be placed within a very short angular distance of the limb.

[2] Young, *The Sun*, p. 82.

The bright lines of the flash spectrum have their sources at different levels of the chromosphere, and so do not all flash out simultaneously; and by a study of their duration or of the length of their arcs on an objective-prism spectrogram, it is possible to acquire information concerning the heights at which they are formed. Mitchell of the University of Virginia finds in this way that ionized calcium (producing H and K) extends 14,000 kilometers above the Sun's limb; hydrogen produces H$\alpha$ at 10,000 km. and the other Balmer lines at about 8000 km.; helium extends about 7500 km.; neutral calcium (spectral line at 4227 A) about 5000 km.; and other elements, chiefly metallic, to lesser heights. The region below 600 km., in which the majority of Fraunhofer lines have their origin, is often referred to as the **reversing layer**, but there is no definite limit between it and the chromosphere above it.

**Double Reversal.** Young found in 1880 that in the spectrum of the chromosphere observed without an eclipse certain lines, notably those of hydrogen, helium, calcium (H and K), sodium, and magnesium, occasionally showed an appearance which he called **double reversal.** The broad dark line of the solar spectrum has superposed upon it a bright line, and this, in turn, a fine dark line through its center; the three parts probably represent successive levels of vapor. At the base of prominences and over bright flocculi, the H and K lines of calcium and the more prominent lines of hydrogen are always thus doubly reversed. Hale has denoted by subscripts the successive parts of the doubly reversed line; thus, $K_1$ is the broad, dark K line of the Fraunhofer spectrum; $K_2$ is the double bright part in its center; and $K_3$ is the fine dark line separating the components of $K_2$. $K_1$ is due to the absorption of the Sun's white light by the dense calcium lying at the lowest levels, $K_2$ to incandescent calcium above this, and $K_3$ to cooler, rarer calcium vapor at still higher levels. It is $K_2$ which is most frequently used for photographing the bright calcium flocculi with the spectroheliograph. In the spectra of rapidly changing bright flocculi, double reversal sometimes appears in many other of the Fraunhofer lines. By placing the second slit of the spectroheliograph in different portions of the line, photographs are made which show the distribution of flocculi at different levels (Figure 137).

**Pressures in the Solar Envelopes.** Investigations by St. John and by Russell, into which we cannot enter here, lead to the conclusion that the pressure of the gases of the chromosphere, down to about 200 kilometers above the photosphere, is very low, about one ten-millionth that of the Earth's atmosphere at sea level; and that in the reversing layer the pressure rises rapidly and may be as great as one one-hundredth of an "atmosphere" (terrestrial) at the photosphere, at which pressure and at the temperatures there prevailing the gases become sufficiently opaque to give the observed appearance of the Sun's sharply defined limb. The production of a continuous spectrum at this pressure is explained by the presence of a high proportion of free electrons.

The principal forces which act upon a particle of gas at the surface of the Sun are (1) gravity, nearly twenty-eight times as great as on the Earth, acting toward the

Sun's center; (2) gaseous expansion, the result of collisions among the molecules; and (3) the pressure of the Sun's radiation, which acts principally outward. It is believed that the chromosphere is held up against the action of gravity almost entirely by radiation pressure, the gaseous pressure in that region being very low.

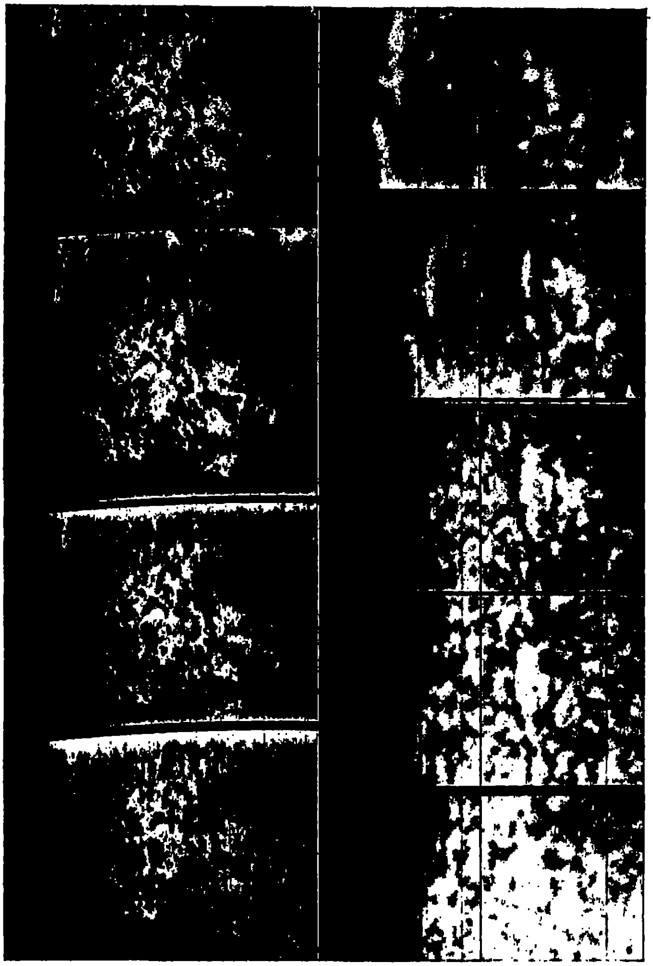


Fig. 137. *Spectroheliograms Taken at Different Levels. Left, Calcium H line, 1919 August 22, Yerkes Observatory; right, hydrogen α line, Mount Wilson Observatory, 1916 May 29. Highest levels are at top of figure.*

**Sunspots.** A sunspot is a relatively cool area in the photosphere, dark only by contrast with its surroundings. The temperature of the darkest part of a spot, as indicated by its spectrum, is about 4000° C. This is about 2000° lower than the temperature of the photosphere, but it still is

so high that the spot would appear to shine brilliantly if it were seen against a dark background. The spot spectrum differs from the spectrum of the photosphere in the strengthening of certain low-temperature lines, notably those of titanium and vanadium, and of bands such as those of the hydrides of calcium and magnesium. A special characteristic of the spot spectrum is a conspicuous Zeeman effect (page 172).

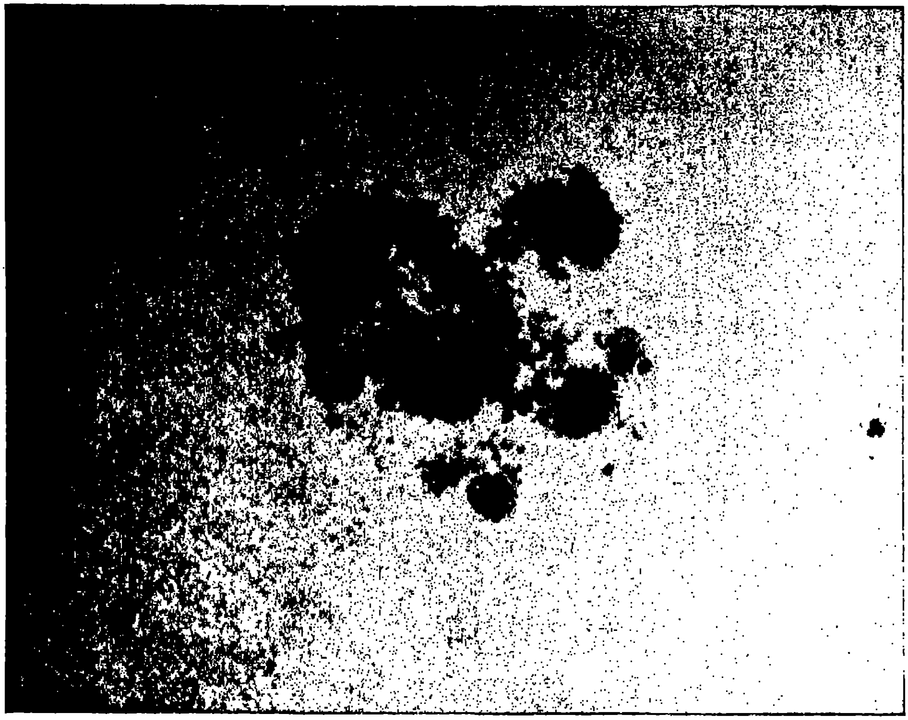


Fig. 138. *Great Spot Group of 1917 August 8. Direct photograph, Mount Wilson Observatory. Black dot shows comparative size of the Earth.*

Most sunspots occur in groups. A typical group begins in two parts separated by 40,000 or 50,000 miles, their line of centers lying nearly east-west but usually with the western (preceding) end a little nearer to the Sun's equator. The group may disappear in a few days or it may grow; in the latter case the two parts enlarge and separate farther, and the eastern portion is likely to break up into smaller spots and disappear, leaving the western part to develop into a single large, round spot which may persist for weeks or months but will eventually disappear by contracting. The inner part of a well-developed spot is darker and is called the **umbra**. A fairly definite boundary separates it from the surrounding **penumbra**,

which is composed of converging filaments (Figures 138, 139). The umbra of a large spot may be large enough to contain the Earth several times over, and the penumbra around a spot group has been known to have a diameter of over 100,000 miles. The area of even so large a group, however, is less than one per cent of the total area of the Sun. Large spots are sometimes seen with no other optical aid than a dark glass, or with the unaided eye when the Sun is shining through mists near the horizon. The Chinese have records of sunspots seen centuries before Galileo.

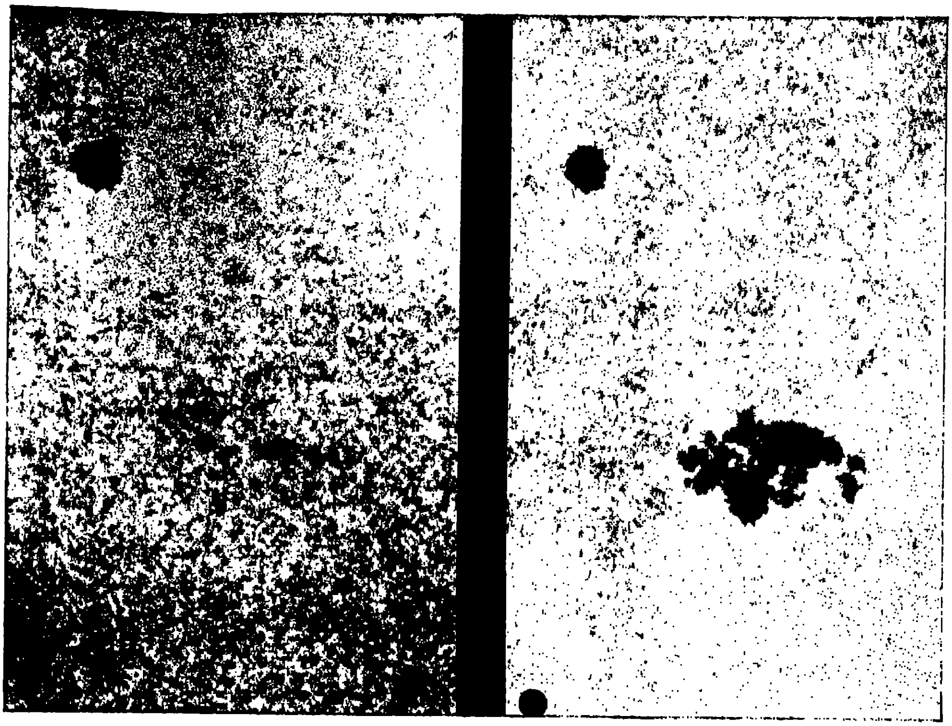


Fig. 139. *Twenty-Four-Hour Development of Sunspot of 1917 August 18-19. Direct photographs, Mount Wilson Observatory.*

Evershed, in India, by measuring the Doppler-Fizeau displacement of lines in the spectra of spots near the limb, showed that the incandescent gases near the photosphere were flowing outward from the umbra, and that the gases of the chromosphere, at higher levels, were flowing inward, a result which was abundantly confirmed by St. John at Mount Wilson.

Sunspots are found in definite zones upon the Sun, chiefly between latitudes 10° and 30° on either side of the equator, though occasionally near the equator itself and in latitudes up to 45°; but never near the poles.

**Prominences.** Solar prominences are clouds of gas, some of which rise to great heights (the greatest recorded height is about 960,000 miles, or more than the Sun's diameter) and extend over similar lengths along

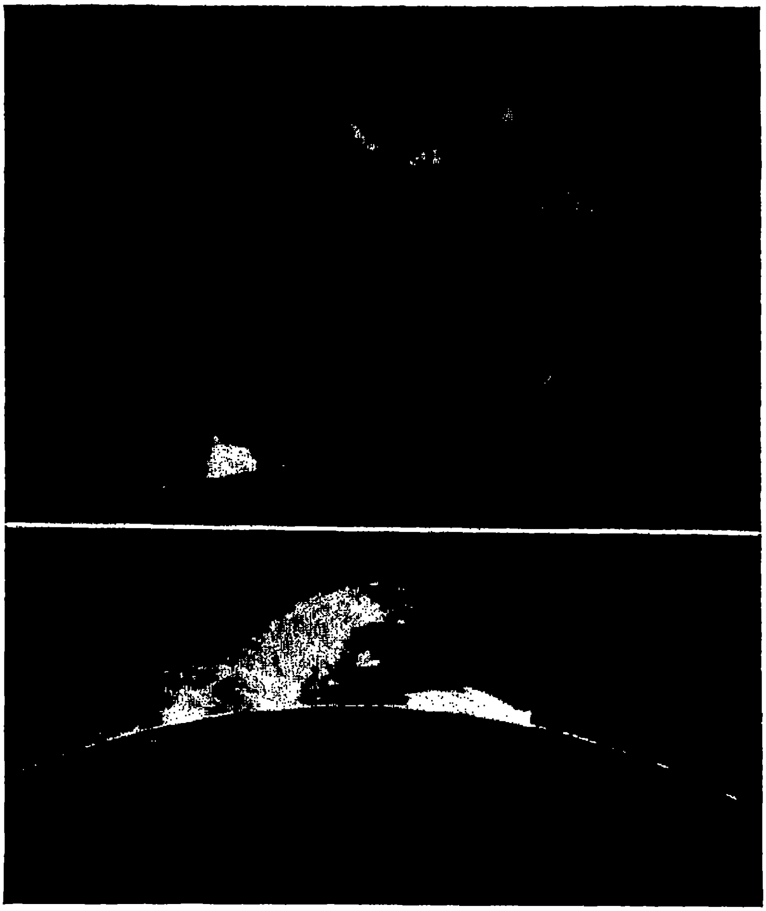


Fig. 140. *Great Solar Prominence of July, 1943, photographed by W. O. Roberts with the Lyot-type coronagraph of the Harvard Observatory at Climax, Colorado. Top, July 17; bottom, July 20.*

the solar disk (Figure 140). In appearance they resemble sheets of flame, and present as great a variety as do real flames. Most small prominences seem to stand directly on the chromosphere, but large prominences often float above it and are connected to it by columns like the trunks of a banyan tree. All are subject to changes which, in good seeing, may often be detected in a few minutes, and the gases in some move with speeds up to 200 miles per second. Pettit has discovered the curious fact that these

speeds remain constant over considerable intervals, then change abruptly as if by a sudden impulse, and at once become constant again at the new value. The spectra of prominences consist of bright lines which always include the Balmer hydrogen series, the stronger lines of helium, and the H and K of ionized calcium; and those of bright, active prominences often display metallic lines. Prominences having the form of spikes or curved jets often overlie sunspots, and all types are most numerous in the spot zones, though some are seen outside these zones and even at the poles. When situated over the disk of the Sun instead of at the limb, prominences may be detected as Hα flocculi.

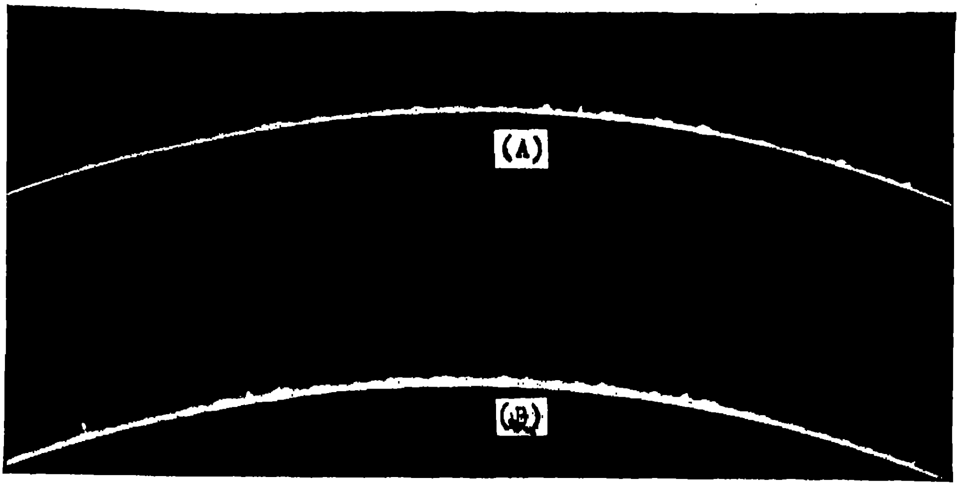



Fig. 141. *Chromospheric Spicules near North Pole of Sun, 1944 February 3, Photographed by W. O. Roberts with the coronagraph at Climax, Colorado. Time interval between (A) and (B), about eight minutes. (B) shows an entirely new set of spicules, those of (A) having disappeared.*

**Chromospheric Spicules.** Small, bright, spike-like prominences have been noticed on eclipse photographs and otherwise for many years. In 1943 W. O. Roberts made motion pictures of these "spicules" at the Colorado solar station of the Harvard Observatory and found that they are short-lived and very numerous, especially near the Sun's poles and in other regions where sunspot and prominence activity is least. The brightest and largest spicules appear to be the longest-lived; they attain heights of about 10″ (4500 miles) and persist about ten minutes. From the number of spicules on the Swarthmore eclipse photographs, Mohler has estimated that the number of spicules on the entire Sun at any one time is about two million (Figure 141).

**Chromospheric Flares.** Not infrequently, there has been seen with the spectrohelioscope or recorded with the spectroheliograph on moving

film a sudden brightening of a portion of a bright flocculus near a sunspot, a brightening so intense and so rapid that in less than ten minutes the area, as seen in hydrogen light, becomes much the brightest spot on the Sun. Such flares never last more than a few hours and usually they subside almost as rapidly as they have developed, leaving the appearance of the area much the same as it was before the flare appeared. They are of special interest because it has been known since 1936 that they are often accompanied by "fadeouts" in short-wave radio transmission. The solar flare and the terrestrial fadeout are perceived simultaneously; hence it appears that the disturbance travels with the speed of light. The most probable explanation seems to be that the fadeout is caused by an increase of the ionization of the upper terrestrial atmosphere by a sudden flood of ultra-violet radiation emitted strongly by hydrogen and helium in the Lyman region of the spectrum—radiation which fails to reach the observer directly because of the ultra-violet opacity of the Earth's atmosphere.

**Vortical Motion in the Gases Surrounding Sunspots.** The features in the vicinity of sunspots are often arranged in curved lines like those formed by iron filings around a magnet. These details have been seen by direct visual observation, but appear much more clearly in hydrogen light, especially in the light of $H\alpha$. In 1908, when red-sensitive photographic plates first became available, Hale applied them at Mount Wilson to the photography of the Sun in $H\alpha$ and found that the vortical structure of the $H\alpha$ flocculi was very marked. By comparing successive photographs of a series, he found that in some cases the masses of gas were in obviously rapid motion along the curved lines; for example, on a series of red-sensitive plates extending from May 29 to June 4, 1908, a dark hydrogen flocculus, many thousands of miles long, was seen to be sucked into a neighboring spot (Figure 142). Just before its disappearance in the spot its velocity exceeded 100 kilometers a second. Deslandres, at Meudon, by applying a very narrow second slit to the exact center ($K_3$) of the calcium line, has since shown a structure in the calcium flocculi similar to that of $H\alpha$ flocculi, and so it appears that the observed vortices exist at high levels, thousands of miles above the spots. In general, the direction of whirl is opposite for two vortices situated in the northern and southern hemispheres of the Sun.

These phenomena led Hale to the conclusion "that a sunspot is a solar storm, resembling a terrestrial tornado, in which the hot vapors, whirling

at high velocity, are cooled by expansion, thus accounting for the observed changes of the spectrum lines and the presence of chemical compounds."[3]

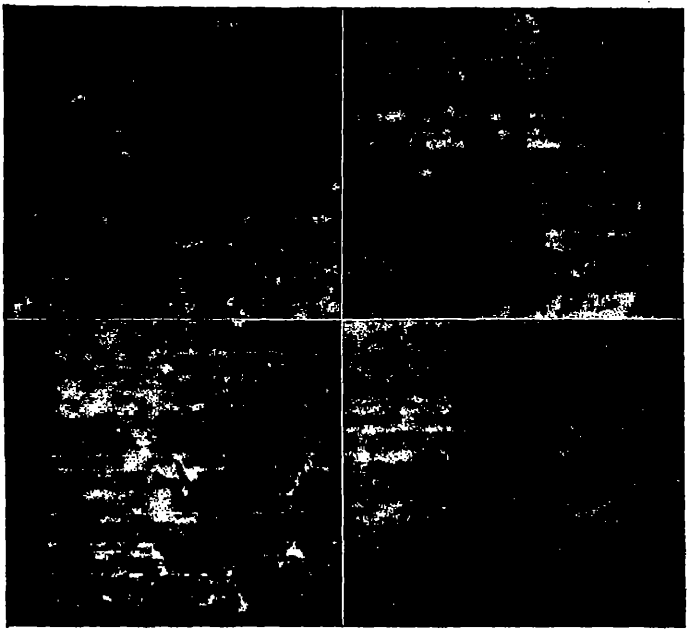


Fig. 142. *Hα Spectroheliograms Made by Hale at Mount Wilson in 1908, Showing Dark Hydrogen Flocculus Drawn into Sunspot. Upper left, June 2, 6:10 A.M.; lower left, June 3, 4:58 P.M.; upper right, June 3, 5:14 P.M.; lower right, June 3, 5:22 P.M.*

**Magnetic Properties of Sunspots.** Immediately after the discovery of solar vortices, Hale found that a sunspot has the properties of a bar magnet, as if a magnetized steel bar were thrust radially into the Sun through the center of the spot.

The reasoning by which he was led to this discovery and the method of making it are of great interest to one acquainted with the principles of physics. In 1876 Rowland had found that an electrically charged disk, when rapidly rotated, produced a magnetic field, showing that, as Maxwell had previously predicted, a moving electric charge was equivalent in its magnetic effects to a current flowing along a wire. For some years previous to Hale's discovery it had been known that hot carbon and hot metals emit negatively charged particles (electrons). It occurred to Hale that there might be a preponderance of positive or negative charges in the gases forming a solar vortex, and that, if so, the rapid whirling of these gases would produce magnetic effects similar to those of a current in a colossal helix, making the vortex into a gigantic electromagnet.

[3] Hale, *Ten Years' Work of a Mountain Observatory*, p. 27.

As early as 1892, Young had noticed that certain lines were doubled in the spectra of sunspots, and Hale suspected that this doubling was a Zeeman effect caused by the magnetic field of the vortex. Using the tower telescope and spectrograph in connection with the appropriate Nicol prisms and Fresnel rhombs for studying polarized light, he found that, when the sunspot was near the center of the disk so that the light reaching the spectrograph emerged radially from the Sun, the components of the double lines were circularly polarized, while plane polarization appeared when the spot was near the limb, the light then emerging nearly at right angles to the lines of magnetic force. Thus his suspicion that the sunspots were magnets was confirmed, and a method was provided for determining their polarities and measuring their field strengths.

Observations made at Mount Wilson of the Zeeman effect in several thousand spot-groups have shown that in the majority of cases the two spots of a pair, or the clusters at opposite ends of a stream, are of opposite magnetic polarity. Many spots which occur singly are preceded or followed in the solar rotation by groups of faculae or flocculi, in which magnetic effects conforming to this rule have been detected; to these regions Hale gave the name of "invisible spots." The arrangement of polarities is opposite in the northern and southern solar hemispheres; that is, when the preceding spot of a pair lying north of the Sun's equator exhibits "south polarity"—i.e., like the south-seeking pole of a magnetic needle on the Earth—the preceding spot of a southern pair shows "north polarity," and vice versa.

**The General Magnetic Field of the Sun.** Like the Earth, the Sun is a great magnet with magnetic poles near its poles of rotation. Its field is about a hundred times as strong as the Earth's. These facts were indicated early in the twentieth century by Hale and his colleagues, who measured the Zeeman widening of spectral lines originating in various parts of the solar disk, and have been confirmed in recent years by Babcock, who studied the same effect with improved equipment.

**The Eleven-year Cycle.** In 1843 Schwabe, a German amateur astronomer, showed from a record of observations of sunspots which he had kept during the preceding twenty-seven years that the spottedness of the Sun was variable in a period which he placed at ten years. A most laborious search by Wolf of Zürich through all available records made since Galileo's discovery of the spots in 1610 confirmed Schwabe's discovery, and systematic observations made since at Greenwich and elsewhere have placed the fact of the periodicity beyond all doubt. At a time of minimum spottedness the face of the Sun may be unspotted for months at a time, whereas at maximum it is almost never without spots. The interval between times

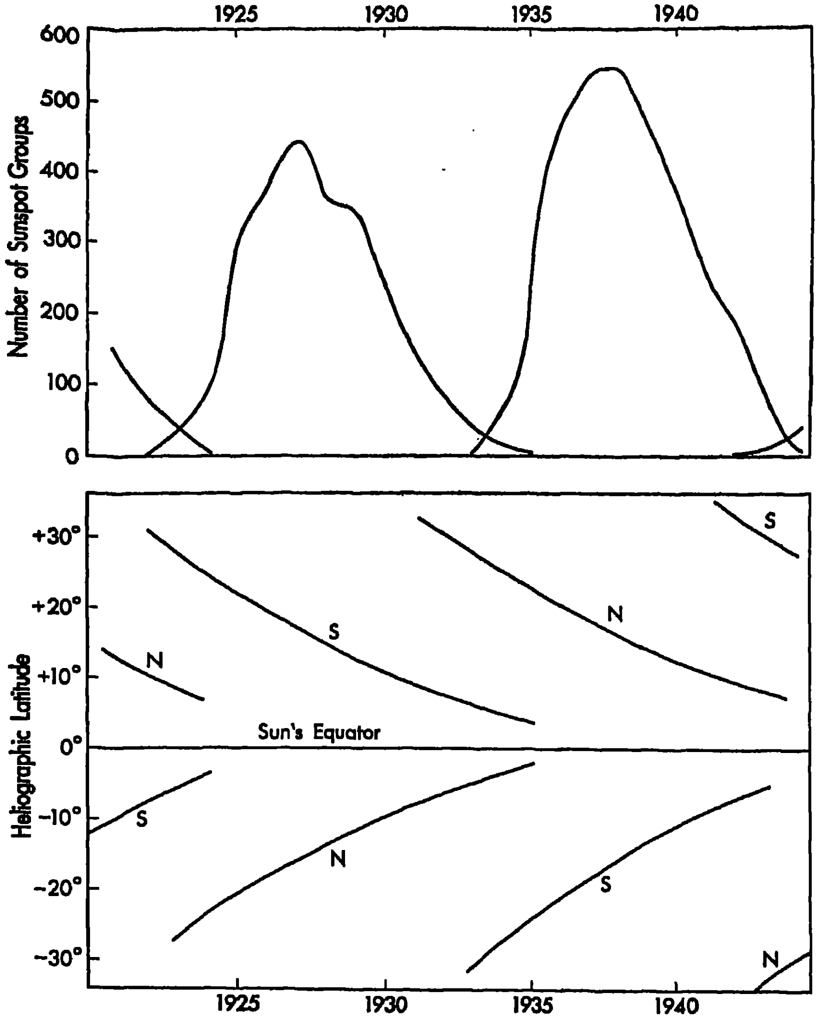



Fig. 143. *The Number, Latitude, and Magnetic Polarity of Sunspots. The upper curves give the number of spot groups observed yearly. The lower curves show that the sunspots of a new cycle appear in high latitudes during the time of minimum solar activity with opposite magnetic polarities in the northern and southern hemispheres. As the cycle progresses, the mean latitude of the spots in each hemisphere decreases continuously while the distribution of magnetic polarities remains unchanged. In the high-latitude spots of the next cycle, which begin to develop more than a year before the last low-latitude spots of the preceding cycle have ceased to appear, the polarities are reversed. The letters N and S printed on the curve indicate the polarity of the preceding (western) spots of each group.*

of minimum spottedness averages 11 years instead of 10, but the "regularity is very irregular," the actual observed interval having had a range of at least four years. The time of descent from maximum to minimum is longer than that of the rise from minimum to maximum, the former averaging 6.5 years and the latter 4.6. In this important respect the sunspot curve resembles the light curves of variable stars of the Cepheid and long-period types (Chapter 18).

It has already been remarked (page 191) that the spots are confined to certain zones. A peculiar relation between their latitude and the time of their occurrence was brought out by Carrington and Spoerer about the middle of the nineteenth century and has become as clearly established as the periodicity. The spots belonging to a new eleven-year cycle appear at the outer limits of the spot zones, and as the cycle progresses they are found farther and farther toward the solar equator. At spot maximum they are seen mostly about latitude $\pm 14°$, while at minimum a few spots of the expiring cycle are seen near the equator and at the same time a few of the beginning cycle appear at high latitudes. It has been found at Mount Wilson that the magnetic polarities of bipolar spot groups are reversed at the end of a cycle, and thus the whole interval required for the changes in the spots, including magnetic properties, appears to be 22 years instead of 11. These relations are illustrated in Figure 143, which is based on data compiled at the Mount Wilson Observatory under the direction of Nicholson.

In addition to the number, latitude, and magnetic properties of sunspots, several other phenomena, both solar and terrestrial, are certainly known to fluctuate in the eleven-year period. Such are the numbers of faculae and of prominences; the form of the solar corona (Figure 149); the number and brilliance of polar aurorae observed on the Earth; the range of variation of the Earth's magnetism; the quality of radio reception (established by Stetson about 1930); and the frequency of magnetic storms.

Suspected more or less confidently of similar eleven-year variations are the intensity of the Sun's radiation (Abbot); the percentage of ultra-violet in the Sun's light (Pettit); the rate of growth of trees (Douglass); the mean annual temperature and rainfall at various localities; and even financial crises, the price of wheat, and the quality of the coats of fur-bearing animals. No satisfactory explanation of the eleven-year cycle has yet been offered.

The **polar aurora**, often referred to in our hemisphere as the Northern Lights, is one of the most mysteriously beautiful of terrestrial phenomena. As seen in

North America and Europe, its most common form is an arch of soft light which appears above the northern horizon, usually early in the night, and from which extend needle-like streamers toward the zenith (Figure 144). These streamers are

Fig. 144. *Common Form of Polar Aurora.*

not still for a moment, but pulsate and quiver, at the same time varying in brightness. Sometimes the aurora assumes the appearance of beautifully folded curtains (Figure 145) which wave as if in a gentle wind. On rare occasions, the auroral light fills the entire visible sky, in which case the streamers usually converge toward the

Fig. 145. *Auroral Curtains.*

magnetic zenith (not the true zenith), where they curve spirally to form an auroral corona, which may persist for hours while the streamers continue their mystic pulsations. The color of the light is usually apple-green, but rose, lavender, and violet tints are not uncommon. The spectrum (Figure 146) consists of bright lines and bands, the brightest of which, a line in the green ($\lambda 5577$), is a forbidden line of

oxygen. There are other forbidden oxygen lines, also bands due to molecular nitrogen, and there is no doubt that they all originate high in the Earth's atmosphere. The auroral spectrum may be detected, as was first noticed by Slipher, on almost any clear night whether an aurora is visible or not. Some of the nitrogen bands in the red often interfere with the astronomical use of red-sensitive photographic plates.


Fig. 146. *Spectrum of the Aurora, Photographed by V. M. Slipher at Flagstaff, 1928 July 7.*

Aurorae are most frequently seen near the time of sunspot maximum, and the most brilliant aurorae appear on occasions when the largest spots are turned toward the Earth. It is probable that the aurora is due to the impact of streams of electrons arriving from the Sun and impinging upon the upper regions of the air, having been directed in the last few hundred miles of their journey by the magnetic field of the Earth.

It is well known that the needle of the magnetic compass does not, in general, point due north; in the eastern part of the United States, for example, it points several degrees west of north, while in the western part it points east of north. Moreover, its direction continually changes through a range of a few minutes of arc, the most conspicuous change being a diurnal oscillation. Soon after the discovery of the periodicity of sunspots, it was found that a similar—in fact, almost identical—periodicity existed in the range of the compass variation and also in the strength of the Earth's magnetic field.

A **magnetic storm** is a sudden violent disturbance of the Earth's magnetism, in which the compass needle oscillates, sometimes through an arc of two or three degrees within an hour or two, and in which the fluctuations in the strength of the Earth's field are so great as to induce currents which interfere seriously with the operation of telegraph lines. Magnetic storms often occur during brilliant auroral displays, and usually coincide in time with the presence of very large spots near the center of the visible disk of the Sun.

**The Dawn Flash.** Slipher discovered that, shortly before the beginning of dawn, when the Sun's rays strike the upper atmosphere which is much too tenuous to reflect a perceptible twilight, new bright molecular nitrogen bands (called the dawn flash) appear in the sky spectrum. They are accounted for by the fluorescence of nitrogen molecules in the extreme ultra-violet solar radiation which does not penetrate the main body of the air and which is also invoked to explain the luminosity of comets.

## The Rotation of the Sun.

Immediately after the telescopic discovery of sunspots by Galileo in 1610, it was noticed that they moved across the disk of the Sun, and they were thought by some to be planets seen in transit (the idea of *spots* on a *celestial* body being obnoxious); but Galileo

refuted this opinion by pointing out that they remained behind the Sun the same length of time as in front of it, and were therefore on the solar surface. Galileo's German contemporary, Christopher Scheiner, made a fair determination of the rotation period from a record of the position of sunspots obtained by projecting the Sun's image on a screen; but the first accurate determinations were made by Carrington and by Spoerer about 1850. The work of these observers not only gave an accurate description of the position of the Sun's axis, but brought out the remarkable fact that spots near the equator travel around the Sun in a shorter time than spots in higher latitudes. They found that the Sun rotates in the same direction as the Earth about an axis whose north pole is in $\alpha = 19^h\,04^m$, $\delta = +64°$, or midway between Polaris and Vega; that the plane of the solar equator is therefore inclined 7° to the plane of the ecliptic; that spots near the equator indicate on an average a sidereal rotation in about twenty-five days; and that 30° on either side of the equator the period is twenty-six and one-half days and at 45° about twenty-seven and one-half. They found also that, in general, each individual spot had a motion of its own in both latitude and longitude, the periods mentioned above being averages only.

The path of a sunspot across the disk appears slightly concave northward in summer and autumn, and convex northward in winter and spring. This is because of the inclination of the axis, the north pole leaning 7° toward the point occupied by the Earth on September 7. On June 3 and December 5 the spot paths are straight, for then the Earth is in the plane of the Sun's equator.

The apparent, or **synodic**, rotation period of the Sun is a little longer than the sidereal, being twenty-seven and a quarter days for a spot at the equator. This is because the Earth advances, during a sidereal rotation of the Sun, nearly a twelfth of the way around its orbit, so that the spot must turn through more than 360° to come back to the same apparent position. The relation between the sidereal and synodic rotation periods is similar to that between the sidereal and synodic months (page 125).

Since sunspots are found only in limited zones of the Sun's surface, they cannot be used to study the rotation of all parts of the solar globe. Faculae offer no advantage over spots, for their distribution is about the same and they cannot be well observed except near the limb. There are, however, two other methods of approach to the problem: by the motions of the spectroheliographic flocculi, and by Doppler-Fizeau displacements of lines in the spectrum of the limb. The results obtained by Hale and Fox from calcium flocculi agree well with those given by spots; but those

derived from hydrogen flocculi, while indicating about the same rate of rotation at the equator as that shown by spots, seem to show a smaller retardation at higher latitudes.

When high-dispersion spectra of the east and west limbs of the Sun are confronted, a relative shift of the lines is immediately evident, the lines belonging to the west limb being shifted toward the red (Figure 128, page 168). This displacement was first used for studying the solar rotation in 1893 by Duner in Sweden, who observed visually. More accurate observations have been made photographically by a number of observers. Probably the most exhaustive study is that of Adams at Mount Wilson, who finds an average period of 24.6 days at the equator, and 33.3 days at latitude $\pm 80°$. Different spectral lines give distinctly different velocities; the lines of lanthanum, titanium, and iron give smaller velocities than the calcium line at $\lambda 4227$, while H$\alpha$ indicates a higher velocity. These differences are probably due to differences of level of the sources of light, the hydrogen that emits H$\alpha$ being higher, and the metals lower, than the non-ionized calcium which emits $\lambda 4227$.

The rate of rotation in different heliographic latitudes may be conveniently represented by a formula. If we represent by $\xi$ the daily heliocentric angular motion and by $\phi$ the heliographic latitude, the results of Carrington, as reduced by Faye, give

$$\xi = 14\overset{\circ}{.}37 - 3\overset{\circ}{.}10 \sin^2 \phi$$

and Adams' results give

$$\xi = 14\overset{\circ}{.}61 - 3\overset{\circ}{.}99 \sin^2 \phi.$$

The effect of the unequal rotation periods in different latitudes is illustrated in Figure 147, which is constructed according to the formula of Adams. Suppose a row of fifteen spots, one for every 10 degrees of latitude from $-70°$ to $+70°$, are arranged upon the same meridian of the Sun as in the left-hand drawing. After one rotation in latitude 45°, they will have arranged themselves as in the right-hand drawing.

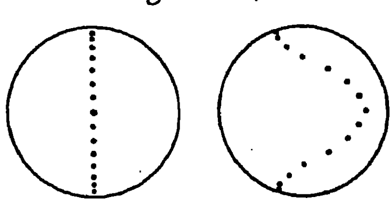


Fig. 147. *Rotation of the Sun.*

**The Solar Corona.** The Sun's corona is a pearly-white atmosphere which extends at least 300,000 miles all around the Sun; some of its streamers have been known to reach to a distance of 5,000,000 miles. Although its total light, as measured at the 1925 eclipse, is about half that of the full Moon, the light per unit area is so small that the detection of the corona without an eclipse is exceedingly difficult. Its extreme tenuity is shown by the fact that the great comet of 1882 passed directly

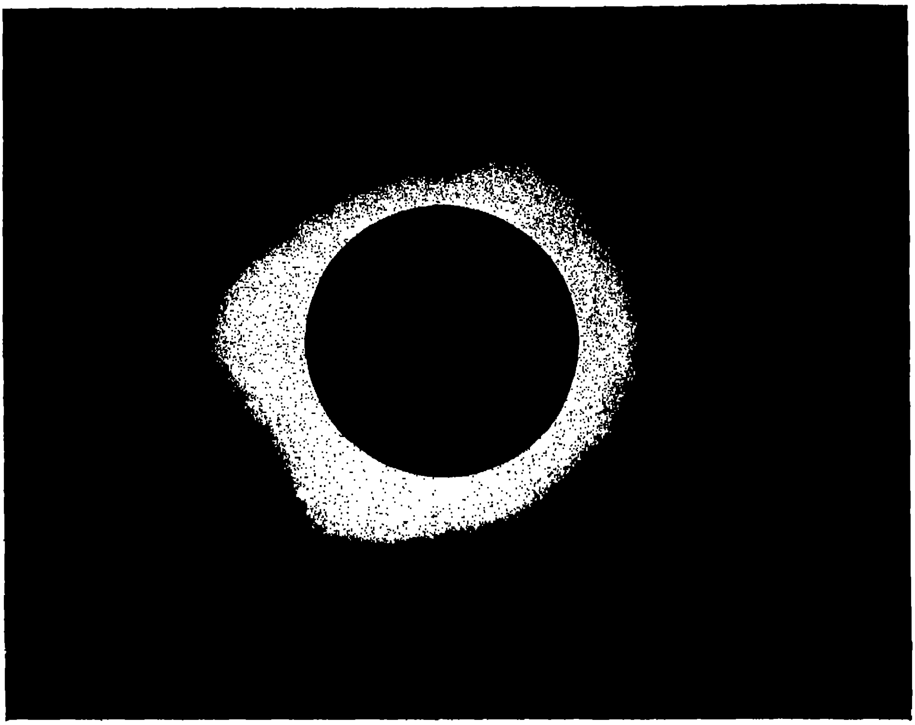
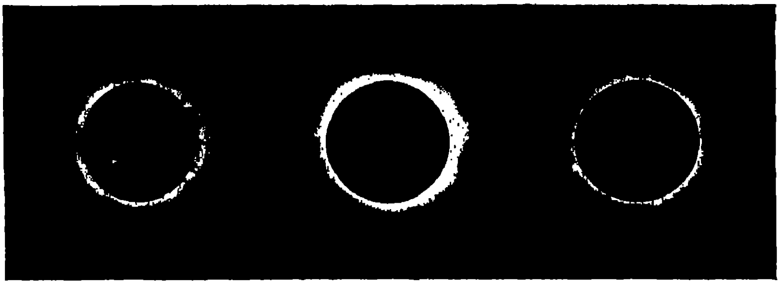


Fig. 148. *The Solar Corona, Photographed with Camera of 40-Foot Focal Length by the Lick Observatory Expedition to Wallal, Australia, 1922 September 22.*

through it without any perceptible change in its speed. Arrhenius estimated its average density as the equivalent of one dust particle to every fourteen cubic yards.

The form of the corona varies with the progress of the eleven-year cycle (Figure 149). At spot minimum, fine rays arranged like the straws in a



1 2 3

Fig. 149. *The Changing Corona. (1) At sunspot maximum. Lick Expedition to Chile, 1893. (2) Intermediate type. Slocum, Middletown, 1925. (3) At sunspot minimum. Barnard and Ritchey, North Carolina, 1900.*

sheaf of wheat cluster about the Sun's poles, while from the spot belts and equatorial regions extend broad streamers to enormous distances. At spot maximum, the distribution of the corona around the Sun is more uniform, few pronounced rays are seen, and no portion of the corona extends so far from the limb as at spot minimum. The inner part of the corona is much the brightest, more than half the total light coming from the region within about 3′ of the limb of the Sun.

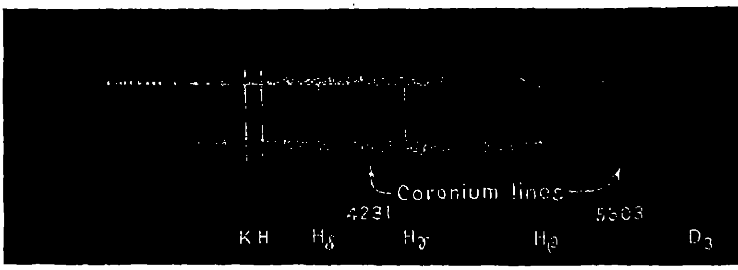



Fig. 150. *Spectrum of the Eclipsed Sun, Photographed by V. M. Slipher with Slit Spectrograph, 1918 June 8. The bright lines belong to the chromosphere and the corona, the light of which was diffused by a hazy sky so that they cross the dark image of the Moon.*

The spectrum of the corona (Figure 150) consists of three superposed parts. First, there is a continuous spectrum, possibly caused by the incandescence of fine solid or liquid particles. Second, the dark lines of the solar spectrum appear, but only faintly, as if partly obliterated by the continuous spectrum first mentioned. These are probably due to reflected sunlight for they are more conspicuous in the light coming from points considerably distant from the Sun, where the light of incandescence would be feebler in proportion to reflected light. Third, there is a spectrum of bright lines, which must be due to luminous gases within the corona. The brightest visible line, first seen by Young at the eclipse of 1869, is in the green at λ5303. About thirty other bright lines have been found, of which two, in the ultra-violet, are more intense than the green line.

The difficult problem of observing the corona without an eclipse was solved in 1930 by Lyot, who scrupulously excluded dust and extraneous light from his telescope and spectroscope, occulted the Sun's image with an opaque disk in the focal plane, contrived to expel even the light diffracted at the edge of this disk, and used his equipment at the top of the Pic du Midi where the sky is exceptionally clear. He succeeded in seeing prominences directly and mapped the outline of the brightest parts of the corona by studying the distribution around the solar image of the light of the coronal spectrum lines. Lyot's methods have since been used suc-

cessfully at Arosa, Switzerland, and at the solar station of the Harvard College Observatory at Climax, Colorado.

**Helium, "Nebulium," and "Coronium."** The light element helium is so named because its existence was discovered by means of its bright lines in the spectrum of the solar chromosphere. Its brightest line, the yellow $D_3$ ($\lambda$ 5875), was first noticed at the eclipse of 1868. Nearly thirty years afterward, the chemist Ramsay found a gas in the rare mineral cleveite which showed the same spectrum and thus discovered helium as a terrestrial element. Helium was long believed to be exceedingly rare on the Earth, but about 1918 it was found to be emitted copiously from certain gas wells in Texas and Oklahoma, and has from that time been used in the United States largely for inflating airships, being non-inflammable and nearly as light as hydrogen.

It was long expected that, similarly, new elements would be chemically discovered which would yield the unidentified lines of the gaseous nebulae and the corona, and these hypothetical elements were referred to as nebulium and coronium; but when the elements of low atomic number (Table 5) had all been discovered and none had met the test, it became practically certain that nebulium and coronium were only familiar elements with unusual spectra. The nebular lines have, in fact, been identified by Bowen and Wyse mainly as forbidden lines of singly and doubly ionized oxygen and nitrogen. The coronal lines are attributed by Edlén to certain atoms, chiefly iron, in a state of ionization so advanced as to have deprived them of about half their electrons.

**The Rate of the Sun's Outpour of Energy.** The rate at which the Sun radiates energy into space is indicated by the quantity known as the **solar constant of radiation,** which is defined as the number of calories which would be received from the Sun each minute upon a surface one centimeter square, if the surface were exposed perpendicularly to the Sun's rays outside the Earth's atmosphere, at the Earth's mean distance from the Sun. Its value, derived by Abbot of the Smithsonian Institution from a vast number of observations made at many stations in different parts of the Earth, averages 1.94, but varies by about 6 per cent.

The word **calorie,** as here used, denotes the amount of energy required to raise the temperature of one gram of pure water from 15° C. to 16° C. It is equal to $4.18 \times 10^7$ ergs, and the solar constant is equivalent to $1.35 \times 10^6$ ergs per square centimeter per second. The energy of the Sun comes to us in the form of waves of a great range in length, a part residing in the infra-red or "heat" waves. This energy has not the form of heat as it passes through space, and assumes that form only when it is absorbed by some non-transparent material. This may be strikingly demonstrated by converging sunlight upon a piece of dark cloth by means of a convex lens made of ice. The Sun's rays heat the ice very little—if it were perfectly transparent to all of them, they would heat it not at all—nor does the ice cool the rays; but they are absorbed by the cloth and so transformed into heat, which is a motion of the molecules, and the cloth is set afire. Dark objects absorb radiation and transform it into heat more readily than do light-colored objects, and this is

why, in summer, light-colored clothing is more comfortable than black. Neither a perfectly transparent body nor a perfect reflector, if either existed, would absorb radiation or be warmed by sunlight.

The determination of the solar constant involves two different problems: the measurement of the rate at which energy is received at the surface of the Earth after it has passed through the air, and the determination of the amount absorbed by the air during passage. The first is solved with the instrument known as a **pyrheliometer**, invented by Pouillet about 1838, in which the Sun's radiation is allowed to fall upon a body of known mass and absorbing power, and the rise of temperature in a known interval of time is noted. Corrections must be made for imperfect absorption of the Sun's energy and for radiation of energy from the pyrheliometer. The absorption of the air is more difficult to determine. Information about it is obtained by comparing pyrheliometer readings made at sea level with those made on the tops of mountains, above a part of the atmosphere, and also from readings made at different times of day, with "high Sun" and "low Sun." Since the absorption is different for different wave lengths, it is necessary also to compare the intensity of the radiation in different parts of the spectrum; for this, use is made of the **bolometer**, an exquisitely delicate instrument invented by Langley at Allegheny about 1880, the operation of which depends upon the change of electrical resistance produced in a thin strip of platinum when radiation falls upon it.

To put the Sun's radiation in more familiar terms, we may compute the rate at which it could melt ice. The appropriate computation shows that the Sun's radiation would, at the Earth's distance, melt in one minute a sheet of ice 0.0267 cm. thick if placed perpendicular to the Sun's rays and if all the energy were absorbed by the ice. Since the intensity of radiation varies inversely as the square of the distance, to obtain the intensity at the Sun's surface we must multiply the intensity at the Earth's distance by the square of the ratio of the distance from the Earth to the Sun (150,000,000 km.) to the radius of the Sun (700,000 km.). The square of this ratio is about 46,000. Therefore, at the photosphere, the solar radiation would melt in one minute a shell of ice $46,000 \times 0.0267$ cm., or about 12 meters (39 feet) thick!

One horsepower is the equivalent of 10,700 calories per minute; hence, at the Earth's distance each square centimeter receives $1.94 \div 10700 = 0.00018$ horsepower, or the radiation falling on a square meter, if utilized in a perfect engine (existing engines are far from perfect), would yield 1.8 horsepower. At the surface of the Sun, each square *centimeter* develops over 8 horsepower continuously.

The above computation involves the assumption that the Sun radiates at the same rate in all directions just as other luminous bodies do—an assumption against which there is not the slightest evidence. On the same assumption, the Earth receives only about $1/2 \times 10^9$ part of the whole amount of energy produced by the

Sun, and only about $1/10^8$ is received by all the bodies of the Solar System combined. A very small amount must be absorbed by the stars, and the remainder of this vast output of energy, to the best of our knowledge, travels forever outward into space.

**The Effective Temperature of the Sun's Surface.** The value of the solar constant affords a direct determination of the Sun's effective temperature by the Stefan-Boltzmann law (page 169). The result is 5750° K (K signifying absolute temperature in Centigrade degrees).

Other methods depend upon observations with the bolometer, which give a curve whose ordinates are proportional to the energy of the Sun's radiation and whose abscissae represent wave-lengths in the solar spectrum. The form of this curve depends somewhat upon the portion of the solar disk which is observed, but when corrected for absorption in the atmosphere of the Earth it resembles the Planck curves shown in Figure 126. The abscissa of the peak is near $\lambda$4700 angstroms (0.000047 centimeter), so that Wien's law gives for the effective temperature 6150° K. The curve itself conforms closely to the Planck curve for 7000° in the infra-red, but falls below the curve for 6200° in the violet. The discordance of the results may well be due to the fact that they represent radiations emanating from different levels of the Sun where the temperatures may be expected to differ widely.

**The Sun's Interior.** Eddington has given a picture of the inside of a star (page 377) which applies as well to the particular star we call the Sun. Near the center, the pressure due to the vast weight of overlying material amounts to thousands of tons to the square inch. The temperature is estimated at 20,000,000° Centigrade. At this temperature the atoms are largely ionized—stripped of their outer electrons—so that they are packed much more closely than is possible in ordinary gas, thus accounting for the Sun's high average density; and the radiation is transcendently intense and consists (Figure 130) mostly of very short waves (X-rays) to which the solar material is opaque, so that the energy escapes to the surface only by a slow process of transformation to greater wave-lengths.

**The Source of the Sun's Energy.** Calculations based on the laws of physics show conclusively that the Sun's vast production of energy cannot be explained by combustion (if it were composed of pure coal and oxygen its consumption at the present rate of outpour would last only about a thousand years), or by the cooling of matter previously heated, or by the fall of meteoric bodies into the Sun.

At the end of the nineteenth century it was generally conceded that the **contraction theory** of Helmholtz afforded an adequate explanation. A contraction of the

Sun is the equivalent of a fall of each particle of its substance toward the center, and in falling the particles must generate heat. From Abbot's value of the solar constant, it may be shown that a contraction of the Sun's radius of 37 meters annually would account for the energy radiated, while the change in the Sun's diameter would not become perceptible from the Earth in several thousand years. Lane proved further that the radiation of energy produced by this contraction would not be as rapid as its generation, and that therefore, paradoxically, the Sun would actually grow hotter until its contraction proceeded so far that it was no longer gaseous. However, if the Sun has been contracting in the past at the rate required by the Helmholtz hypothesis, it must have been as large as the Earth's orbit as recently as 20,000,000 years ago, and under such circumstances the Earth could not have supported life if indeed it could have existed. Geologists find evidence of life on the Earth as much as 300,000,000 years old and place the age of the Earth's crust at not less than 1,000,000,000 years; hence the Helmholtz theory must be regarded as inadequate.

There is now little doubt that the source of energy of the Sun and of the other stars lies in the nuclei of their atoms. Modern physical theory shows that matter and energy are ultimately identical. On the basis of the theory of relativity, Einstein has shown that the energy contained in a body of mass $m$ is

$$E = mc^2$$

where $c$ is the speed of light and the units are those of the centimeter-gram-second system. That is, $c^2 = 9 \times 10^{20}$ and a gram of matter possesses 900 billion billion ergs of energy. Two ways are conceivable in which the energy stored within an atom might be liberated: by the coalescence and mutual destruction of a proton and an electron, and by the transmutation of one element into another. The first process, which, if it occurred, would afford a greater supply of energy than any other that has been thought of, was suggested by Eddington and regarded favorably in the 1920's, but it is not known to occur anywhere and there is no theoretical reason why it should depend upon a high temperature or why it should happen in the stars if not on Earth. The possibility of the second process was recognized at the same time, and about 1940 Bethe of Cornell showed that the Sun's radiation probably originates in the transmutation of hydrogen into helium through what he calls the carbon cycle.

The atomic weight of hydrogen on the chemists' scale is 1.008; that of helium, 4.004. If four atoms of hydrogen are transmuted into one atom of helium, 0.028 of the hydrogen will disappear as energy. That is, for each gram of hydrogen transmuted, $0.028 \times 9 \times 10^{20}$ or 25 billion billion ergs of energy will be released. In the carbon cycle, which theory shows can be sustained at the internal solar temperature of 20 million degrees, a series of nuclear reactions takes place in which, by

capture of protons (hydrogen nuclei), carbon nuclei form isotopes of nitrogen, carbon, oxygen, and helium. When the cycle is complete the carbon is restored and ready to start over with more hydrogen; the oxygen and nitrogen have appeared only to disappear; and the net change is the consumption of four protons and the emergence of one helium nucleus and a relatively prodigious quantity of radiation.

On the basis of the estimated amount of hydrogen in the Sun and the rate of progress of the carbon cycle (about 6 million years per atom of helium), it is calculated that the Sun will radiate some 10 or 12 billion years longer; that toward the end, as more hydrogen is consumed, it will get hotter so that the temperature of the Earth (if there is an Earth at that remote time) will rise some 400°; and that the Sun (and Earth) will then rapidly cool down so that after a few million years more the Sun will emit very little light.

## EXERCISES

1. In what direction do sunspots appear to be carried across the disk by the Sun's rotation?

2. What must be the diameter, in miles, of a group of sunspots which subtends an angle equal to the apparent separation of the two stars composing $\epsilon$ Lyrae (about 3′)?

*Ans.* 81,000

3. What is the apparent diameter of a sunspot the size of the Earth?

*Ans.* 17″.6.

4. If the Sun's effective temperature were 15,000° K, like that of some stars in the Pleiades, what would be the value of the solar constant?

*Ans.* Over 75 calories per sq. cm./per min.

5. What would then be the wave-length ($\lambda_{max}$) of the point of greatest intensity of the solar spectrum?

*Ans.* 0.000019 cm. or 1900 A.

# CHAPTER 10

## THE PATHS OF THE PLANETS

**The Planets.** A planet may be distinguished from a star in three ways: First, the stars twinkle and the planets usually do not; this rule, however, is far from infallible. Second, when magnified by a telescope, the planets show disks of perceptible area, while the stars appear as glittering points. This distinction holds for all the principal planets, but fails for most of the many minor planets, or asteroids. Third, and most important, the stars maintain practically the same relative positions for years, whereas a planet changes its position among them perceptibly from night to night or, seen in a telescope, in the course of a few hours or even minutes.

The word planet is derived from a Greek word meaning *wanderer*, and is so applied because of the third characteristic just mentioned. The ancients recognized seven planets: the Sun, the Moon, Mercury, Venus, Mars, Jupiter, and Saturn. The Sun and Moon are not now so classed, but modern astronomy places the Earth among the planets and has discovered three others, Uranus, Neptune, and Pluto, which are more distant than the farthest of the ancient planets; and besides these more than 1500 little bodies called minor planets, or asteroids. The word as now applied means an opaque body that shines by reflected sunlight and moves around the Sun in a nearly circular orbit.

**Apparent Motions of the Planets upon the Celestial Sphere.** The apparent motions of the planets are not so simple as those of the Sun or Moon. The Sun seems to move with nearly constant speed and always toward the east in the great circle called the ecliptic, which is practically fixed among the stars. The Moon's apparent motion is also eastward and in a great circle that is slightly inclined to the ecliptic. Although the apparent path of each of the planets (some of the asteroids excepted) lies near the ecliptic, their motion is very different from that of the Sun or

Moon, being zigzag or looped—toward the east for a considerable period, then toward the west for a shorter time, and then eastward again. The long, eastward motion is called **direct** and the short, westward motion, **retrograde**. For example, the apparent motion of Mars from 1928 August 1 to 1929 May 1, through the constellations Taurus and Gemini,

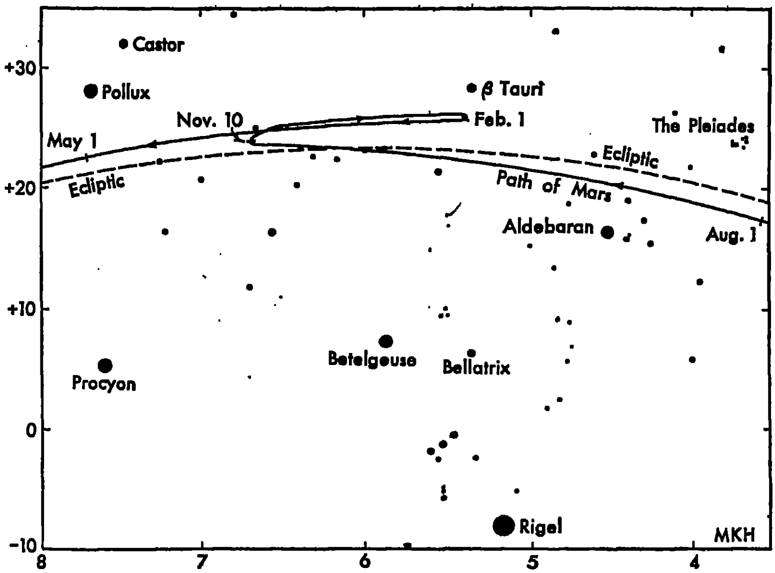



Fig. 151. *Apparent Path of Mars in 1928–1929.*

is shown in Figure 151. From 1926 December 8 until 1928 November 10, this planet moved directly, but its motion became reversed on the latter date, and remained retrograde until 1929 February 1, when it again became direct. Mars then continued to move directly until 1930 December 19, going eastward entirely around the zodiac and passing the region shown in Figure 151 in the autumn of 1930; then it traversed a loop in Cancer, about 25° farther east, in the winter of 1930–1931. Observation through the centuries has shown this planet's apparent motion to be continuously of this description: direct for nearly two years, around the zodiac and about 40° over; then retrograde for about 70 days over an arc of 16° (on the average); and then direct again.

Since the Sun also appears to move among the stars, the motions of a planet relative to the Sun and relative to the stars are necessarily different. The direct motion of all but two of the planets is always slower than that of the Sun, so that they drop relatively westward and cross the meridian of

e earlier each night as counted by solar time. The two Mercury and Venus, which drop rapidly west of the Sun ..... .etrograde motion and then slowly overtake it as they move , thus appearing at one time as "morning stars" on the west side .e Sun and then as "evening stars" on the east.

›lanet's apparent position in the sky with reference to the Sun is described in the same terms that are used in the case of the Moon (page 125).

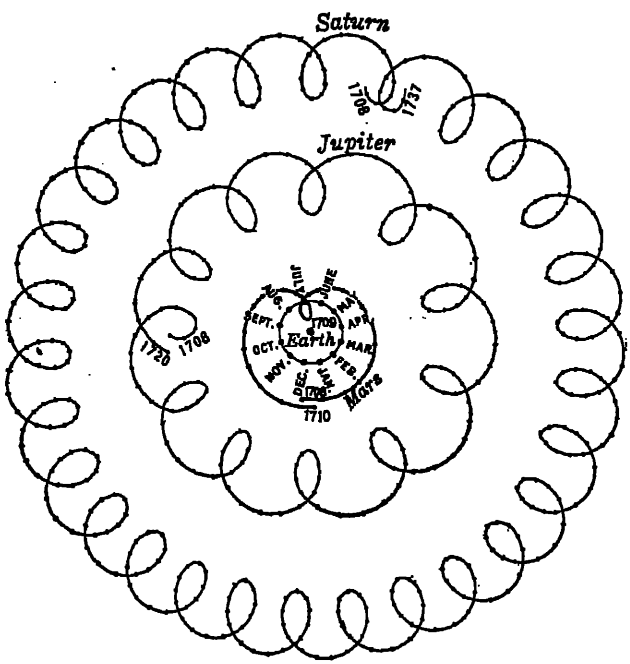



Fig. 152. *Geocentric Paths of Mars, Jupiter, and Saturn.* (*From Proctor's* Old and New Astronomy.)

Mercury and Venus never reach opposition, or even quadrature; their greatest possible elongations are respectively 28° and 47°. The conjunction reached by either of these planets during its retrograde motion is called **inferior** conjunction, for then the planet is between the Earth and the Sun; **superior** conjunction occurs during direct motion, when the Sun is between Earth and planet. Opposition, with the Earth between Sun and planet, is reached by each of the other planets about the middle of its retrograde motion. The time occupied by a planet between successive oppositions or successive superior conjunctions is called its **synodic period.**

**Apparent Geocentric Motion in Space.** The motions just described are the apparent motions on the celestial sphere and take no account of the changing distance of the planets from the Earth. In the cases of some of the planets, especially Mars, the brightness changes greatly with the planet's position, being greatest for this planet near opposition and least near conjunction. This was interpreted centuries ago as being due to change of distance, and the correctness of this interpretation is proved

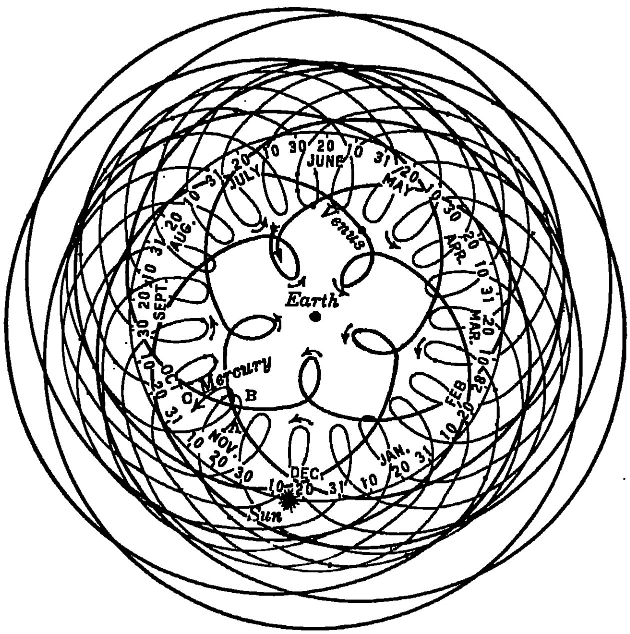



Fig. 153. *Geocentric Paths of Mercury and Venus. (From Proctor's* Old and New Astronomy.)

by the telescope, which shows that the apparent diameter is greatest when the planet is near opposition or inferior conjunction.

We have seen (page 107) how the apparent geocentric path of the Sun in space can be pictured by drawing radiating lines from a point to represent the Sun's direction from the Earth at different times, and cutting off the lines to a length inversely proportional to the Sun's apparent diameter; and that the curve so indicated is a nearly circular ellipse with the Earth at one focus. The Moon's geocentric path is also an ellipse and has only a slightly greater eccentricity. If a curve is similarly constructed to represent the geocentric path of a planet, it appears as an intricate series of loops, as shown in Figures 152 and 153.

**The Ptolemaic System of Planetary Motions.** Many philosophers among the ancient Greeks attempted a logical description of the motions of the heavenly bodies. Most of them assumed the Earth to be stationary, but some, among whom was Aristarchus of Samos (310–250 B.C.), taught that both the Earth and the bodies now classed as planets revolved around the Sun. After Aristarchus the principal promoters of theoretical astronomy were the Alexandrians Apollonius (*c.* 230 B.C.), Hipparchus (190–120 B.C.), and Claudius Ptolemy (A.D. 100–170). Probably they did not intend their theories to be accepted unquestioningly as the true physical description of the world, but rather to be used as mathematical tools for computing the positions of the planets at any time.

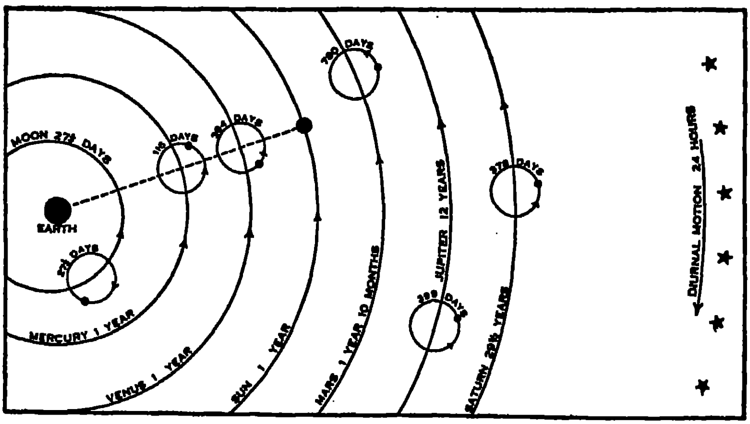



Fig. 154. *The Ptolemaic System.* (*Adapted from* Dante and the Early Astronomers, *by M. A. Orr.*)

Ptolemy composed a thorough compendium of the astronomy of his time, which he called the *Mathematical System of Astronomy*, but which came to be known as Μεγίστη Σύνταξις, the Great System. During the centuries of semi-barbarism that stifled Europe after the decline of the great school at Alexandria, this work was honored and preserved by the Arabs, who prefixed their article *al* to the title and modified it to Almagest, the name the book bears to this day. The description of the planetary system given in Ptolemy's Almagest, known as the Ptolemaic system, exercised a profound influence on literature, science, and religion, which is abundantly evident, for example, in the great epic poems of Dante and Milton.

According to the Ptolemaic system, the Earth is fixed at the center of the universe. (Ptolemy argued that, if the Earth moved, falcons and other birds would be left behind when they flew into the sky.) Around it revolves the Sun in a period of a year in a slightly eccentric circle. Each of the planets revolves, in a small circle called an epicycle, around a point which in turn revolves around the Earth in a large circle called a deferent. The deferents of Mercury and Venus lie within the orbit of the Sun, and the centers of their epicycles lie always on a straight line joining Sun and Earth; this explains their apparent oscillations relative to the Sun. The deferents of the other planets lie outside the Sun's orbit. Within all the other orbits revolves the Moon in an epicycle, backward, in a period of one sidereal month, while its epicycle revolves forward in the same period, in a deferent that surrounds the Earth eccen-

trically. Outside all the orbits Ptolemy places the sphere of the fixed stars, and beyond this is the **Primum Mobile,** which furnishes the motive power that keeps the whole intricate machine turning westward in the diurnal motion while the planetary motions go on inside. The general principles of the system, and the way in which it explained the observed motions, can be seen in Figure 154.

**The Copernican System.** The Ptolemaic system was virtually undisputed for fourteen centuries, until the Polish churchman Nikolaus Kopernik (1473–1543), or Copernicus (the Latin form of his name), published his book *De Revolutionibus Orbium Caelestium*, in which he showed that the

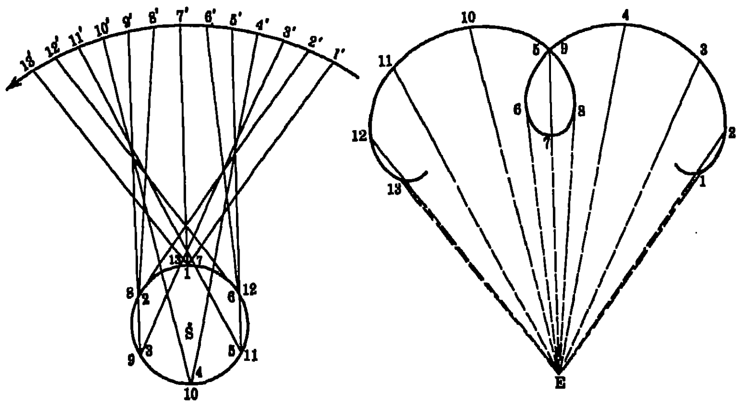


Fig. 155. *Copernican Explanation of the Looped Path of a Planet.*

observed motions of the celestial bodies could be explained more simply and reasonably by placing the Sun at the center of the system and supposing the Earth to be merely one of the planets revolving around it. The looped geocentric orbits of the planets are easily explained on the Copernican theory in this way: In Figure 155 (left side) let $S$ represent the Sun, the small circle the orbit of the Earth, and the large arc a part of the orbit of another planet, say Jupiter. The Earth would occupy the positions 1, 2, 3, etc., at intervals of two months; and Jupiter, whose angular velocity is only a twelfth that of the Earth, would at the corresponding times occupy the points 1′, 2′, 3′, etc. To us who live on the smoothly moving Earth, it seems that we are stationary while Jupiter is seen in the directions and at the distances represented by the lines 11′, 22′, 33′, etc. Hence, if we construct a drawing like that at the right in Figure 155, making the Earth stationary at $E$ and drawing lines parallel to the lines 11′, etc., and of the same length, the locus of the ends of these lines will represent the apparent geocentric path of Jupiter, which thus proves to be looped like the observed path.

It is to be noted that Copernicus's theory explains the observations that had been made up to his time as well as does Ptolemy's, but no better; but it has the advantage of greater simplicity and is free from the difficulty of making the stars, which were by that time known to be very distant. revolve daily in enormous orbits.

Without questioning the circularity of the planets' orbits, Copernicus was able to make very fair estimates of their relative distances by considering their apparent positions at different times with reference to the Sun. The case of Venus is an especially simple example. When this planet is at her greatest elongation of 47° (Figure 156), our line of sight $EV$ is tangent to her orbit (assumed circular) and so makes a right angle with her radius vector $SV$. The ratio of Venus's distance from the Sun to the Earth's distance from the Sun, $SV/SE$, is clearly the sine of 47°, and a trigonometric table shows this to be 0.73.

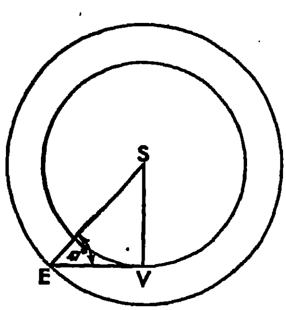



Fig. 156. *Venus at Greatest Elongation.*

Copernicus' great book was published in 1543, the year of its author's death; it is said that a copy fresh from the press was placed in his hands as he lay dying. His fellow churchmen disapproved of his ideas but were not greatly disturbed. Martin Luther sneered, more truthfully than he perhaps suspected, *Der Narr will die ganze Kunst Astronomiae umkehren* (The fool will upset the whole science of astronomy); Philipp Melanchthon referred sarcastically to *Sarmaticus astronomus, qui movet terram et figit solem* (The Sarmatian astronomer who would move the Earth and fix the Sun), and thus fathered a phrase which, unknown to him, was later to be placed on a monument to Copernicus; and in 1616 Copernicus' own Roman Catholic Church placed the *De Revolutionibus* on its Index of Prohibited Books, there to remain until 1835.

Tycho Brahe (1546–1601). Three years after the death of Copernicus there was born a Danish nobleman, Tyghe Brahe, whose name is usually given in the Latin form of Tycho. Although learning of any kind was then considered beneath the dignity of the nobility, Tycho was attracted to astronomy at the age of fourteen by the fulfillment of a prediction of an eclipse, and, overcoming his aristocratic scruples, became so great an astronomer (and astrologer) that the king of Denmark established for him a great observatory on the island of Hven, near Elsinore, which Tycho furnished with the most accurate instruments ever made before the introduction of the telescope. With these instruments and the help of many assistants, Tycho made a long and accurate series of observations of the positions of the stars and planets which formed the greatest contribution to observational astronomy up to his time. Upon his observa-

tions of the planets Kepler based his great discovery of the laws of planetary motion, which firmly established the Copernican system and paved the way to Newton's discovery of the law of gravitation.

Tycho rejected the Copernican system, partly on theological grounds and partly because his most careful observations failed to show any parallactic displacement in the stars. Instead, he substituted a "Tychonic" system in which the Sun revolved around the immovable Earth while the planets revolved around the Sun. This system was not generally accepted.

Galileo (1564–1642). The heliocentric theory received support from the brilliant discoveries of the great Italian, Galileo Galilei, when he applied the telescope to the observation of the sky. One of the first discoveries he made was that of the four bright satellites of Jupiter, which in their orbital motion around that planet exemplify almost exactly the Copernican motions of the planets around the Sun.

The news of the discovery soon spread and excited the greatest interest and astonishment. Many, of course, refused to believe it. Some there were who, having been shown them, refused to believe their eyes, and asserted that although the telescope acted well enough for terrestrial objects, it was altogether false and illusory when applied to the heavens. Others took the safer ground of refusing to look through the glass. One of these who would not look at the satellites happened to die soon afterwards. "I hope," says Galileo, "that he saw them on his way to heaven." [1]

The most powerful blow to the geocentric theory of Ptolemy was given by Galileo's observation of Venus. According to the Ptolemaic system, as we have seen, both Venus and Mercury were always nearly between the Earth and Sun, and so could never present to the Earth so much as half of their illuminated hemispheres—in other words, could never exhibit the gibbous phase. Copernicus had predicted that, if human sight could ever be sufficiently enhanced, these two planets would show the same phases as the Moon. Galileo observed Venus in the gibbous phase, and, having learned caution, announced the fact in an anagram which, after he had followed the planet until it took the crescent form, he translated by interchanging the letters. The original sentence read "*Haec immatura a me iam frustra leguntur.* o y" (These unripe things are now read by me in vain). The translation was "*Cynthiae figuras aemulatur mater amorum*" (The mother of the loves imitates the form of Cynthia); or, Venus goes through the same phases as the Moon.

[1] Lodge, *Pioneers of Science*, p. 104.

In his old age Galileo was tried by an ecclesiastic court composed of men who no doubt were pious and conscientious, but to whom the idea of the Earth's not being the immovable center of the universe was so repugnant that they convicted Galileo of heresy; he escaped severe punishment only by publicly abjuring the belief that he knew to be true. Forty years earlier, Giordano Bruno had been burned alive for similar heresies.

**Kepler (1571–1630) and the Laws of Planetary Motion.** The great German mathematical astronomer, Johannes Kepler, was a pupil of Tycho Brahe and held friendly correspondence with Galileo. Impressed with the rationality of the Copernican theory, he concerned himself with numerical questions relating to the planets, such as, Why are there just six planets? What law determines their distances from the Sun? Why do the outer planets move more slowly than the inner? and Why does the speed of any given planet vary?

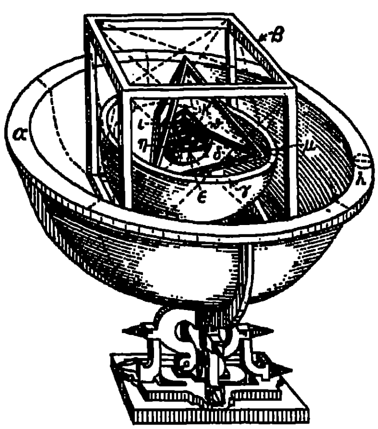



Fig. 157. *Kepler's Conjecture concerning Planets and Geometry.*

At one time he thought he had solved the first two questions, and it was this solution which attracted the attention of Tycho and brought Kepler into contact with that great man. It is shown in solid geometry that there are just five possible regular solids: those having four, six, eight, twelve, and twenty sides. Kepler found that, if the heliocentric orbit of Saturn were imagined to lie on the surface of a sphere and a cube were inscribed in this sphere, the sphere inscribed in the cube would nearly fit the orbit of Jupiter; if a regular tetrahedron were inscribed in the sphere of Jupiter it would about contain the sphere of Mars; and, with fair approximation, a dodecahedron might be inserted between the spheres of Mars and the Earth, an icosahedron between those of the Earth and Venus, and an octahedron between those of Venus and Mercury. Figure 157, copied from the frontispiece to Volume I of Kepler's *Collected Works*, illustrates this idea.

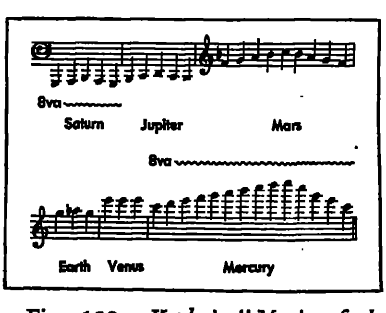



Fig. 158. *Kepler's "Music of the Spheres."*

These relations, however, were not sufficiently exact to satisfy Kepler, and so, following the ancient Pythagorean notion of the Music of the Spheres, he imagined that the pitch of the note sung by a planet might depend upon the planet's velocity. He thus made many attempts to discover the celestial harmonies, but found nothing better than the example shown in Figure 158.

Finally, after years of incredible labor and many wrong guesses, by using the extensive series of observations of the planets recorded by Tycho, Kepler arrived at the following important conclusions, which are known as Kepler's laws of planetary motion:

I. *Each planet moves in an ellipse which has the Sun at one of its foci.*

II. *The radius vector of each planet passes over equal areas in equal intervals of time.* This is known as the law of areas.

III. *The cubes of the mean distances of any two planets from the Sun are to each other as the squares of their periodic times*, or,

$$a_1^3 : a_2^3 :: P_1^2 : P_2^2$$

where the $a$'s denote the mean distances and the $P$'s the sidereal periods of any two planets.

The third law, which Kepler called the Harmonic Law, especially delighted him, and he wrote of it, "The die is cast, the book is written, to be read either now or by posterity, I care not which; it can await its reader; has not God waited six thousand years for an observer?"

**Elements of a Planet's Orbit.** The orbit of a body that revolves around the Sun is most conveniently and accurately described by means of certain numbers known as the **elements** of the orbit. Those most commonly used are the following:

1. The longitude of the ascending node, $\Omega$
2. The inclination of the orbit plane to the plane of the ecliptic, $i$
3. The longitude of perihelion, $\pi$; or else the "argument of the latitude of perihelion," $\omega$
4. The semimajor axis, $a$
5. The eccentricity, $e$
6. The mean heliocentric longitude, $L$, or the mean anomaly $M$, of the planet at a specified epoch; or else the time of perihelion passage, $T$
7. The sidereal period, $P$, or mean daily motion, $\mu$

The definition of these elements will be assisted by reference to Figure 159. The plane *EKLI* represents the plane of the Earth's orbit (or of the ecliptic), and *ORBT* the plane of the orbit of the planet in question. Their line of intersection $NN'$, which always passes through the center of the Sun, is the **line of nodes**. The planet passes from the south to the north side of the ecliptic at the point $n$, which is the **ascending node.**

Let the line $S\Upsilon$ be drawn from the Sun toward the position of the vernal equinox on the celestial sphere. The angle between this line and the line of nodes, measured from the vernal equinox toward the ascending node, is $\Omega$, the **longitude of the ascending node.** The angle between the two planes is $i$, the **inclination.** These two elements define completely the position of the orbit plane.

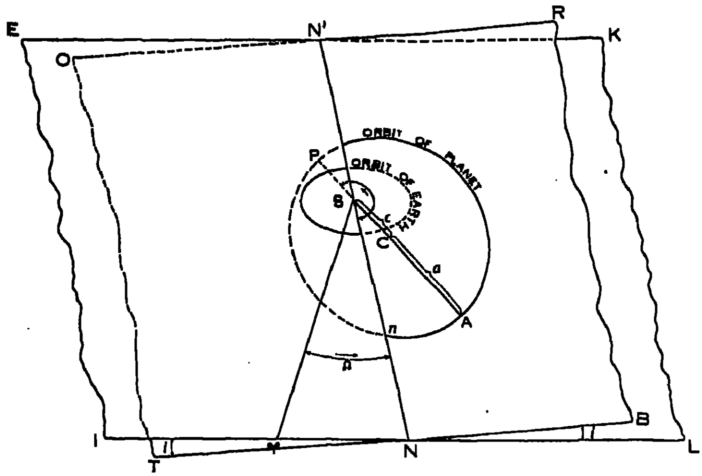



Fig. 159. *The Elements of an Orbit.*

The orientation of the orbit within its plane may be described by the angle $\omega$, measured in the plane of the orbit and in the direction of the body's motion (eastward in the case of each planet, but often westward in the case of a comet) between *SN* and *SP*, *P* being the **perihelion.** For the principal planets, however, it is customary to substitute for this element the "**longitude of perihelion,**" $\pi$, which (strictly speaking not a longitude at all) is the sum of the two angles $\Omega + \omega$.

The **semimajor axis,** $a$, or **mean distance** of the planet from the Sun, defines the size of the orbit. It may be expressed in miles or kilometers, but for most purposes it is better to use the "astronomical unit" of distance, which is the semimajor axis of the Earth's orbit.

The **eccentricity,** $e$, defines the shape of the orbit. It is a pure number, defined as the ratio of $c$, the Sun's distance from the center of the orbit (or half the distance between the foci) to $a$, the semimajor axis; that is, $e = c/a$.

The above five elements completely describe the orbit itself; but to determine the position of the planet at any time it is necessary to know its

position at some specified time, which is given by the sixth element, and also the **time of revolution,** $P$, or, what is usually more convenient in computation, the **mean daily motion,** $\mu$, which is simply 360° divided by the number of days in $P$. In the case of a planet which is so small that its mass may be neglected, this last element is superfluous, since it can be computed from $a$ by Kepler's harmonic law; but where the mass is appreciable as compared to that of the Sun, the harmonic law is not quite exact.

The position of a planet in its orbit is described by two coördinates: $r$, the length of the **radius vector,** or line joining the planet and the Sun; and $v$, the **true anomaly,** the angle made by the radius vector with the line of apsides, counted from the perihelion in the direction of the planet's motion. In a circular orbit, the calculation of these two coördinates is extremely simple, for $r$ is constant and $v$ changes uniformly with the time; but in an elliptic orbit both $r$ and $v$ change in conformity with the law of areas, and the problem of their calculation, known as **Kepler's problem,** is a matter of some difficulty.

Use is made of an imaginary **mean planet** which, coinciding with the true planet at perihelion, moves with a uniform angular velocity equal to the mean angular velocity of the true planet. The planet's **mean anomaly** is the anomaly of this fictitious body, and its **mean longitude** is the "longitude" of the fictitious planet, counted, like the "longitude" of perihelion, in the plane of the ecliptic from the vernal equinox to the ascending node, and in the plane of the orbit from the ascending node to the place of the mean planet.

**Kepler's Method of Determining the Form and Size of an Orbit.** Since the discovery of the law of gravitation and the development of modern mathematical methods, all the elements of the orbit of a planet can be very approximately determined, by a computation that in skilled hands requires less than a day, from three observations of the planet's right ascension and declination made on different dates. This method was not available to Kepler, who investigated the planetary orbits and discovered his three great laws by a much more laborious process.

As we have seen, there were available to Kepler the numerous observations of the apparent positions of the planets recorded during many years by Tycho Brahe and his assistants at the great observatory on the island of Hven. The first step was to determine accurately the planet's sidereal period, which could be done in two ways: (1) by observations of the time of conjunction or opposition, and (2) by observations of the time of node passage.

The **sidereal period** is the time of one complete orbital revolution, from a given direction among the stars to the same direction again, as

seen from the Sun; the **synodic period** is the time from opposition to opposition or from conjunction to conjunction as seen from the Earth. The synodic periods are variable because of the varying speed of the planets in different parts of their orbits; but if we represent by $S$ the *mean* synodic period of a planet in days, by $P$ its sidereal period, and by $E$ the sidereal period of the Earth (365.25), then

$$\frac{1}{S} = \frac{1}{P} - \frac{1}{E} \quad \text{if} \quad E > P, \text{ or}$$

$$\frac{1}{S} = \frac{1}{E} - \frac{1}{P} \quad \text{if} \quad E < P$$

for each side of the equation represents the average daily gain of the Sun upon the planet in their apparent eastward motion, expressed in fractions of a circumference. Oppositions, quadratures, and conjunctions of the planets having played an important part in astrology, records of such observations made long before the time of Tycho were available; from these $S$ could be found, and $P$ could then be determined from one of the above equations.

When a planet is at its node, its latitude is zero as seen from either the Earth or the Sun, for it is then exactly in the plane of the ecliptic. One sidereal period later, it will again be at the node and, although its longitude as seen from the Earth will be different because the Earth occupies a different point of its orbit, the planet's latitude will again be zero. Hence, the interval between two successive times when the planet is seen crossing the ecliptic in the same direction is its sidereal period.

Having the sidereal period, Kepler determined the planet's distance from the Sun at different points of its orbit by triangulation. His first thorough investigation was that of the orbit of Mars, the period of which is 687 days. Suppose that Mars is at the point $C$ of its orbit (Figure 160) when the Earth is at $A$. After 687 days Mars will be back at $C$, while the Earth will have made one complete revolution and a large part of another, and be at $B$. In the triangle $ASB$ the sides $SA$ and $SB$ are radii of the Earth's orbit, and the angle $ASB$ can be found from the interval of time; hence it is possible to compute trigonometrically the side $AB$ and the angles $SAB$ and $SBA$. Tycho's observations furnished the angles $SAC$ and $SBC$, the planet's elongations when the Earth was at $A$ and $B$, respectively; from these, by subtracting the computed angles $SAB$ and $SBA$, he obtained the angles $CAB$ and $ABC$; and from these and the side $AB$

he computed the sides *AC* and *BC* of the triangle *ABC*, giving the distances of Mars from the Earth in terms of the Earth's distance from the Sun. Finally, he obtained *SC*, the radius vector of Mars, from the triangle *SAC* or *CBS*, having given two sides and the included angle. From many pairs of observations separated by the interval of 687 days, he was thus enabled to find the distance of Mars from the Sun at many points, to deduce the size and form of the orbit and the speed with which the planet moved, and so to discover his first two laws. The third law resulted later from a comparison of the distances and periods of different planets.

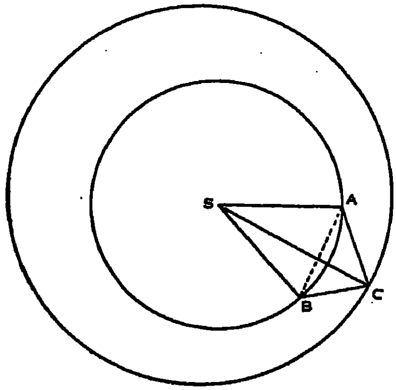



Fig. 160. *Kepler's Method of Determining the Orbit of a Planet.*

**Orbital Elements of the Principal Planets.** The elements of the orbits of the principal planets for the year 1945 are given in Table 6. They are subject to slow changes due to perturbations, but the effect of these will be unimportant for many years.

Table 6

ORBITAL ELEMENTS OF THE PRINCIPAL PLANETS[a]

| Name of Planet | Symbol | Mean Distance, $a$ | Eccentricity, $e$ | Inclination, $i$ | Longitude of Ascending Node, ☊ | Longitude of Perihelion, $\pi$ | Mean Heliocentric Longitude of Planet, 1945, January 0.0 | Sidereal Period, $P$ | Mean Daily Motion, $\mu$ |
|---|---|---|---|---|---|---|---|---|---|
| Mercury........ | ☿ | 0.387 | 0.206 | 7°.0 | 47°.7 | 76°.6 | 120°.5 | $88^{d}$ | 4°.09 |
| Venus......... | ♀ | 0.723 | 0.007 | 3.4 | 76.2 | 130.8 | 36.0 | 225 | 1.60 |
| Earth.......... | ⊕ | 1.000 | 0.017 | | | 102.0 | 99.8 | 365 | 0.99 |
| Mars........... | ♂ | 1.524 | 0.093 | 1.9 | 49.1 | 335.0 | 267.4 | $1^{y}9$ | 0.52 |
| Jupiter......... | ♃ | 5.203 | 0.048 | 1.3 | 99.9 | 13.4 | 164.4 | 11.9 | 299″ |
| Saturn......... | ♄ | 9.539 | 0.056 | 2.5 | 113.2 | 92.0 | 97.1 | 29.5 | 120 |
| Uranus........ | ♅ | 19.191 | 0.047 | 0.8 | 73.7 | 169.8 | 76.8 | 84.0 | 42 |
| Neptune....... | ♆ | 30.071 | 0.009 | 1.8 | 131.2 | 44.1 | 184.0 | 164.8 | 22 |
| Pluto.......... | ♇ | 39.457 | 0.249 | 17.1 | 109.6 | 223.5 | 158.3 | 247.7 | 14 |

The table contains many interesting facts. Note, for example, that the range in mean distances, from Mercury to Pluto, is about a hundredfold

[a] Abridged from *The American Ephemeris* for 1945.

and the range of periods about a thousandfold, thus illustrating Kepler's third law in simple numbers. Expressed in miles, the mean distance of Mercury is about 38,000,000, and that of Pluto more than 3,600,000,000. The Sun's light, which arrives at the Earth in 500 seconds, requires more

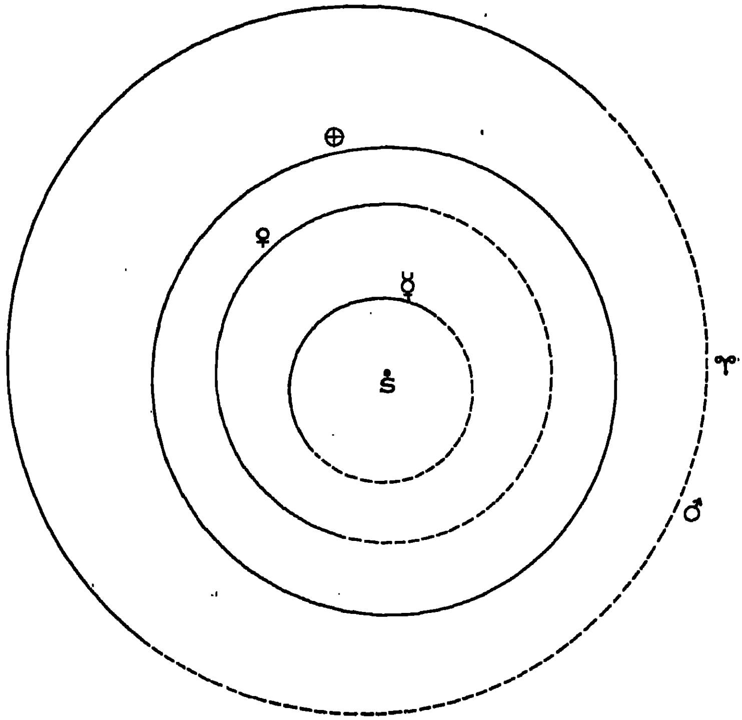



Fig. 161. *Orbits of the Four Inner Planets.*

than four hours to travel to Neptune and nearly seven hours to reach Pluto when at aphelion. The age of an earthly octogenarian is less than half of a Neptunian "year" and less than a third of the period of Pluto. The values of $e$ and $i$ show that all the orbits, except the smallest and the largest, are nearly circular and lie nearly in the same plane.

The orbits are represented in Figures 161 and 162, in projection upon the plane of the ecliptic. None of the orbits approach each other closely; although Pluto, at perihelion, is nearer than Neptune to the Sun, the inclination of its orbit is such that it is then more than 800 millions of miles from Neptune's plane.

**Bode's Law.** The approximate mean distances of the planets from the Sun may be conveniently remembered by a relation first pointed out by Titius but now

commonly known as Bode's law. If we write a series of 4's and add to them the numbers 0, 3, $3 \cdot 2 = 6$, $6 \cdot 2 = 12$, $12 \cdot 2 = 24$, etc., thus:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| 0 | 3 | 6 | 12 | 24 | 48 | 96 | 192 | 384 |
| 4 | 7 | 10 | 16 | 28 | 52 | 100 | 196 | 388 |

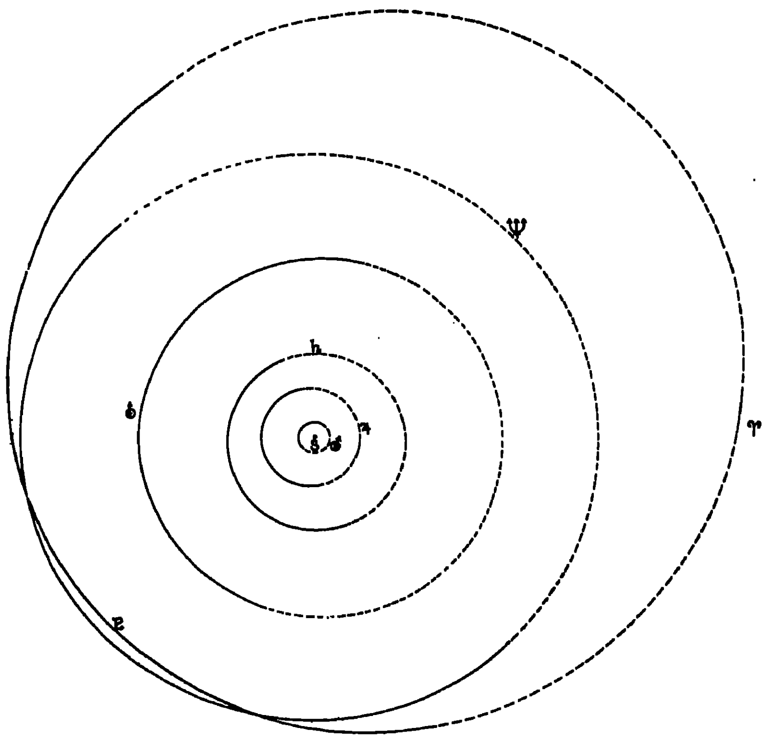


Fig. 162. *Orbits of the Five Outer Planets.*

we get a series of numbers which are approximately ten times the mean distances of the planets in astronomical units. The fifth term of the series represents well the average distance of the asteroids, and the last term approximates the mean distance of Pluto; but the "law" fails for Neptune. There is no known reason for the "law" and it may be merely a coincidence.

**The Symbols of the Planets.** The symbols which are shown in Table 6, and which are often encountered in astronomical literature, are mostly of ancient origin and are supposed to be conventionalized pictures of objects associated with the deities for whom the planets are named. The symbol for Mercury represents the Caduceus, a wand with two serpents twined around it, which was carried by the messenger of the gods. Venus, the planet of love and beauty, is symbolized by a hand mirror; Mars, planet of war, by a shield and spear; and Saturn, slowest of the ancient planets, by a sickle, corresponding to the scythe of Father Time. The symbol of Jupiter is perhaps a thunderbolt or the letter Z, initial of Zeus. That of

the Earth is probably a globe showing the equator and central meridian, and is of more recent origin. The symbol of Uranus is said to represent the heavens (Uranus was god of the sky), and Neptune is represented by his familiar trident. The last three are of course modern.

**The Sun as Seen from Different Planets.** As an observer recedes from the Sun, both its apparent diameter and the intensity of its light and heat become less, the apparent diameter being inversely proportional to the first power of the distance and the intensity to the square. As seen from Mercury, the Sun's diameter is about two and one-half times as great as it appears from the Earth; as seen from Neptune, its diameter is only a little more than a minute of arc, and to the naked eye it would appear as an intensely bright star without perceptible disk.

Taking the intensity of the Sun's light and heat at the Earth's distance as unity, we have the following values of the intensity at the mean distances of the various planets:

| ☿ | ♀ | ⊕ | ♂ | ♃ | ♄ | ♅ | ♆ | ♇ |
|---|---|---|---|---|---|---|---|---|
| 6.7 | 1.9 | 1.00 | 0.43 | 0.04 | 0.01 | 0.003 | 0.001 | 0.0006 |

It may well be imagined that there exists a great diversity of climate.

**The Earth as Seen from Other Planets.** The Earth, being an opaque body, must shine by reflected sunlight and, to an observer on one of its nearest neighbors, must present much the same appearance as the other planets do to us. The most favorable view would be obtained from Venus. When the Earth and Venus are nearest together, at a distance of some 26,000,000 miles, the latter planet is at inferior conjunction, nearly or (on rare occasions) quite directly between us and the Sun, so that her dark side is turned toward us and at the same time the eye is dazzled by the light of the Sun. For an observer on Venus at the same time, the conditions for viewing the Earth would be the reverse of these: the Earth would be at opposition with its illuminated side turned full upon the observer and would appear as a star about six times as bright as Venus appears to us. The Moon would also be plainly seen as a fainter star passing from side to side of the Earth in the course of a month to a distance of about half a degree.

From Mars the view would be less favorable, since for the Martians the Earth, like Venus and Mercury, would seem to oscillate from side to side of the Sun and would reach a "greatest elongation" of 48°, or about the same as that of Venus as seen from the Earth. When Mars and the Earth

are nearest together, the Earth would be at inferior conjunction, and near that time a Martian with a good telescope might see both the Earth and the Moon in the crescent phase.

As seen from Jupiter, the Earth is never more than 12° distant from the Sun, and it is likely that a race of Jovians, if equipped with eyes and telescopes similar to ours, would be unaware of the existence of this little globe. This deplorable state of ignorance is of course still more likely to prevail among any possible inhabitants of more distant planets, or their satellites.

## EXERCISES

1. Construct a drawing of the complete geocentric path of Jupiter on a scale of one centimeter to the astronomical unit by the method described on page 213, assuming for Jupiter a distance from the Sun of five astronomical units and a period of twelve years.

2. What would be the period of a planet having a mean distance from the Sun one-fourth that of the Earth?

*Ans.* $\frac{1}{8}$ of a year, or 45.7 days

3. What must be the mean distance of a planet in order that it will revolve around the Sun in 216 years?

*Ans.* 36 astronomical units

4. If the Earth's mean distance from the Sun were doubled, what would be the value of the constant of aberration?

*Ans.* $14''.6$

5. Calculate the mean synodic periods of Venus and Jupiter from their sidereal periods given in Table 6, assuming 1 year = 365.25 days.

*Ans.* Venus, 586 days; Jupiter, 399 days

6. What is the phase of Venus at greatest elongation?

*Ans.* The quarter

7. What is the elongation of Mars as seen from the Earth when the Earth as seen from Mars is in the quarter phase?

*Ans.* Quadrature

# CHAPTER 11

## THE LAW OF GRAVITATION

**Isaac Newton (1643–1727).** Kepler's laws of planetary motion were simply descriptive statements of the behavior of the planets. Neither Kepler nor anyone else had given an explanation of the force that causes the planets to move in just this way and no other. It was generally supposed that, to keep a planet moving, some "projectile force," acting along a tangent to the orbit, was required. Kepler seems to have been somewhat inclined to attribute this force to the will of a supernatural being—perhaps an angel that had charge of each planet—or to invisible spokes that radiated from the Sun and pushed the planets along. It remained for the great English mathematical philosopher Sir Isaac Newton to show that the planetary motions were but manifestations of a universal principle and to derive the mighty generalization known as the Law of Gravitation.

The contributions of Newton to astronomy, mathematics, and physics are numerous and exceedingly important. Before reaching the age of twenty-four he had discovered the binomial theorem, formed his theory of colors, founded the branch of mathematics which he called "fluxions" and which has grown into the modern calculus, and laid the basis of the law of gravitation. It was not, however, until 1687 that he published his immortal work *Philosophiae Naturalis Principia Mathematica*, commonly known as Newton's *Principia*, the appearance of which probably marks the greatest forward step ever made in physical science.

**Newton's Laws of Motion.** The science of mechanics consists of theorems built upon certain laws or principles just as geometry is built upon its familiar axioms. The motions of bodies as ordinarily observed, whether they be the planets or familiar bodies at the surface of the Earth, may be calculated precisely by the system of mechanics built up by Newton. He used as a basis the axioms of Euclidian geometry together with three principles that are known as Newton's laws of motion because they

were first definitely stated by Newton, although they were at least pa understood by Galileo and, before him, by the great artist-inventor Leonardo da Vinci. These laws of motion are:

I. *Every body continues in a state of rest, or of uniform motion in a straight line, unless it is compelled to change that state by a force impressed upon it.*

II. *When a force acts on a body, the state of rest or motion is changed in the direction in which the force acts; and the rate of change of momentum is proportional to the force.*

III. *To every action there is an equal and opposite reaction.*

**Comment on the First Law.** If a block of wood resting on a level floor is given a sufficient impulse, it will slide along the floor, but if then left to itself it will come quickly to a stop because of the friction between it and the floor. If the floor is made very smooth, the block will slide farther before coming to a stop because the friction is less. If the block is placed on smooth ice, a given impulse will carry it farther still. We cannot remove from a body all forces such as friction, gravity, etc., and so cannot completely verify the law experimentally; but experiment shows that the more nearly we eliminate the forces acting on a body, the more nearly does its motion conform to the law. Newton assumed that a body moving in free space, unacted upon by any force whatever, would travel forever uniformly in a straight line. Simply to keep moving, then, a planet or any other body needs no force such as the "projectile force" of Aristotle. What requires an explanation is the curvature of the planet's path and the variation of its speed.

**Comment on the Second Law.** **Momentum** is defined as the product obtained by multiplying mass by velocity; and these words too have technical meanings. The **mass** of a body is what gives it inertia; it is sometimes defined as the quantity of matter in the body. Mass is expressed in such units as grams and pounds, but it is not the same as **weight**, which is the pull of gravity. Strictly, one should speak not of "a weight of one pound" but rather of "the weight of a body whose mass is one pound." At any place on the Earth's surface, the weights of all one-pound bodies are the same, but if one of these bodies were carried to the Moon its weight would become much less although its mass would still be a pound.

**Velocity**, as defined in mechanics, is not mere speed, it is *directed* speed. Velocity includes both the rate at which the distance from some point of reference is changing and the direction of the motion; a change of velocity may involve a change of speed or a change of direction, or both. A car driven around a circular race track is continually changing its velocity even though its speed is constant. The rate of change of velocity is called acceleration.

That which, according to the second law of motion, is proportional to the force is the rate of change of the product mass $\times$ velocity; or, since the mass of an ordinary body does not change, the force is proportional to mass times the rate of change of velocity, or, finally, to mass times acceleration.

For centuries it was supposed, in agreement with Aristotle, that a ten-pound body falls faster than a one-pound body. Galileo, by dropping similar bodies of unequal mass from the Leaning Tower of Pisa, showed that they fall with equal speed and equal acceleration. While the force (weight) acting on the ten-pound body is ten

times greater, the body's inertia is also ten times greater; and so a tenfold force is both necessary and sufficient to give it the standard acceleration of 32 feet per second per second. The fact that all bodies, such, for example, as a bullet and a feather, do not fall a given distance *in air* in the same time is due to the difference of the resistance of the air caused by their different shape or density. If dropped in a tube from which the air has been exhausted, the feather and the bullet fall in the same time.

In the statement of the second law, nothing is said about the condition of rest or motion of the body at the time the force is applied, nor of other forces that may be acting at the same time. It is therefore implied that, whether the body is at rest or in motion, and whether other forces are acting or not, a given force produces the same change of momentum that it would produce if it alone acted on the body at rest. Thus, if a body at $A$ (Figure 163) is acted upon simultaneously by a force which would cause it to move in one second to $B$ and by another which would send it in one second to $C$, it must, at the end of one second, arrive at a point $D$ which is at the distance $AB$ from $A$ as measured in the direction of the first force and at the distance $AC$ from $A$ as measured in the direction of the second force; that is, $D$ is at the opposite corner of a parallelogram of which the lines $AB$ and $AC$ are adjacent sides. The important principle of the parallelogram of forces is thus a corollary to the second law of motion.

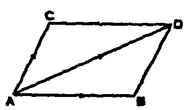



Fig. 163. *The Parallelogram of Forces.*

**Comment on the Third Law.** Suppose a man on a raft that floats freely in still water pulls on a rope attached to a similar raft that is heavily loaded, say with scrap iron. Both rafts will move, and the more lightly loaded raft will move faster. If there were no friction or other forces except the pull on the rope, the product of the mass of each raft (plus its load) times its rate of change of velocity would be the same—the "action" of the man is met by an equal and opposite "reaction" on the part of the inanimate scrap iron. A little reflection will disclose a similar balance wherever forces are applied.

The first two laws are sufficient for a discussion of the results of applying various forces to one body; the third is needed for studying the motions of a system of bodies.

**The Theorem of Areas.** As an example of Newton's methods and the important results to which they led we may prove the following theorem:

*To Prove: If a body moves subject to no forces except one that is directed always toward the same fixed point* O, *the line joining the body to* O *must pass over equal areas in equal intervals of time.*

**Proof.** Suppose, first, that the body is subject to no forces whatever, so that, by the first law of motion, it will move uniformly in a straight line. Let the line of its motion be $AC$ (Figure 164) and let it move the distance $AB$ in one second. In the next second it will travel an equal distance $BC$. Let $O$ be any point not in the line $AC$ and draw $OA$, $OB$ and $OC$. The triangles $OAB$ and $OBC$, having equal bases $AB$ and $BC$ and the common altitude $OK$, are equal in area.

Suppose, however, that upon arriving at *B* at the end of the first second, the body is acted on by an instantaneous force—like a blow from a hammer, say—acting along the line *BO*, and that this force is of such magnitude that, had the body been at rest, it would have been sent to *D*

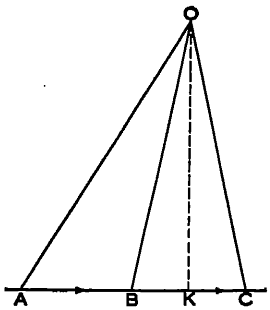



Fig. 164.

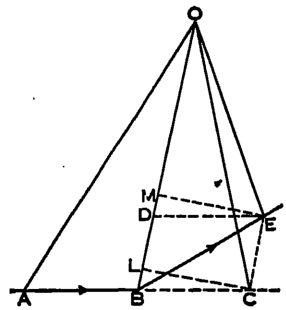



Fig. 165.

*The Theorem of Areas.*

(Figure 165) in one second. According to Newton's second law, this instantaneous force will produce a change of motion parallel to the line *BO*, causing the body to be displaced in one second a distance *BD* in the direction of that line; but it will not cause it to lose its original motion which, acting alone, would carry the body in one second from *B* to *C*. The actual path is therefore *BE*, the diagonal of the parallelogram of which *BD* and *BC* are adjacent sides. Draw *OC* and *OE*. The triangles *OBC* and *OBE* have the common base *OB* and equal altitudes *CL* and *EM*, the distance between the parallel lines *BD* and *CE*. Therefore triangle *OBE* is equal in area to triangle *OBC*, which we have already shown to be equal to *OAB*.

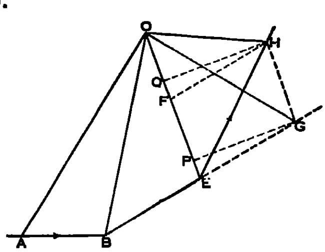



Fig. 166.

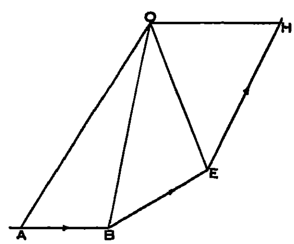



Fig. 167.

*The Theorem of Areas.*

In the third second, if the body were left to itself, it would, according to the first law of motion, move from *E* to *G* where $EG = BE$ (Figure 166); but let it again be acted upon, when at *E*, by an instantaneous force directed toward *O* and of such magnitude as to carry it to *F* had it been at rest at *E*.

It will travel to $H$, the opposite corner of the parallelogram whose sides are $EG$ and $EF$. The triangles $OEH$ and $OEG$ are equal in area since they have the common base $OE$ and equal altitudes $GP$ and $HQ$.

We now have (Figure 167):

$$\Delta OAB = \Delta OBE = \Delta OEH.$$

That is, if the body is subject to no forces except one which acts instantaneously toward $O$ at the end of each second, the lines joining $O$ to the positions occupied by the body at the instants of action of the force form triangles of equal area. Our proof will hold as well if we suppose the force to act at intervals of half a second or a millionth of a second, or at intervals that are smaller than any assignable quantity, however small. In the limit, the intermittent, instantaneous forces are replaced by a continuously acting force (still supposed to be always acting along the line joining the body to $O$), and the path of the body becomes a smooth curve of a form depending on the way in which the force varies. Yet, however small the interval at which the force is supposed to act, the area described in any unit of time, such as one second, will be the same as that described in any equal unit since it is the sum of the same number of equal areas; and hence, however the force may vary in intensity, so long as it acts always along the line joining the body to $O$ (it may be directed away from $O$ as well as toward $O$), that line will pass over equal areas in equal intervals of time. Q.E.D.

We have placed no limitations upon the force except that it shall always act along the line joining the body to $O$. It may be of constant or variable intensity, or it may be intermittent, or alternately attractive and repulsive; it may vary in intensity directly as the distance, or inversely as the square of the distance, or according to any other law, or even capriciously; and the law of areas proved above must still hold.

This theorem illustrates one application—the only one that we shall attempt to give in detail—that Newton made of his laws of motion to the study of mechanics. He proved also that the converse is true: If a body be found moving in such a way that the line joining it to any point passes over equal areas in equal intervals of time, then the only force (or the resultant of all the forces) acting on the body must be directed toward (or away from) the point. Kepler, as we have seen, had already shown that each planet moves in that way in regard to the Sun—i.e., that its radius vector describes equal areas in equal times. The conclusion

was therefore obvious that *the only force required to explain the motion of any one of the planets is a force directed toward the Sun.*

**Newton's Inferences from Kepler's Laws.** By mathematical processes based upon the laws of motion as in the above example, Newton made an important inference from each of the laws of Kepler. These inferences, including the one just stated, are:

1. From Kepler's law of areas: *The force that controls the motion of each planet is directed toward the Sun.*

2. From Kepler's first law: *The Sun's attraction for a given planet varies inversely as the square of the planet's distance from the Sun.*

3. From the harmonic law: *The Sun's attraction varies from one planet to another inversely as the square of the distance.*

The second statement means that, as a planet approaches perihelion, the intensity of the attraction rapidly increases; if the distance is halved, the attraction is increased fourfold. It depends both on the fact that the orbit is an ellipse and on the fact that the Sun occupies its *focus*. If, for instance, Kepler had found that the Sun was in the center of the planetary ellipses, Newton must have concluded that the attraction varied directly as the first power of the distance instead of inversely as the square.

The third statement means that the attraction for a planet of given mass depends only on the distance and not on any other properties of the planets such as temperature, chemical constitution, etc. If Mars were placed at the Earth's distance from the Sun, the Sun's attraction for it, except for the difference of mass, would be the same as it is in the case of the Earth. Newton showed, however, that for planets of appreciable mass, the harmonic law is only approximately true; the precise relation is

$$a_1^3 : a_2^3 : : P_1^2 \, (M + m_1) : P_2^2 \, (M + m_2)$$

where $M$ is the mass of the Sun and the subscripts refer to the different planets. In every case in the Solar System, $M$ is so much greater than $m$ (the mass of the Sun is about a thousand times that of even Jupiter, the greatest planet) that Kepler's statement of the law, in which the masses are omitted, is very nearly correct.

**The Law of Gravitation.** It is related that Newton was sitting one day in his garden, reflecting upon the force that bends the planets from straight paths and upon the force that holds the Moon in its orbit around the Earth, when his attention was diverted by an apple falling from a nearby tree. It occurred to him that the fall of the apple and the divergence of the

Moon and planets from straight lines might be manifestations of the same force. Becoming convinced that this was true, he was eventually led to the Law of Gravitation, which states:

**Every particle of matter in the universe attracts every other particle with a force that varies inversely as the square of the distance between them and directly as the product of their masses.**

This great law may be expressed very simply in symbols if we let $m_1$ and $m_2$ be the masses of any two particles, $d$ the distance between them, $F$ the force of their attraction, and $G$ a constant; thus:

$$F = G\frac{m_1 m_2}{d^2}.$$

Newton did not state the law of gravitation in just these words in the *Principia*, nor does it appear that he explicitly extended it beyond the Solar System; but the whole of celestial mechanics, including Newton's own contribution, is in harmony with the law as above stated, and the elliptical motion of double stars shows that the law holds between them as between the Sun and planets.

**Gravity and Gravitation Distinguished.** **Gravitation** is the universal attraction of every particle of matter for every other, wherever located. **Gravity** is the resultant effect, at the Earth's surface, of the Earth's gravitation and the centrifugal force caused by its rotation. Gravity is measured by the acceleration produced in a freely falling body, which is about 32 feet (981 cm.) per second per second; that is, a body falling under the influence of gravity alone increases its velocity by 32 feet per second during each second of its fall. Starting from rest, it falls 16 feet in the first second, 48 feet in the next, 80 feet in the third, and so on. This acceleration is commonly denoted by $g$. The **weight** of a body is its mass multiplied by $g$. The value of $g$ is greater at the poles than at the equator by about 1/190 of itself; hence, an object that weighs 190 pounds at the pole would weigh only 189 at the equator (if weighed on a spring balance). One pound in 289 of this difference is due to centrifugal force; the remainder, about one pound in 555, is due to the difference of the attractive power of the Earth, which depends on the fact the equator is farther from the center of the planet than is the pole.

**The Earth's Form Deduced from Pendulum Experiments.** A method of determining the Earth's form independently of measurements of its size (page 82) utilizes observations made in different parts of the world upon a swinging pendulum. This method was discovered in the year 1672, when the French astronomer Richer was sent by Louis XIV on an expedition to the tropical island of Cayenne. He took with him a clock whose pendulum had been regulated to beat seconds in Paris, but

which he found from astronomical time-determinations to be losing about two and a half minutes a day in Cayenne, although the pendulum had not been altered. It is well known that the time of swing of a simple pendulum is

$$T = \pi\sqrt{\frac{l}{g}}$$

where $l$ is the length of the pendulum. Since it was known that $l$ had not changed, the change in $T$ must have been due to a variation in $g$.

The part of the variation in $g$ in different latitudes that is due to centrifugal acceleration may be readily calculated, and the difference between this and the observed variation is the effect upon gravity of changing distance from the Earth's center. From a study of this effect in many localities the Earth's form has been deduced, and the results agree well with those of the geodetic method.

**Test of the Law of Gravitation by the Motion of the Moon.** By means of his principles of "fluxions" Newton was able to prove the difficult proposition that, if the law of gravitation be true, a homogeneous sphere of any size attracts an exterior particle (or sphere) inversely as the square of the distance from its center; that is, as if the whole mass of the sphere were concentrated at its center. He was then in a position to test the law of gravitation by comparing the distance which an apple falls in the first second with the distance that the Moon "falls" toward the Earth, or in other words deviates from a straight line, in the same time.

At a given distance from the Earth the Moon and the apple would, according to the second law of motion, fall at the same speed just as did Galileo's unequal weights at the Leaning Tower of Pisa; but, as Newton knew, the Moon is about sixty times as far from the center of the Earth as is the apple, and hence, if they are both controlled by an attraction that varies inversely as the square of their distances from the center of the Earth, the fall of the apple in the first second should be $60^2$, or 3600 times the Moon's departure from a straight path in an equal time. The apple falls in the first second 16.1 feet, or 193 inches; hence the Moon should depart in one second from a straight line 1/3600 of this distance, or 0.0535 inch.

The actual deviation may be easily determined from a knowledge of the Moon's distance from the Earth and the time of its revolution. In Figure 168 let the circle, of radius $R$, represent the orbit of the Moon, and let the Moon move from $A$ to $B$ in one second. Draw the diameter $AP$ and the chord $BP$, and draw $BN$ at right angles to $AP$. The fall of the Moon toward the Earth in one second is the distance $BM$ or its equivalent $AN$. The arc $AB$ is in reality so short that it may be taken as a straight line, being one side of a right triangle whose hypotenuse is $AP$, and therefore a mean proportional between $AN$ and $AP$; that is,

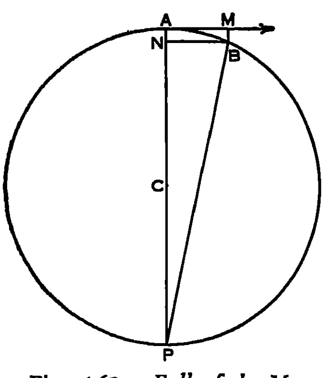


Fig. 168. *Fall of the Moon Toward the Earth.*

$$AN = \frac{\overline{AB}^2}{AP}.$$

But $AB$, the distance traveled along the orbit in one second, is $2\pi R/T$, where $T$ is the number of seconds in a month; and, as $AP = 2R$,

$$AN = \frac{2\pi^2 R}{T^2}.$$

To compare the fall of the Moon with that of the apple, we must have $T$ in seconds and $R$ in inches. Newton knew the value of $R$ in Earth-radii (60), but is said to have been misled at first by an erroneous value of the Earth's radius (possibly Snell's, page 80). His work was delayed several years, partly by the difficulty of questions relating to the attraction of spherical bodies; and in the meanwhile a more accurate value of the Earth's radius was published by Picard. With the resulting value of $R$, Newton shows in the *Principia* a close agreement between the speed of falling bodies and that of the Moon's departure from the tangent.

In Newton's day modern methods of mathematical analysis were unknown and he developed his theorems by the comparatively cumbrous processes of elementary geometry, aided by his principles of "fluxions." Celestial mechanics was brought to a high state of perfection in the latter half of the eighteenth century by the French mathematicians Lagrange and Laplace who applied to Newton's laws the methods of the calculus.

**Detection of Gravitation Between Small Bodies.** A number of experiments have been performed whereby the attraction between two bodies of much smaller mass than that of the Moon or planets has been detected and measured. In 1740 Bouguer detected the deviation of the plumb line caused by the attraction of Mount Chimborazo in South America, and in 1774 Maskelyne made a similar observation of greater accuracy at Mount Schiehallien in Scotland, from which he deduced the mass of the Earth by comparing its attraction with that of the mountain. A more accurate experiment fulfilling the same purpose is that of the **torsion balance**; first performed by Lord Cavendish in 1798, it is known as the Cavendish experiment.

Cavendish's torsion balance consisted of a light rod about six feet long which was supported in a horizontal position by a slender wire, and which carried at each end a lead ball about two inches in diameter. Suspended at the same level with these balls and on opposite sides of the rod were two 12-inch balls, also of lead. Any rotation of the rod around the supporting wire could be observed by small telescopes directed to small mirrors attached to the ends of the rod. The whole apparatus was enclosed in a case to prevent disturbance by air currents.

Figure 169 is a diagram of the apparatus as seen from above. Suppose the normal position of the rod was $xx$. With the large balls in the positions $W_1W_1$, their attraction, which was resisted only by the twist of the

slender wire, brought the rod into position $x_1x_1$. The large balls were then turned to $W_2W_2$, and their attraction brought the rod into the line $x_2x_2$. The total angle moved through by the rod from position 1 to position 2, which is observed by the telescope, is four times the angle through which it would be moved by the attraction between one small ball and one large one. This angle is proportional to the force of attraction and therefore to the resistance due to the torsion of the wire, and the factor of proportionality can be determined by observing the time of swing when the balance is disturbed slightly from its position of equilibrium and then released, the large balls $W$ being removed for this purpose. The stiffer the wire the more rapid are the vibrations of the balance.

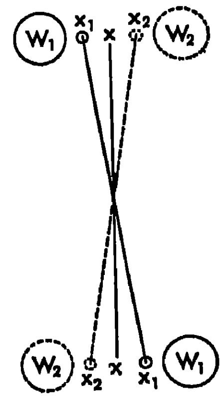



Fig. 169. *Method of Using the Torsion Balance.*

Cavendish was able with this apparatus to detect and measure the attraction between the balls, but the accuracy of the measurements was impaired by the effect of air currents which could not be entirely eliminated by the enclosing case. In 1895 Boys used gold balls 5 mm. in diameter, attracted by lead balls 10 cm. in diameter. The rod of his torsion balance was a mirror 24 mm. long, which was suspended by an exceedingly fine fiber of spun quartz. The gold balls were suspended from the mirror at different levels so that the lead balls would not interfere with one another, and the balance was enclosed in an airtight case from which the air was partially exhausted. In 1930, the experiment was repeated with great care and precision by Heyl at the U. S. Bureau of Standards.

**Determination of the Constant of Gravitation.** If we let $f$ represent the attraction between the large and the small ball, $d$ the distance between their centers, and $B$ and $b$ their respective masses, we have by the law of gravitation

$$f = G\frac{Bb}{d^2}$$

or, solving for G,

$$G = \frac{fd^2}{Bb}.$$

The masses $B$ and $b$ can be determined by weighing the balls, $d$ can be measured, and $f$ is determined by the experiment; hence the equation gives the numerical value of $G$, the constant of gravitation, which is believed to

have the same value throughout the universe. The value determined by Heyl is $6673 \cdot 10^{-11}$ C.G.S. units, which means that two spheres, each having a mass of one gram, if placed with their centers one centimeter apart, attract each other with a force of 6673/100,000,000,000 of a dyne.

The dyne is the C.G.S. unit of force, and is defined as the force required to give a mass of one gram an acceleration of one centimeter per second per second. The *weight* of a body at the Earth's surface, which is the force of attraction between the Earth and the body, gives it an acceleration, commonly denoted by $g$, of 981 cm. (about 32 feet) per second per second; hence, the weight of a body whose mass is one gram is 981 dynes.

The force of gravitation acting between bodies of ordinary size is exceedingly small when compared, for instance, with the attraction between two magnets. An illustration involving larger masses will perhaps make this clearer. Suppose two spheres, of mass $10^{11}$ grams each (iron balls 100 feet in diameter would have about this mass), placed with their centers one kilometer ($10^5$ cm., 0.62 mile) apart. No matter what buildings or other obstacles intervene, the two spheres will attract each other, but the attraction will be only 66,640 dynes, a force equal to the weight of a body whose mass is 68 grams, or less than three ounces. Even if all friction and other forces that might affect their motion were eliminated, either ball could be kept from moving by a slight pressure of the finger. If perfectly free from other forces, the balls will begin to approach each other, but so slowly that a microscope will be required to detect the motion. At the end of an hour, each ball will have moved only about 4.4 centimeters, and will then be moving at the rate of a fortieth of a millimeter per second. This speed will be accelerated at a slowly increasing rate, and at the end of eighty-four hours the two balls will come together at a relative velocity of 2.1 cm./sec.

**Determination of the Mass of the Earth.** Let $E$ be the mass of the Earth, and $w$ the weight of the small ball used in the Cavendish experiment. The centers of the Earth and the ball are a distance $r$ apart, where $r$ is the radius of the Earth. The weight $w$ is simply the force of the attraction between the ball and the Earth, and is found by multiplying the mass $b$ of the ball by the quantity $g$; also, it is given by the law of gravitation,

$$w = G\frac{Eb}{r^2}.$$

Solving for $E$ gives

$$E = \frac{wr^2}{Gb}$$

or, substituting the value of $G$ found in the preceding section,

$$E = \frac{wr^2B}{fd^2}.$$

From this equation, expressing the forces $w$ and $f$ in dynes, the mass $B$ in grams, and the distances $r$ and $d$ in centimeters, we can compute $E$ in grams. The result is $6 \cdot 10^{27}$ grams, or $6 \times 10^{21}$ metric tons.

The Earth's mass and that of the vastly larger Sun are so great that, despite the smallness of $G$, the force of their mutual attraction is sufficiently strong to pull asunder a solid steel rod nearly 3000 miles in diameter; and yet, because of the Earth's great inertia, this force causes the Earth to deviate from a straight line only about one-ninth of an inch while traveling eighteen miles along its orbit.

**Calculation of the Superficial Gravity of a Heavenly Body.** Since a sphere attracts an external body as if its mass were all situated at its center, if $m$ represent the mass of the Earth, $r$ its radius, and $g$ its attraction for unit mass at its surface, the law of gravitation gives

$$g = \frac{Km}{r^2}$$

where $K$ is a factor of proportionality. Similarly, if $m'$, $r'$, and $g'$ represent the corresponding quantities for any other sphere,

$$g' = \frac{Km'}{r'^2}.$$

Dividing the second equation by the first, we have

$$\frac{g'}{g} = \frac{m'r^2}{mr'^2}.$$

The quantity on the left has already been defined as the **superficial gravity** of the sphere (page 128). It may readily be computed by the last equation when the body's mass and radius are known in terms of the Earth's.

**The Problem of Two Bodies.** The following problem has been completely solved, first by Newton and later by other mathematicians who used modern and more powerful methods.

*Given the masses of two particles or spheres which are subject to their mutual gravitational attraction and to no other force; given also their positions and velocities at any moment; to determine the orbits which they will follow and their positions at any other time.*

Strictly speaking, the conditions of this problem are never fulfilled in nature, since every particle is attracted by *all* others instead of by a single one; but they are fulfilled approximately in the case of each planet and the Sun, because the attraction of the latter vastly preponderates over that of

the other planets and the stars, and probably still more nearly in the cases of some double stars. The details of the solution of the problem cannot be given here, but some of the important results may be mentioned, as follows:

1. The position of the center of mass (page 128) of the two bodies is unaffected by their attraction, and it remains at rest or in uniform rectilinear motion.

2. The two bodies will follow orbits around their common center of mass which are similar in form but of size inversely proportional to their masses, the more massive body moving in the smaller orbit. The orbit of each body relative to the other will be a curve similar to its orbit relative to the center of mass, but of the size proportional to the *sum* of the masses of the two bodies.

Observations of the right ascension and declination of Sirius made with meridian circles before 1844 showed that this brightest of stars was moving slowly in a little orbit upon the celestial sphere, and the motion was attributed by Bessel to the attraction of an unseen companion. In 1862, during a test of the new eighteen-inch

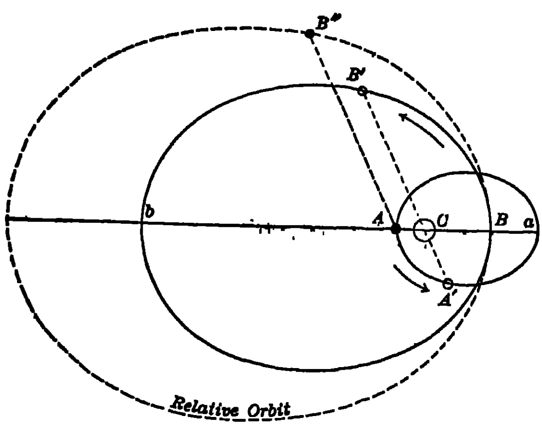



Fig. 170. *Orbits of Sirius and Its Companion.*

objective of the Dearborn telescope, the companion was seen by Clark, and it has since been observed entirely around its orbit which it requires fifty-two years to circumvolve. Figure 170 shows the orbits of the two stars, of which the fainter possesses a mass about two-fifths that of the brighter. *C* is the center of mass, and the continuous curves are the absolute orbits of the two bodies, the smaller ellipse belonging to the larger star, *A*. The orbit of the small star relative to the bright one, as determined by micrometer measures of distance and position angle, is shown by the dotted ellipse.

3. The orbits of the two bodies will not necessarily be ellipses, but must in every case be one of the species of curves known as **conic sections**,

of which the ellipse is a member; and the center of mass will be at the focus of the conic.

The conic sections are so called because each may be defined as the intersection of a plane with a right circular cone, the different forms resulting from different positions of the cutting plane. A right circular **cone** (Figure 171) is defined as the surface generated by a straight line which passes always through a fixed point *V* called the **vertex,** while one point of the line travels around a circle whose center *C* is at the point of intersection of a perpendicular drawn from *V* to the plane of the circle. This perpendicular, *VC*, is called the **axis** of the cone. If this surface is cut by a plane passing at right angles to the axis, as at *NM*, the curve of intersection is a **circle** of a size depending on the angle of the cone and the distance of the cutting plane from the vertex. If the plane is gradually inclined from this position the curve of intersection becomes an **ellipse,** as at *FE*; and the eccentricity of the ellipse increases until the plane makes an angle with the axis equal to half the vertical angle of the cone, as at *DP*, when the ellipse becomes a **parabola.** If the cutting plane is still further inclined, as at *YH*, it cuts the cone above the vertex as well as below, and the curve becomes a **hyperbola.** The circle and parabola are thus seen to be limiting cases of the ellipse. Parabolas as well as circles are all of the same shape, though of different sizes; ellipses and hyperbolas have various shapes as well as sizes. If the cutting plane passes through the vertex of the cone, the circle and ellipse each degenerate to a point; the parabola narrows to a line and the hyperbola becomes a pair of crossed lines.

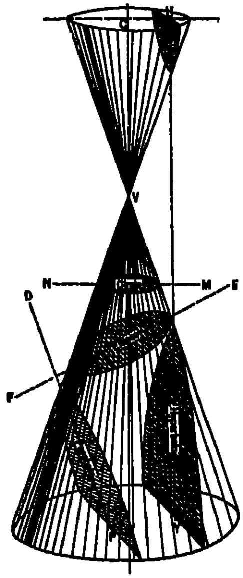


Fig. 171. *The Conic Sections.*

The properties of conics were worked out from the above point of view by Apollonius of Perga about 230 B.C. In modern analytic geometry the curves are treated by means of their equations in rectangular or polar coördinates. The equation of a conic is always a quadratic.

The ellipse is a closed curve, returning into itself as does the circle. The parabola does not return into itself, but extends to infinity. A body moving around the Sun on a parabola, as many comets appear to do, makes but one visit to the Sun in all eternity. The parabola may be regarded as an ellipse with its second focus infinitely removed from the first; the distance between its foci is therefore infinite and so is its major axis, and its eccentricity is exactly 1. The hyperbola consists of two infinite branches. The second focus is behind the first, and the eccentricity is greater than 1. In the case of an attractive force varying inversely as the square of the distance, the body will move on the branch

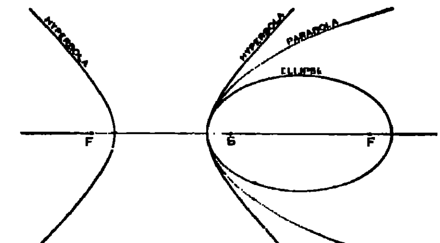


Fig. 172. *Confocal Conics.*

of the hyperbola that is concave to the center of the attraction; if the force were repulsive, it would move on the convex branch. The ellipse is the locus of points the *sum* of whose distances from the foci is constant; the hyperbola is the locus of points the *difference* of whose distances from the foci is constant. In Figure 172 are shown an ellipse, a parabola, and a hyperbola which have a common focus at $S$.

**Speed in the Relative Orbit.** In the solution of the two-body problem it is shown that the speed of one of the bodies in its orbit around the other is always the same at a given distance $r$, irrespective of the direction of one body from the other, and is given by the formula

$$V^2 = k^2(m_1 + m_2)\left(\frac{2}{r} - \frac{1}{a}\right)$$

where $a$ is the semimajor axis of the orbit and $k$ a constant. For a given pair of bodies, $m_1$ and $m_2$ are also constant, and we may write

$$V^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right).$$

If the orbit is circular, $a = r$ and $V^2 = \mu/r$; if it is parabolic, $a = \infty$ and $V^2 = 2\mu/r$. The velocity of a body in a parabola at a given distance from the Sun is therefore $\sqrt{2}$ times that of a body moving in a circle at the same distance. Meteors are observed to enter the Earth's atmosphere at an average speed of about twenty-six miles a second, which is $\sqrt{2}$ times eighteen miles per second, the orbital speed of the Earth. It is therefore inferred that these bodies travel on orbits that are either parabolas or very long ellipses.

If the velocity $\sqrt{2\,\mu/r}$, which is called the **parabolic velocity**, is represented by $U$, the equation for the velocity in an orbit of semimajor axis $a$ becomes

$$V^2 = U^2 - \frac{\mu}{a}$$

from which

$$a = \frac{\mu}{U^2 - V^2}.$$

Therefore, if a planet were launched in an orbit at a given distance $r$ from the Sun, the length of the major axis of the orbit would be greater the greater the speed of projection, for an increase in $V$ decreases the denominator and hence increases $a$. If $V = U$, the orbit is of course parabolic; if $V < U$, it is elliptic, and if $V > U$ it is hyperbolic. Figure 173 shows a number of orbits described by planets projected from the same

point in the same direction, but with different speeds. If projected in different directions with the same speed, the bodies would move in orbits of the same length but of different eccentricities and lines of apsides, as shown in Figure 174.

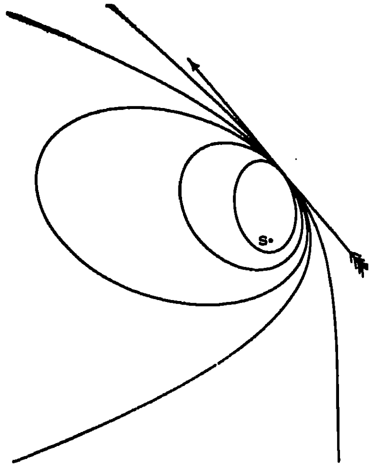



Fig. 173. *Orbits of Bodies Projected in the Same Direction with Different Speeds.*

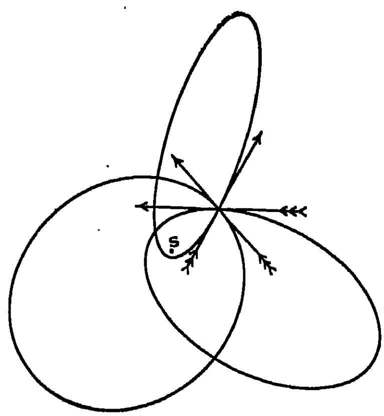



Fig. 174. *Orbits of Bodies Projected with the Same Speed in Different Directions.*

**Projectiles Near the Earth; Escape of Atmospheres.** When a ball is tossed into the air, it forms with the Earth a two-body system, and, if there were no friction, the ball would move along an ellipse having the center of the Earth as its distant focus and the highest point to which it is thrown as its apogee. Since the ball is stopped by the Earth's surface after describing only a short arc, the orbit is sensibly a parabola. If the speed of projection is increased, the major axis of the orbit is also increased, and the ball rises higher. The greatest speed yet attained by a projectile from a gun (the German "Big Bertha" of 1918) is less than a mile a second, and the greatest height of projection about twenty-four miles—very small in comparison with the dimensions of the Earth. If the speed were increased until it equaled the parabolic velocity due to the Earth's attraction, the projectile would move on a parabola and depart an infinite distance from the Earth, never to return. The parabolic velocity at the surface of a planet is therefore sometimes called the **velocity of escape.** It is equal to $\sqrt{2\,\mu/\rho}$, where $\rho$ is the radius of the planet and $\mu$ is to be computed from its mass. The velocities of escape for various bodies of the Solar System are given in Table 7.

Table 7 [1]

| Body | Velocity of Escape |
|---|---|
| Earth | 11,180 meters, or 6.95 miles, per sec. |
| Moon | 2,396 " " 1.49 " " " |
| Sun | 618,200 " " 384,30 " " " |
| Mercury | 3,196 " " 1.99 " " " |
| Venus | 10,475 " " 6.51 " " " |
| Mars | 5,180 " " 3.22 " " " |
| Jupiter | 61,120 " " 38.04 " " " |
| Saturn | 37,850 " " 23.53 " " " |
| Uranus | 23,160 " " 14.40 " " " |
| Neptune | 20,830 " " 12.95 " " " |

We are assured by physicists that any body of gas, such as the atmosphere of a planet, consists of molecules which are in rapid motion, each molecule moving in a straight line until it collides with another molecule or with the wall of the containing vessel. The mean velocity of the molecules depends on the temperature and pressure of the gas, but that of any individual may be indefinitely increased or lessened by collisions with others. A molecule near the upper limit of an atmosphere may thus attain a velocity exceeding the velocity of escape. The absence of an atmosphere on the Moon, and on other bodies for which the velocity of escape is small, may be explained on this basis.

**Rockets.** The foregoing discussion pertains to bodies, such as projectiles shot from guns, which are given an impetus and then are left to follow a trajectory under the action of gravitation and of no other force. It is otherwise with a rocket, a device that might be described as a gun in reverse. A rocket is in principle a tube containing a rapidly combustible fuel such as gasoline and oxygen, with a nozzle in the rear through which the products of combustion escape; it is the reaction of the rearward blast of these exhaust gases that propels the rocket forward. Thus the propulsive force, instead of ceasing at the beginning of the motion, continues as long as the fuel lasts. The problem of causing a rocket to escape from the Earth is one of stocking it with enough fuel.

**Determination of the Mass of a Heavenly Body.** In the mathematical treatment of the problem of two bodies, it is shown that

$$m_1 + m_2 = \frac{a^3}{P^2}$$

where the sum of the masses $m_1 + m_2$ is expressed in terms of the Sun's mass, the mean distance $a$ in astronomic units, and the period $P$ in years. This formula gives at once the combined masses of a planet and its

[1] From Moulton's *Celestial Mechanics*, p. 48.

satellite, or of the components of a double star, when the period and mean distance have been determined by observation. As the planets Mercury and Venus have no known satellites, their masses can be determined only by their attractions for other planets or for occasional comets that pass near them, and are not very accurately known.

The mass of the Sun is found in terms of that of the Earth-Moon system by a simple application of Kepler's third law as modified by Newton; for if $M$ is the sum of the masses of the Earth and Sun, $m$ of those of the Earth and Moon, $A$ and $a$ the semimajor axes of the orbits of the Earth and Moon, and $P$ and $p$ the lengths of the sidereal year and the sidereal month, then

$$M : m :: \frac{A^3}{P^2} : \frac{a^3}{p^2}.$$

**The Problem of Three Bodies.** The problem of the motion of three mutually attracting bodies is just as determinate as that of two—that is, a given set of initial conditions leads as certainly to a definite result; but the problem is of such difficulty and complexity that only in special cases is even the most powerful of modern analysis capable of producing formulae by which this result can be computed. The simplest two of these special cases were solved by Lagrange; in one, the three bodies remain always in a rotating straight line, and in the other they remain at the vertices of an equilateral triangle.

If an infinitesimal body were launched with the proper velocity at a point about a million miles from the Earth on the side directly opposite the Sun, and then not disturbed by any outside force, it would remain in that relative position and, like the Earth, revolve around the Sun in a period of one year. The motion, however, would be *unstable*, and if the body were disturbed ever so slightly—and it certainly would be disturbed by the attraction of the other planets and the Moon—it would depart from the position indefinitely. The dim light known as the Gegenschein (page 276) may, as suggested by Moulton, be due to sunlight reflected from tiny bodies which, moving through interplanetary space, are caught temporarily at the antisolar point by a sort of "dynamic whirlpool."

The equilateral triangle solution is *stable*, and examples are found in the motion of certain of the asteroids known as the Trojan group (page 275). Each of these asteroids revolves at the mean distance of Jupiter from the Sun and at the same time oscillates slowly about a point equally distant from the Sun and Jupiter.

**Perturbations.** The practical problem of calculating the positions of bodies in the Solar System is that of $n$ bodies and would be entirely unmanageable were it not for the fortunate circumstance that one of the

bodies, the Sun, so far exceeds the others in mass that, in a first approximation to the orbit of any one of the planets, the attraction of the others may be neglected. This first approximation, which, as we have seen, is an ellipse, may then be corrected by computing the effects of the attractions of the other planets at as many points of the orbit as desired. The deviations of the planets from exact elliptic motion are called **perturbations.**

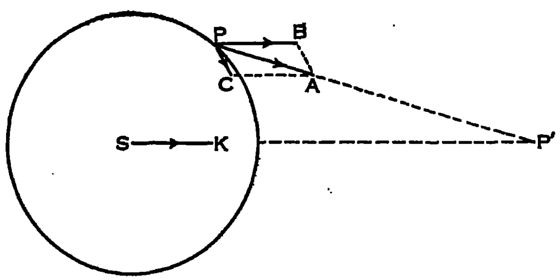



Fig. 175. *Perturbations.*

The perturbing effect of a third body upon the relative motion of two depends upon the *difference* of the acceleration produced in the two bodies by its attraction—difference in direction as well as in amount; for if the third body moved both the others in the same direction and at the same rate, it could produce no change in their *relative* position. To make this clear, let $S$ (Figure 175) represent the Sun and $P$ a planet revolving around it, and let a third body, say another planet, be at $P'$. $P'$ will attract both $S$ and $P$, producing in them accelerations inversely proportional to the squares of their distances. Let the lines $PA$ and $SK$ represent, in magnitude and direction, the acceleration produced by $P'$ in $P$ and $S$, respectively. Both will be directed toward $P$, and $PA$ will be the greater. The acceleration $PA$ is, by the second law of motion, equivalent to two accelerations that may be represented by the sides of a parallelogram of which $PA$ is the diagonal, and we are at liberty to choose the length and direction of one of these sides. Let it be $PB$, equal and parallel to $SK$. If $PB$ alone acted on $P$, it would change the relative position of $P$ and $S$ not at all, for the two bodies would then be pulled in the same direction and by the same amount; hence, the *perturbative* action of $P'$ upon $P$ is represented by $PC$, the other side of the parallelogram; and this component is what is meant by the *difference* of the accelerations produced by $P'$ in $P$ and $S$.

As may be inferred from the above discussion, it is not a difficult problem to compute the perturbations produced by a third body in a given

relative position of the three bodies; but as the bodies move in their orbits, the perturbations continually change in both direction and amount, and to determine accurately the path of $P$ they must be found for *every* position. What is worse, $P'$ is perturbed by the attraction of $P$ so that it also departs from an elliptic orbit, and these perturbations of $P'$ produce perturbations of the second order in $P$, and so on *ad infinitum*. Since, however, the perturbations of the planets are small because of the small masses and great distances of the perturbing bodies, their positions may, over long periods of time, be computed as accurately as they can be observed.

**The Regression of the Moon's Nodes: Precession.** The regression of the Moon's nodes (page 124) is a perturbation of the Moon's motion produced by the Sun. It has just been shown that the perturbative action of $P'$ on the motion of $P$ is directed toward a point of the line $SP'$, and it may be shown that this is the case whatever the position of $P$ on its orbit around $S$ (cf. page 253 and Figure 178). In the problem we are now considering, $S$ is the Earth, $P$ the Moon, and $P'$ the Sun, and the perturbative action of the Sun is always directed to the plane of the ecliptic. Consider the Moon's motion as it approaches the node, as in Figure 176 at $A$. If unperturbed, it would cross the plane of the ecliptic at $N$, which would thus be the ascending node; but the Sun's perturbative action causes it to move more nearly at right angles to the ecliptic and so to cross at $N'$, farther west, and the node is thus moved backward. After node passage, the action is again toward the ecliptic; the inclination is restored after undergoing a slight temporary disturbance, and the node is still moved backward.

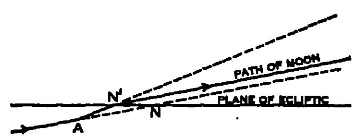


Fig. 176. *Regression of the Moon's Nodes.*

Precession may similarly be explained by the perturbative action of the Moon (and Sun) on the protuberant matter at the Earth's equator. If the Earth were a homogeneous sphere, there would be no precession. Suppose it were such a sphere, except for a single mountain at its equator. The perturbative action of the Moon upon this mountain would be directed toward the plane of the Moon's orbit, and as the Earth's rotation brought the mountain across this plane the crossing point (the equinox) would be moved backward as in the regression of the Moon's nodes. Instead of a single mountain, the spherical core of the Earth is surrounded by a ring of matter constituting the equatorial bulge, and every particle of this ring is affected in the way just described. Since the particles are fastened together to form the rigid Earth, this means a precessional motion such as was described in Chapter 5.

**Perturbations of the Earth.** The perturbations of a planet are classed as **periodic** and **secular.** The former set the planet forward or back or sidewise by slight amounts and run through their changes in a century or less. The secular perturbations are more conveniently regarded as changes in the orbit itself, and operate in one direction for thousands or millions of years, or indefinitely. The periodic perturbations of the Earth

never exceed about 1′ as seen from the Sun—about 30,000 miles. Its principal secular perturbations are as follows:

1. The line of apsides is revolving eastward at a rate that would, if continued, carry it entirely around in about 108,000 years; but it will not continue always at the same rate.

2. The eccentricity of the orbit, which is now 0.016, is diminishing and will continue to do so for some 24,000 years, when it will be about 0.003; then it will increase for some 40,000 years, but will never exceed about 0.07.

3. The plane of the orbit is slowly changing its position, resulting in a change of the obliquity of the ecliptic. The value of the obliquity is now 23° 27′, and is diminishing at the rate of 0″.5 a year. This decrease will continue about 15,000 years, after which the obliquity will increase. It oscillates in this manner about 1°.5 on either side of the mean.

**Perturbations of the Moon and Planets.** The perturbations of the Moon, produced principally by the powerful attraction of the Sun, are much greater than those of the Earth; and the **lunar theory**, or mathematical treatment of the Moon's motion, forms an important branch of astronomy in itself. The complexity of the subject is illustrated by an equation which alone covers no less than 250 quarto pages in Delaunay's *Théorie du Mouvement de la Lune.* This work, published in 1860–1867, is now superseded by the monumental *Tables of the Motion of the Moon,* published in 1919 by Brown of Yale and used by the national almanacs in the computation of the Moon's hourly ephemeris.

We have already noted two of the most important of these lunar perturbations, the regression of the nodes and the advance of the line of apsides. Another famous one is the **evection,** which was known in the time of Hipparchus and is so large that it may set the Moon forward or back about a degree and a quarter. This is a periodic perturbation with a period of 31.81 days. Other important periodic perturbations are the **variation,** the **annual equation,** and the **parallactic inequality.**

The diminution of the eccentricity of the Earth's orbit, noted above, results in a slight increase of the orbital speed of the Moon known as the **secular acceleration of the Moon's mean motion.** The theoretical amount of this perturbation, according to Brown, is about 6″ a century, but Fotheringham, from a comparison of ancient and modern eclipses, finds a value of about 10″ a century. The discrepancy is believed to be due not to a real change of the Moon's motion, but to a gradual slowing of our fundamental timepiece, the Earth, produced largely by the friction of the tides. There are still other minute discordances between theory and observation, which Brown attributes to an alternate but irregular expansion and contraction of the Earth, amounting to only a few feet and imperceptible by direct observation, but sufficient to produce a slight irregularity in the Earth's rotation.

The most famous of planetary perturbations is perhaps the **long inequality** of Jupiter and Saturn. Five of Jupiter's periods are nearly equal to two of Saturn's, so that these planets are repeatedly brought to their nearest approach in nearly the

ame part of their orbits and their mutual perturbations are repeated. Since, how-ver, the commensurability of their periods is not exact, the perturbation is periodic, aving a period of about 900 years.

**The Discovery of Neptune.** One of the most dramatic events in the istory of astronomy was the discovery of the planet Neptune. Uranus 'as discovered in 1781 and its orbit was computed from observations nade during the next few years; but it refused to follow the orbit com-uted for it, even when the perturbations of all the known planets were iken into account. By 1845 the difference between computation and bservation amounted to the "intolerable quantity" of nearly two minutes f arc—almost enough to perceive without a telescope—and it had by nat time long been suspected that Uranus was perturbed by an unknown lanet. Two young mathematicians, Adams in England and Leverrier in rance, undertook the difficult and laborious task of locating the dis-irber by solving the problem of perturbations inversely. Each assumed distance from the Sun of about 38 astronomic units, in accordance 'ith Bode's law; this proved to be incorrect, but the orbits calculated by iem were sufficiently in accordance with the true path of the new planet ) determine its direction from the Earth; this was all that was needed to nd it in the sky. Adams secured his result first, but was unable to waken much interest among English astronomers. Neptune was first :cognized in 1846 by Galle, a German astronomer to whom Leverrier ad communicated his result, and who found the planet within half an our, less than a degree from the point that Leverrier had indicated.

**The Discovery of Pluto.** The success of Adams and Leverrier naturally led to forts to extend the boundaries of the known Solar System still farther. Even after ie perturbations of Neptune were taken into account, the observations of Uranus d not agree perfectly with theory, although the discrepancies have never exceeded .′5, a very small quantity compared with the "intolerable" 2′ of 1845. The problem as attacked by a number of investigators, of whom the most determined and orough was Percival Lowell. He based his computations on the motion of Uranus r, in his opinion, Neptune itself could not be used as a guidepost to a farther planet ecause it had traversed less than half its orbit since its discovery and the available oservations did not go far enough back. Before 1905 he had made estimates of the obable position of the trans-Neptunian planet, and in that year he inaugurated a iotographic search by his assistants at the Lowell Observatory, the general plan which was to compare two plates covering an identical region of the sky and made veral days apart, and to note any object which had changed its apparent place nong the stars in the interval. In 1915, a year before his death, Lowell published e results of mathematical work on the subject which, with the help of a corps of omputers, he had been conducting for years. His investigation disclosed two

possible solutions which placed the planet in either of two opposite regions of the zodiac, with considerable uncertainty as to its exact location in either, and "indicated for the unknown a mass between Neptune's and the Earth's; a visibility of 12–13 magnitude; and a disk more than 1″ in diameter."

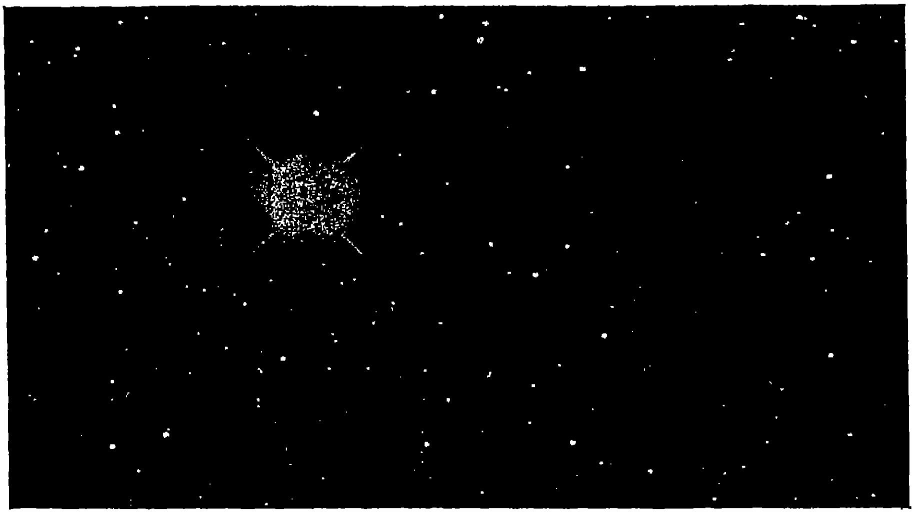


Fig. 177. *Apparent Path of Pluto near the Time of Discovery, Plotted by C. O. Lampland from Photographs Made by Him with the 42-Inch Reflector of the Lowell Observatory. The photograph shown was made March 4, 1930, and the image of Pluto is the white dot marked by two black dashes, just above the figure 4. The great white image is that of the star δ Geminorum. The four rays extending from it are caused by diffraction around the thin steel supports of the secondary mirror of the telescope. Scale of photograph: 1 mm. = 9″.*

It soon became evident that no distant planet as bright as this was in either of the regions indicated by Lowell's analysis, but the search was continued with improved instruments and methods. On January 21, 1930, C. W. Tombaugh, working at the Lowell Observatory under the direction of Slipher and Lampland, found on photographs taken by him in one of the predicted regions an object which in apparent path and rate of motion conformed approximately to Lowell's predictions of the trans-Neptunian body. After a few weeks' watching, the planetary character of the object and its location beyond the orbit of Neptune were clearly established. The new planet, which was soon named Pluto, is unexpectedly faint (fifteenth photographic magnitude) and shows no perceptible disk; its detection among the faint stars examined by Tombaugh, which are literally millions in number, is a triumph of skill and persistence.[2]

While there is no question of the honor due Lowell and his associates for the discovery of Pluto through systematic photographic search, the view that Lowell could be said to have discovered the planet *mathematically* has been disputed. For

[2] The photographs were made with a new 13-inch triplet lens and were studied with a *blink microscope*, a contrivance by means of which two plates, placed side by side, are viewed with magnification and in rapid alternation.

many years, W. H. Pickering had worked on the problem by methods less e than Lowell's but taking account of the discrepancies of Neptune's motion as those of Uranus', and had predicted a position not differing greatly from J In 1919, prompted by a paper of Pickering's, Humason photographed a region on three nights at Mount Wilson, but his examination of the cent of the plates, near the ecliptic, failed to reveal a new planet. After Tomba covery, Humason's plates were reëxamined by Nicholson, who found about 3°.5 from the ecliptic. The position derived from these photograp with the Lowell Observatory positions and others which were soon re old photographs taken for various purposes elsewhere, made possibl mination of the elements of the orbit, which are in fair agreement with thes by Lowell and by Pickering. The mass of Pluto, however, proves to be less than that of the Earth, and it is held by Brown and other authorities that the discrepancies of motion of Uranus and Neptune could not possibly be due to the attraction of so feeble a body. If this is true, the appearance of the planet so near the predicted positions must be regarded as an astonishing coincidence.

**The Theory of Relativity.** It is the great triumph of the law of gravitation that it explains not only the regularities of the motions of the planets, as described by Kepler's laws, but also their minute and irregular departures from elliptic motion, so that calculation is abundantly confirmed by observation. In only one case is an exception to this known, and that is the line of apsides of Mercury, which advances at the rate of 574″ per century, 43″ faster than it should according to the law of gravitation and the positions of known planets. On this account it was at one time proposed to amend the Newtonian law by substituting the exponent 2.00000016 for 2; but this would introduce perturbations in the other planets for which we have no observational evidence, notably in the motion of the line of apsides of Venus.

What many regard as the greatest advance in physical theory since Newton is the **theory of relativity**, developed principally by the physicist Einstein in papers published in Germany 1905 and 1915. Many unsuccessful attempts had been made to detect the Earth's motion relative to the luminiferous ether. For example, it had been reasonably expected that the velocity of light would appear to be slightly different when measured along and across the line of the Earth's orbital motion; but many experiments known to be capable of detecting the theoretical difference, notably a famous experiment first performed in 1887 by Michelson and Morley, had all failed. The only possible explanation seemed to be that the measuring apparatus became shorter when placed in the line of the Earth's motion than when placed at right angles to this position—an effect known as the Fitzgerald contraction. The principle of relativity goes much further

than this, and assumes that mass, length, and time are all relative, their value depending on the speed with which the observer is moving, but changing very little for velocities that are small compared with the velocity of light—which even planetary velocities are. Newton's law is expressed in terms of mass, distance, and time (time being included in the conception of force), and hence, in the light of the principle of relativity, it is ambiguous. Einstein derived a new law of gravitation (which cannot be stated in simple terms, but which, for the bodies of the Solar System, differs but minutely from the Newtonian law); and he showed that, if it is true, the orbit of a planet at Mercury's distance must revolve 43″ per century even if unperturbed.

Astonishing deductions are made from the theory of relativity. It appears that the universe is four-dimensional, its dimensions being length, breadth, thickness, and *time;* and this "space-time" is curved in a fifth dimension, the curvature being greatest in the neighborhood of bodies of greatest mass. Although the human mind cannot picture such a state of affairs, it can treat it adequately by mathematical methods, and the theory has, as far as it can be tested, received striking confirmation. Einstein showed that, according to the principle of relativity, a ray of light which passes near a body of the mass and dimensions of the Sun should be deviated by 1″.74. At the total eclipse that was visible in Africa and Brazil in 1919, an unusually fine opportunity existed for testing this prediction, for the Sun at the time was among the bright stars of the Hyades, so that their light had to pass near the Sun to reach the Earth. These stars were photographed during the eclipse by Eddington and Davidson, who later photographed the same star field with the same instruments when the Sun was in another part of the sky. A comparison of the plates revealed the fact that, during the eclipse, the stars were apparently shifted in a direction and by an amount to fulfill very nearly Einstein's prediction. This result was confirmed at the Australian eclipse of 1922 by Campbell.

The relativity theory is further confirmed by the fine structure of the lines in the spectra of the elements, which under very high dispersion are found to consist of groups of lines. This is explained by relativists by a revolution of the orbits of the electrons within the atom somewhat analogous to the motion of Mercury's line of apsides. Another predicted effect is a slight redward shift in the spectral lines of a massive, luminous body like the Sun. Many of the Fraunhofer lines certainly are thus shifted, but the Einstein effect is so complicated with effects of pressure, radial velocity, etc., that its detection is very difficult. (See also pages 208, 380, and 471.)

**Elementary Theory of the Tides.** The cause of the periodic rise and fall of the water of the ocean, known as the tides, may be regarded as a case of perturbations due to the action of the Moon and the Sun. The connection of the tides with the Moon is evident from the fact that the average interval between successive high waters is $12^h\ 25^m$, which is exactly half the average interval between successive upper culminations of the Moon.

Their connection with the Sun is shown by the occurrence of spring tides, the highest tides of the month, near the time when the Moon is in syzygy, that is, in the line joining the Earth and the Sun.

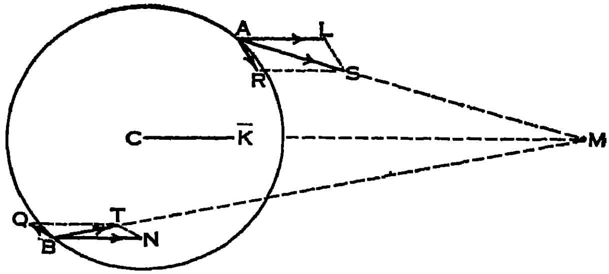



Fig. 178. *Tide-Raising Forces at Opposite Points.*

In Figure 178, let the center of the Earth be at *C* and that of the Moon at *M*, and consider the perturbative action of the Moon on drops of water situated at *A* and *B*. The tide problem can be considered in the same way as the problem of the perturbations of a planet which was treated in Figure 176. Since the lithosphere of the Earth is rigid,[3] it moves as a single body and the Moon's effect on it is the same as if its mass were concentrated at *C*; but each drop of the ocean being free to move independently, the drops at *A* and *B* may be moved relatively to the lithosphere and to the other drops. Let *CK* represent the acceleration produced by the Moon's attraction in the solid Earth, and let *AS* and *BT* be the accelerations produced in *A* and *B*. Since *A* is nearer the Moon than is *C*, while *B* is farther away, *AS* will be longer than *CK* and *BT* will be shorter. Resolve *AS* and *BT* as was done with the planet's attraction in Figure 176, taking in each case one side of the parallelogram equal and parallel to *CK*. The other side, *AR* or *BQ*, will represent the Moon's perturbing acceleration on the drop of water, or what is called the tide-raising acceleration. It is important to note that, since the diagonal of the lower parallelogram is shorter than its horizontal side, the other side is here directed toward the left; that is, the tide-raising force is directed away from the Moon. The tide-raising acceleration in different parts of the Earth's circumference is shown by the arrows in Figure 179, by which

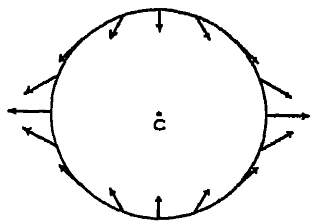



Fig. 179. *The Two Tidal Bulges.*

[3] It is not absolutely rigid, and the Moon creates in it minute "Earth-tides" which have been detected and measured by Michelson.

it is seen that the tidal forces of the Moon tend to heap the water up in two places on the Earth's surface situated at opposite ends of a diameter—one directly under the Moon and the other on the opposite side of the Earth. In fact, the tendency of the Moon's attraction is to deform the spherical Earth into a prolate spheroid with its longest axis directed toward the Moon.

When the Moon is in syzygy, its tidal bulges are added to those of the Sun, and we have spring tides; when it is at quadrature, the solar tides occur between the lunar ones, resulting in a smaller difference between high and low water (neap tide).

Suppose that the Earth consisted of a spherical, rigid lithosphere entirely covered by a frictionless fluid, each particle of which yielded immediately to the tide-raising force of the Moon. This covering would take the form of a prolate spheroid, the longest axis of which would pass always through the Moon's center, as suggested above. Let the lithosphere rotate every twenty-four hours about an axis, as in Figure 180. A point $O$ on its surface will be brought successively under the two tidal bulges and so will experience high water twice in each interval between upper culminations of the Moon—that is, once in every $12^h\ 25^m$. In general, the heights of the two daily tides will be unequal; but twice a month, when the Moon is in the plane of the equator, this **diurnal inequality** will vanish.

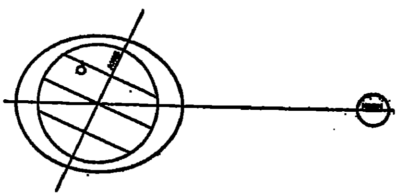



Fig. 180. *The Semi-Diurnal Tide.*

In the actual case, the water of the ocean is not frictionless, and so the tidal bulge is partly carried forward by the Earth's rotation. Moreover, when a protuberance is once formed it has a tendency to travel as a wave with a velocity which depends upon the depth, and the progress of the tidal wave around the Earth is vastly complicated by the varying depth of the sea, the presence of continents and islands, and the irregularity of coast lines. While in most localities the two daily tides and their diurnal inequality are recognizable, as are also the spring and neap tides, the actual prediction of the time and height of high water cannot be made from astronomical data alone, but depends also upon local observations of the tides.

**Effect of the Tides on the Rotation of the Earth and of the Moon.** The friction of the tides, slight though it is in comparison with the mo-

entum of the Earth, must act as a brake upon the Earth's rotation, and, ntinuing through millions of years, must inevitably tend to lengthen e day.

This tendency is reinforced by the slow deposit of meteoric matter upon the rth, and opposed by the shrinkage of the Earth due to the radiation of its internal t; but both these effects are very slight indeed. A comparison of the recorded ies of ancient eclipses with their times computed from modern observations of motions of the Earth and Moon indicate that the day actually is lengthening at ite of about one second in 100,000 years. It is believed that most of this change due to the friction of the tides in shallow seas such as Bering Sea.

Unless the Moon is perfectly rigid, the Earth must create tidal bulges on it, and the tide-raising effect of the Earth must be greater than that the Moon because of the Earth's greater mass. If in the past the Moon s in a liquid condition and rotated more rapidly than it revolved, the ction of the Earth's tides upon it must have tended to lengthen its rota-n period, and this tendency must have persisted until the Moon always sented the same face to the Earth as it does now. It is, in fact, believed t the Moon's slight departure from perfect sphericity, which results the minute physical libration (page 132), is due to the tidal protuber-ces having become solid and permanent.

**Effect of the Tides on the Revolution of the Moon.** The tides raised on the Earth must affect not only the rotation of the Earth but also the olution of the Moon. Consider Figure 181, in which the lithosphere of Earth is represented as a sphere ro-ing within a hydrosphere which com-tely covers it, its equator and also the ter of the Moon being in the plane the paper. The tidal friction of the on creates bulges in the hydrosphere ich are carried forward by the Earth's ation. As soon as their deepest points no longer in the line of centers of the Earth and Moon, a component the Moon's attraction will pull back upon them toward that line, as y be shown by the reasoning that was applied to Figures 176 and 178. is force will resist the tendency of the Earth's rotation to carry the ges forward, and they will therefore be carried only to certain points nd *B*, where they will be in equilibrium between the frictional forward l of the Earth and the gravitational backward pull of the Moon; they

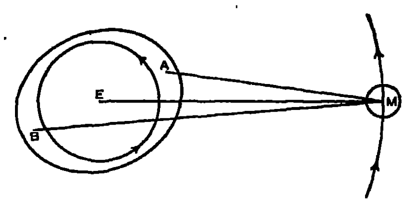


Fig. 181. *Effect of the Tides on the Revolution of the Moon.*

will remain just so far in advance of the Moon while the body of the Earth turns under them; and, as pointed out in the last section, their friction acts as a brake on the Earth's rotation. (This does not mean that the separate particles of water would remain stationary with respect to the Moon; each drop would be carried along by the Earth's rotation but would rise gently to the crest of a wave at $A$ and at $B$ and would sink to a trough at the points midway between.)

The protuberance whose summit is at $A$ will attract the Moon approximately along the line $MA$ and more strongly, mass for mass, than does the lithosphere which is centered at $E$; whereas the other bulge will attract the Moon along $MB$ more feebly than does the lithosphere. But the attraction at $A$ pulls forward on the Moon and makes a greater angle with a line of centers than does the weaker attraction of $B$, which pulls backward; hence, the net effect is to hasten the Moon in its eastward orbital motion. Paradoxical as it may seem, this does not shorten the month but lengthens it; for it increases the centrifugal force due to the Moon's orbital motion, causing the Moon to move farther from the Earth and actually to move more slowly in a larger orbit.

**Evolution of the Earth-Moon System.** At present, the change in the Moon's revolution, like that in the Earth's rotation, must proceed with extreme slowness; but in the remote past, when the Moon was nearer the Earth, the tidal action must have been much greater than now, since the tide-raising force varies inversely as the cube of the distance; and the tides must have been especially potent when both bodies were plastic throughout, as they are believed once to have been. Upon this line of thought is based the famous theory of **tidal evolution**, which was first elaborated by the English mathematician Sir George Darwin late in the nineteenth century.

According to this theory, the Earth and Moon evolved from a single liquid body which rotated on its axis in a little less than five hours. Owing to its rapid rotation, this body was very oblate, and the motion of its particles was continually disturbed by tides raised upon it by the Sun, which of course had half this period. Darwin showed that, granted certain reasonable assumptions, the free period of vibration of this body must have been the same as that of the tides, in which case a vibration of very large amplitude must have been set up; according to the theory, this vibration caused a portion of the body to be separated from the remainder. Probably the smaller portion was not originally all in one piece, but for

some unknown reason it became consolidated into a single body; we know this body as the Moon, and the remainder of the original spheroid is the Earth. The two bodies then had the form of prolate spheroids with their long axes in the same line, and this line was rotating in a period of five hours. But this condition was unstable and the disturbance of the solar tide caused the Moon to fall back a little, whereupon the friction of their enormous mutual tides came into play, and their periods, both of rotation and of revolution, were lengthened, the latter faster. This process, at first rapid, has continued with diminishing speed until the present, when the Earth's rotation period has reached twenty-four hours, while both the rotation and revolution of the Moon are twenty-seven times as slow.

## EXERCISES

1. Two balls, of masses 10 kilograms and 100 kilograms respectively, are placed with their centers 1 meter apart. With what force do they attract each other?

*Ans.* 0.006673 dyne

2. The balls are moved so that their centers are only 50 centimeters apart. What is now their mutual attraction?

*Ans.* 0.026692 dyne

3. What is the weight, in dynes, of the smaller ball?

*Ans.* 9,810,000 dynes

4. Calculate the superficial gravity on the planet Jupiter, whose mass is 318 and radius 11 times that of the Earth.

*Ans.* 2.6

5. Prove that the values of superficial gravity on spherical planets of equal density are proportional to the radii of the planets.

6. If a body weighs 100 pounds at the surface of the Earth, what would be its weight at the surface of an asteroid having the Earth's mean density and a radius $\frac{1}{20}$ that of the Earth?

*Ans.* 5 pounds

7. A lamp with a cylindrical shade, open at top and bottom, stands near a wall. What is the form of the edges of the shadow of the lampshade on the wall?

8. What is the form of the path of the shadow of the top of a pillar cast upon flat, level ground by the Sun on June 21? On March 21? On December 21?

# CHAPTER 12

## THE PLANETS

**Mercury and Venus.** The principal numerical facts concerning the two planets whose orbits lie within that of the Earth are given in Table 8:

Table 8

| | Mercury | Venus |
|---|---|---|
| Mean distance from Sun | 0.39 astron. unit | 0.72 astron. unit |
| Intensity of solar radiation received | 6.7 | 1.9 |
| Orbital velocity | 23 to 36 mi./sec. | 22 mi./sec. |
| Eccentricity of orbit | 0.20 | 0.007 |
| Inclination of orbit | 7°.0 | 3°.4 |
| Sidereal period | 88 days | 225 days |
| Mean synodic period | 116 days | 584 days |
| Diameter | 5″ to 13″ = 3000 miles | 11″ to 67″ = 7575 miles |
| Greatest elongation | 18° to 28° | 47° |
| Distance from Earth in miles | 57,000,000 to 129,000,000 | 26,000,000 to 160,000,000 |
| Mass (Earth's mass = 1) | 0.05 | 0.82 |
| Density (water = 1) | 4.5 | 4.8 |
| Surface gravity | 0.31 *g* | 0.85 *g* |
| Albedo | 0.06 | 0.60 |

Mercury, of all the planets, is nearest the Sun, and so it has the swiftest motion and receives the greatest intensity of radiation. With the exception of Pluto and some of the asteroids, it has the most highly inclined and most eccentric orbit and the least diameter and mass. Venus has the most nearly circular orbit and a diameter most like that of the Earth (about 400 miles smaller).

At its brightest, Mercury appears to the unaided eye as a star nearly as bright as Sirius, but so near the Sun that (as seen from northern latitudes) it is never in the unilluminated part of the sky and hence is never conspicuous.

The preceding sentence, which in the first edition of this book was printed without the parenthesis, brought forth protest from Australia, where Mercury at times

really is visible in the unilluminated part of the sky. The advantage possessed in this respect by southern observers is due to the form and situation of Mercury's orbit. The *greatest* "greatest elongation" (28°) occurs when the planet is at aphelion, and a study of Figure 161 will show that such eastern elongations (evening star) occur in late summer or early autumn, and such western elongations (morning star) in early spring. In either case the Sun is near the equator while Mercury is south of both equator and ecliptic, and so the interval between its setting or rising and the setting or rising of the Sun is longer for an observer in the southern hemisphere than for one in the northern.

Venus is much brighter than Mercury and can be easily seen at midday if the sky is very clear; besides, she attains so great an elongation that she is at times visible, from points in the United States, more than three hours after sunset or before sunrise. Of all the planets (a few of the asteroids excepted), Venus approaches nearest to the Earth, and with the exception of the Sun and Moon and very rare comets she appears the brightest of all the heavenly bodies.

Telescopic observation of these planets is best carried on in the daytime because at night, when visible at all, they are at too low an altitude for good seeing. To the eye they reveal little surface detail; each presents a nearly uniformly illuminated disk, and the few markings that have been reported are vague and exceedingly difficult of detection. The surface of Venus is intensely white, contrasting beautifully with the blue of the sky. That of Mercury is much duller.

Ordinary photographs show as little as do visual observations; but photographs of Venus in ultra-violet light, made by Ross with the Mount Wilson reflectors in 1927, show bright clouds near the cusps and, in other parts of the disk, dark belts which change from day to day and which suggest those of Jupiter (page 276).

It is probable that the visible surface of Venus consists of a dense layer of white clouds which prevent our seeing the solid surface and which also, probably, prevent direct sunlight from reaching that surface. It is not impossible to imagine that Venus is inhabited by beings who, because of this impenetrable mantle of clouds, are ignorant of the existence of the Sun and of all other bodies exterior to their own planet.

The most striking feature of the telescopic appearance of either Venus or Mercury is its change of phase and of apparent diameter (Figure 182) as the planet changes position with respect to the Earth and the Sun. The change of apparent diameter is especially noticeable in Venus, her maximum diameter being about six times the minimum. Her greatest brilliancy

occurs at thirty-six days before and after inferior conjunction, when about one-fourth of her earthward hemisphere is illuminated.

No evidence of an atmosphere on Mercury has ever been found, and there is no doubt that this planet, like the Moon, is without air.

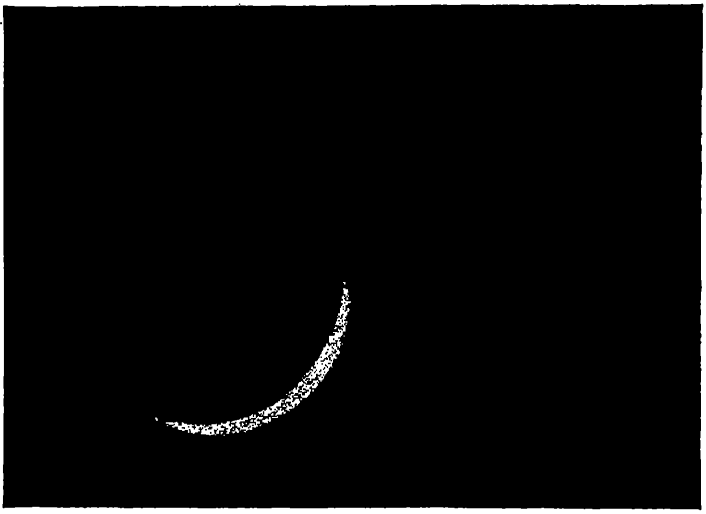


Fig. 182. *The Planets Mercury and Venus, Photographed with the 24-Inch Refractor of the Lowell Observatory by E. C. Slipher. Top, Mercury at greatest elongation; left, Venus at inferior conjunction (by red light); right, Venus at greatest elongation (by ultra-violet light).*

**The Atmosphere of Venus.** That Venus has a fairly dense atmosphere is certain, for it is plainly visible when the planet is near inferior conjunction, between the Earth and the Sun. The illuminated disk then appears as a very thin crescent which is extended beyond the ends of a diameter by thin bright arcs. When the apparent distance of the planet from the Sun is about two degrees or less, these arcs are prolonged until they meet and form a complete ring of light (Figure 182), an appearance due to sunlight illuminating the planet's atmosphere from behind.

The light of any planet, being reflected sunlight, must have passed twice through that part of its atmosphere which lies above the reflecting surface, and so the constituents of the atmosphere may reveal themselves through absorption bands in the planet's spectrum. In the Earth's atmosphere, such bands are produced, mainly in the red and infra-red, by oxygen and water vapor, and in the far infra-red by carbon dioxide. The detection of these gases in a planet's atmosphere is rendered difficult by the fact that the planet's light must pass through our air, so that the planetary bands are apt to be masked by the telluric. This difficulty may be partly circum-

vented in two ways: (1) By comparing spectrograms of the planet and the Moon, taken at equal altitudes so that their light passes through equal depths of the Earth's atmosphere; since the Moon is airless, the planetary constituents should produce a relative strengthening of the bands. (2) By taking advantage of the Doppler-Fizeau effect, which, when the radial velocity of the planet is considerable, displaces the planetary bands and may perceptibly separate them from the telluric bands or at least produce a noticeable widening.

The spectrum of Venus has been carefully studied, especially at the Lowell and Mount Wilson observatories. In the ordinary photographic and visible region it appears identical with the solar spectrum, giving no evidence whatever of any effect of Venus's atmosphere; but in the infra-red, as Adams and Dunham discovered in 1932, numerous heavy bands of carbon dioxide appear, indicating the presence of an astonishing quantity of this gas in the planet's stratosphere. No trace of oxygen or of water vapor has been detected, and if the planet has a supply of these gases it must lie below the cloud layer which limits the penetration of the spectroscope. The composition of the clouds of Venus is unknown.

If the composition of Venus's atmosphere is the same below the cloud layer as it appears to be above, no life such as ours can exist there. Terrestrial plants consume carbon dioxide and liberate oxygen, and animals perform the inverse process. It has been suggested that the present abundance of oxygen in our own atmosphere is due principally to our vegetation. Total absence of oxygen in a planet's atmosphere would imply non-existence both of plants, which cannot live without producing it, and of animals, which cannot live without consuming it. It is of course conceivable, however, that conditions below the cloud layer of Venus are very different from those in the region accessible to observation.

**The Temperatures of Mercury and Venus.** The surface temperatures of the planets have been computed by means of the laws of radiation (page 169) from measures of their radiation made at the Lowell and Mount Wilson observatories with exceedingly delicate thermocouples (page 341).

Though all the visible light of a planet is reflected sunlight, much of its invisible, long-wave radiation is due to its own heat. The proportion of reflected to directly emitted radiation can be determined by transmitting the light through screens which are opaque to certain parts of the spectrum—for example, a cell containing pure water, which is transparent to visible light but opaque to the invisible, long-wave planetary radiation.

From these observations it appears that the temperature of the sunlit side of Mercury is about 350° C.—above the melting point of lead—while that of the dark side is too low to be measured and is probably not much

above the absolute zero (−273° C.). The condition of this airless planet is thus similar to that of the Moon (page 141), except that the temperature of its sunward side is much higher because of its proximity to the Sun.

The temperature of the dark side of Venus is found to be about −25° C., and that of the bright side seems to be only a few degrees higher. Here the measures refer to the upper atmosphere, or upper surface of the bright envelope of cloud. A similar condition is known to exist in the stratosphere, or upper atmosphere of the Earth (page 66), which has an apparently uniform temperature of about −55° C.

**Evidence of the Rotation of Venus and Mercury.** Until 1880 the impression prevailed generally among astronomers that both Venus and Mercury rotated in about twenty-four hours. This impression was based upon the apparent motion of markings which various observers had detected or thought they had detected, mostly with very small telescopes in unfavorable climates and when the planets were at low altitudes. In 1880 Schiaparelli, observing in the daytime under favorable conditions in Italy, found for each of these planets a rotation period identical with its revolution period, indicating that, except for the effect of librations (page 131), it always keeps the same face toward the Sun just as the Moon keeps the same face toward the Earth. Schiaparelli's results were confirmed by a number of observers, especially Lowell, but are disputed by others. The markings on both planets are too indefinite and those of Venus too changeable to permit a reliable value of the period to be obtained by direct observation.

When the image of a rapidly rotating planet is formed so that its equator falls on the slit of a spectrograph, one end of the slit is illuminated by that part of the planet which is approaching the Earth, and the other end by the part which is receding. Therefore, according to the Doppler-Fizeau principle, one end of each line of the spectrum is displaced toward the violet, and the other end toward the red, so that the lines are inclined. This effect is very marked in the case of Jupiter (Figure 190, page 278) and is noticeable in the case of Mars, whose period is slightly more than twenty-four hours. Careful observations of the spectrum of Venus by V. M. Slipher at the Lowell Observatory, and by St. John and Nicholson at Mount Wilson, disclose no inclination of the lines whatever, and make it certain that the rotation period is at least several days. So long a period as Schiaparelli's 225 days could not be detected by this method. Spectrographic evidence on the rotation of Mercury has not been obtained, and the small

disk of this planet and its proximity to the Sun would make the securing of such evidence very difficult.

The fact that the dark and illuminated sides of Venus are nearly at the same temperature is regarded by many as evidence that the rotation and revolution periods are not identical, for if one part of the planet's surface were always in darkness and another part always shone upon, the former should be much colder. Others think that the uniformity of the planet's temperature may be adequately accounted for by convection currents in its atmosphere producing an interchange of warm air from the sunlit side and cold air from the dark. Ross regards the changing parallel bands which he has photographed as an indication of a rather rapid rotation, and, reviewing all the evidence, suggests a compromise of about thirty days as the best estimate of the rotation period of Venus.

In the case of Mercury, the best evidence seems to point to identity of rotation and revolution periods.

**Transits of Venus and Mercury.** On rare occasions, one of these planets passes directly between the Earth and the Sun and appears as a black dot upon the photosphere. Such an occurrence is called a **transit.** The conditions necessary for a transit are analogous to those for an eclipse of the Sun: the planet must be at inferior conjunction (corresponding to new Moon) and must be near the node; that is, both the planet and the Earth must be nearly on the planet's line of nodes and on the same side of the Sun.

Transits of Mercury can occur only in May and November, when the Earth crosses Mercury's line of nodes; those of Venus occur only in June and December. The relations of the year and the synodic period of Mercury are such that transits at a given node occur usually at intervals of thirteen or sometimes of seven years, with transits at the other node between. The November transits are much more frequent than the May transits because Mercury's perihelion is nearly in the direction of the November node and its orbit is so eccentric that its perihelion distance is only about two-thirds its aphelion distance.

Transits of Venus at a given node occur either in pairs eight years apart or singly; the interval between single transits or the midway dates of pairs is 243 years, and transits occur at the other node about halfway between.

The first authentically observed transit was one of Venus, which occurred in 1639. It was predicted by Jeremiah Horrocks, a young English clergyman, and as it occurred on a Sunday his ministerial duties prevented him from seeing the beginning

of the transit; but his friend Crabtree, whom he had informed of the impending phenomenon, witnessed the entrance of the planet on the solar disk, and both young amateurs saw it before the end. It was seen by no other observers. Four later transits of Venus have been observed, the last one in 1882; but the next will not occur until the year 2004.

The dates of transits of Mercury during the second half of the twentieth century are, according to Newcomb:

| | | | |
|---|---|---|---|
| 1953 | Nov. 14 | 1973 | Nov. 9 |
| 1957 | May 5 | 1986 | Nov. 12 |
| 1960 | Nov. 7 | 1993 | Nov. 5 |
| 1970 | May 8 | 1999 | Nov. 15 |

From observations of the times of beginning and ending of a transit of Venus, made at two widely separated stations on the Earth, it is possible to determine the parallax of the Sun, and the transits of the nineteenth century were very thoroughly observed for this purpose; but other methods of determining the solar parallax have proved more accurate. Transits of either planet are of value for determining the planet's position on its orbit at a known time and thus for improving the accuracy of the orbital elements.

**Facts Concerning Mars.** Mars is notable among the planets and stars for its red color. It is inconspicuous during the greater part of its synodic

Table 9

FACTS CONCERNING MARS

| | |
|---|---|
| Mean distance from Sun | 1.52 astronomical units |
| Intensity of solar radiation received | 0.43 |
| Orbital velocity | 15 miles per second |
| Eccentricity of orbit | 0.09 |
| Inclination of orbit to ecliptic | 1°.85 |
| Sidereal period | 687 days |
| Mean synodic period | 780 days |
| Distance from Earth in miles | 35,000,000 to 247,000,000 |
| Mean albedo | 0.15 |
| Mass, Earth = 1 | 0.106 |
| Diameter | 4215 miles = 3″.6 to 24″.5 |
| Density, water = 1 | 3.92 |
| Surface gravity | 0.38 g |
| Rotation period | $24^h\ 37^m\ 22^s.58$ |
| Inclination of plane of equator to plane of orbit | 23°.5 |

period and is invisible for several months near conjunction, when it is beyond the Sun; but at opposition, when it is relatively near the Earth and is above the horizon all night, rising at sunset and reaching its greatest altitude at midnight, it is very bright and conspicuous. The relation of its orbit to that of the Earth is shown in Figure 183. Owing to its rather high eccentricity, its distance from the Sun varies by about 28,000,000 miles from perihelion to aphelion. The longitude of its perihelion is 333°, which

is the heliocentric longitude of the Earth about August 25; hence, when opposition occurs near this date the two planets reach their least possible distance apart, about 35,000,000 miles. At the next opposition, which occurs in October or early November two and one-seventh years later, the distance is about 43,000,000 miles. The greatest distance at opposition (with Mars at aphelion, in February) is about 63,000,000 miles. Close oppositions occur every fifteen or seventeen years (1924, 1939, 1954, 1971). At its closest, Mars appears more than four times as bright as at its least favorable opposition and more than fifty times as bright as when near conjunction.

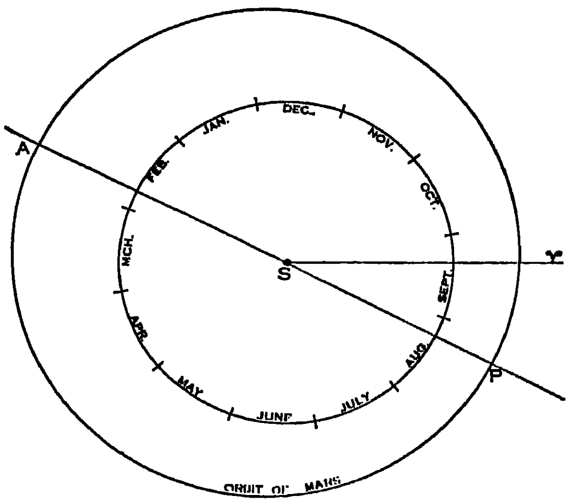



Fig. 183. *The Orbits of Mars and the Earth.*

**Rotation and Seasons of Mars.** The first known telescopic drawing of Mars, which was made by Huyghens in 1659, shows the dark marking which is now called the **Syrtis Major** (see Figure 184). It soon became evident that this and many other markings were of a permanent nature and that they were being carried around by the planet's axial rotation in the direction of the rotation of the Earth and of the Sun. In 1666 Cassini determined the rotation period as forty minutes longer than that of the Earth. The long interval over which observations of the markings have now been made, and especially the definite and permanent character of the markings themselves, have made it possible to determine Mars's rotation period more exactly than that of any other planet. The period given in Table 9 is that determined by Lowell.

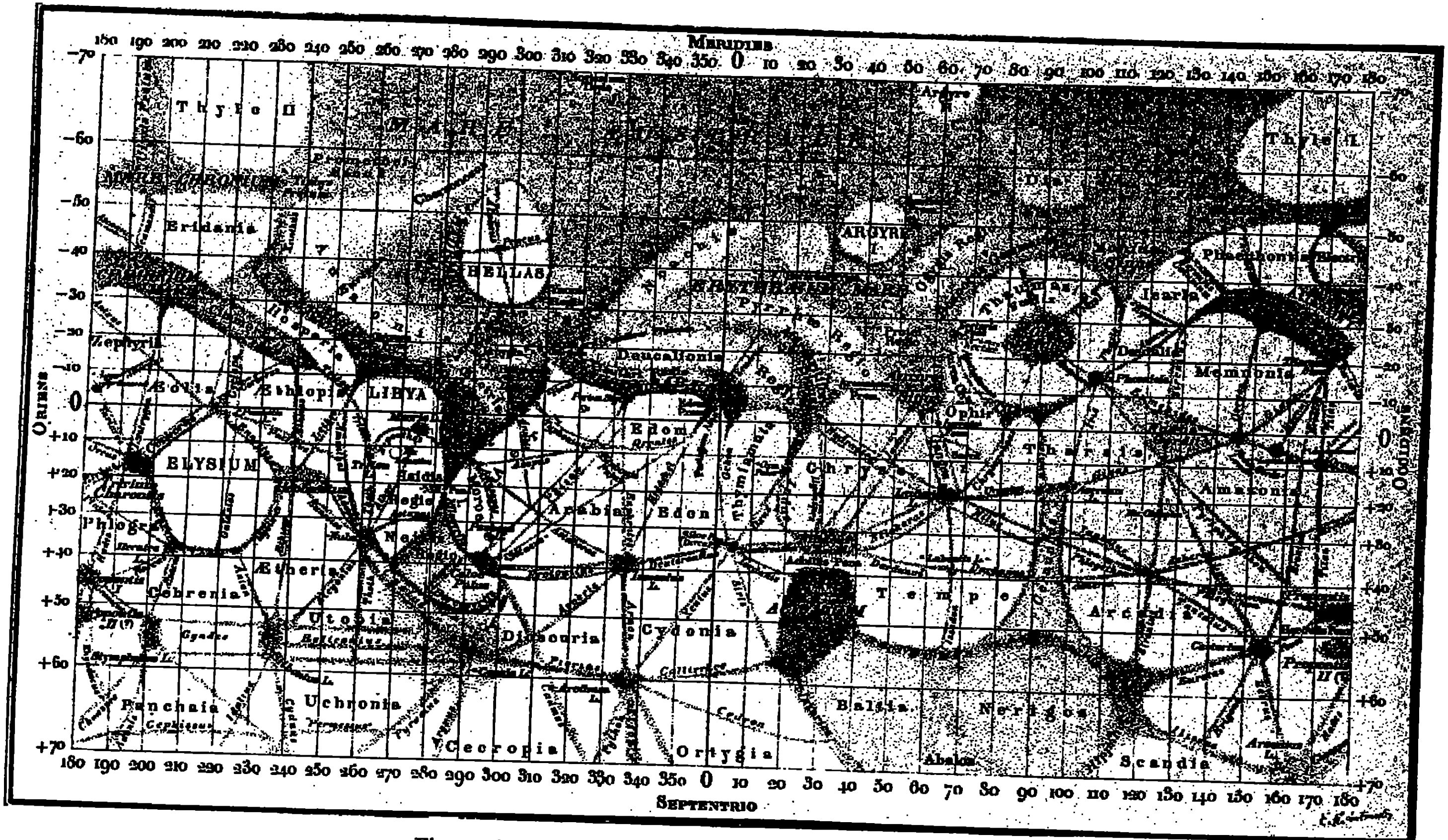



Fig. 184. *Map of Mars, by Flammarion and Antoniadi.*

The north pole of rotation of Mars is in $\alpha = 21^h10^m$, $\delta = +54°5$ to the bright star Deneb ($\alpha$ Cygni) which is therefore the Martian Polar ... The inclination of the planes of Mars's equator and orbit is almost exactly the same as the obliquity of the ecliptic, which is the corresponding angle in the case of the Earth; and hence the Martian climatic zones are similar to those of the Earth, but the Martian seasons are nearly twice as long as the terrestrial. Moreover, the greater eccentricity of Mars's orbit results in a greater difference between the seasons of the two hemispheres; those of the southern hemisphere are more extreme since Mars passes perihelion in the southern summer.

Fig. 185. *The Planet Mars in 1939, Photographed with the 27-Inch Refractor of the Lamont-Hussey Observatory, South Africa, by E. C. Slipher. Left, region of Syrtis Major; right, Sabaeus Sinus, Margaritifer Sinus, Solis Lacus, Fons Juventae; South Polar Cap at top.*

**Telescopic Appearance of Mars.** Since its orbit lies outside that of the Earth, Mars cannot present to us the crescent phase, but at quadrature it is distinctly gibbous and at conjunction and opposition it is full. Because it is full at opposition, when nearest the Earth, it is a more favorable object for study than Venus, whose dark side is turned toward us at inferior conjunction, the occasion of her nearest approach. Although Mars certainly possesses an atmosphere, it is probably of low density as compared with our air, and it is never filled with clouds; and so, when the planet is favorably placed, its surface is plainly visible and is seen to be covered with fine detail (Figure 185).

About three-fifths of the surface is of a reddish-ocher color and upon this are darker regions of permanent form which, seen under the best

conditions of instrument and atmosphere, appear dark green. Around the poles are areas of pure white which are known as the **polar caps.** Usually only one is visible, the other pole being turned away from the Earth. The reddish areas were called "continents" by the early observers of Mars, and the dark areas "seas" or "lakes"; but it is now certain that the "seas" are not water and that there is in fact only little water on the planet.

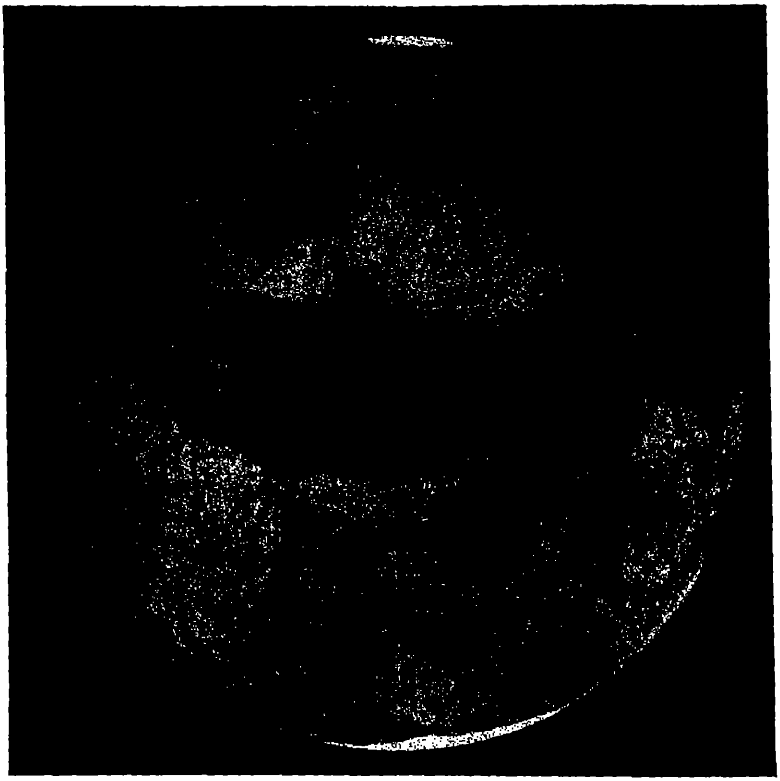


Fig. 186. *Martian Globe, Showing Features Observed Visually with 24-Inch Telescope. (By Percival Lowell.)*

In 1877 Schiaparelli of Milan discovered in the "continents" a network of exceedingly fine lines which he called *canali.* This word was translated "canals," and as the **Canals of Mars** they have ever since been known. Their reality was at first universally doubted, as they were so difficult that they were not seen by other observers. In 1888, however, they were detected by Perrotin at Nice and later by many others, although a few good observers, like Barnard and Antoniadi, have been able to see them only as broad, indefinite shadings or as interrupted lines. In 1892

W. H. Pickering at Arequipa, and in 1894 Lowell and Douglass at Flagstaff, found that the canals were present in the "seas" as well as the continents, thus showing that the former are not bodies of water.

The most complete study of Mars and its canals is probably that carried on at the Lowell Observatory, which was founded in 1894 by Percival Lowell of Boston for the express purpose of studying the planets and their satellites, particularly Mars. It is situated at Flagstaff, Arizona, in an exceptionally fine climate for astronomical observing, and at an altitude of 7000 feet. Lowell observed Mars assiduously at every opposition from the time of the founding of his observatory until his death in 1916, and his work has been supplemented by that of other members of his staff, past and present, particularly Douglass, W. H. Pickering, C. O. Lampland, and E. C. Slipher.

Although, if perfect seeing could be secured, the power of a telescope for showing fine detail on a planet would be proportional to its aperture, such seeing is never actually known; and it is found that, in such conditions as really exist, a moderate-sized telescope is often better for the purpose than a large one, since the larger light beam must traverse a greater area of disturbance in the terrestrial atmosphere. It is the practice of most observers who use large telescopes for observing the planets to stop down the apertures except when the seeing is at its best.

The nomenclature most commonly used for the features of Mars is that founded by Schiaparelli, in which the names are taken from ancient or mythological geography. The surface of the planet is laid off by meridians and parallels, the origin of longitudes being the **Fastigium Aryn** between the branches of the forked **Sabaeus Sinus.** The most prominent markings are perhaps the **Solis Lacus,** the dark "eye" of Mars in longitude 90°, latitude 30° south; and the **Syrtis Major** and **Margaritifer Sinus,** dark horn-shaped markings which extend from the southern hemisphere to middle latitudes in the northern. The most easily detected canal is usually the **Nilosyrtis,** which forms a sort of tail to the Syrtis Major. Maps of Mars are ordinarily printed with south at the top, because the planet is most frequently observed with inverting telescopes (Figures 184, 186).

**Changes in the Features of Mars.** Each of the white polar caps changes markedly, increasing in size when turned away from the Sun and shrinking as it is turned sunward—a behavior first noted by Sir William Herschel, who attributed the caps to snow that formed in winter and melted in summer. The southern cap, which is turned sunward when the planet is near perihelion, disappears completely, but the northern one has not been observed to do so (Figure 187).

Careful, systematic observation discloses seasonal change also in other parts of the planet. From the great quantity of observations that have been made during the last half-century it has become apparent that, in general, the change is as follows: Just after the Martian solstice, when the sunward polar cap has grown small, the hue of the dark areas in its vicinity begins

to deepen. The deepening spreads slowly toward the equator, pervading both "seas" and canals, and finally crosses the equator and may be detected in the low latitudes of the opposite hemisphere. Then, as the planet moves around its orbit and approaches the other solstice, the dark areas of the hemisphere which is turning away from the Sun begin to fade, first near the pole and later in lower latitudes. These changes are suggestive of the vernal quickening and autumnal fading of vegetation.

Fig. 187. *Seasonal Changes in South. Polar Cap, Sabaeus Sinus, and Mare Erythraeum. Photographs by E. C. Slipher, Lowell Observatory.*

On occasions at almost every opposition, bright spots appear at various points of the planet's surface, move across it with speeds large enough to be detected in a few hours' observation, and disappear after a few days. Some of these are white and resemble clouds, while others are yellowish and may be dust storms. Sometimes when Mars is in the gibbous phase one of these clouds is seen in projection beyond the terminator, like a high peak on the Moon (Figure 107, page 136), showing that it lies high above the general surface.

Frequently, large white areas appear at the eastern or sunrise limb, dwindle rapidly, and disappear before reaching the middle, or noon point,

of the disk. Their behavior is suggestive of hoarfrost which, having formed during the Martian night, melts when carried by the planet's rotation into the morning sunlight.

**Climatic Conditions on Mars.** The existence of an atmosphere on Mars is proved beyond a doubt by the occasional presence of floating clouds and the melting and reforming of the polar caps. That it is much rarer than the Earth's air is almost equally certain because of the low surface gravity of Mars and the clearness with which the solid surface is seen. The appearance and behavior of the polar caps suggest strongly that they are frozen water, but the absence of oceans indicates that the total quantity of water on the planet is small.

The bands produced by the Martian atmosphere in the part of the spectrum which has been observed are very feeble, and investigators are not in agreement as to their significance or even their existence. Spectrograms of Mars and the Moon, secured by V. M. Slipher at Flagstaff on exceptionally favorable nights in 1908, show the "a" band distinctly stronger in the spectrum of Mars, indicating the presence of water; and Very, who measured Slipher's spectrograms photometrically, estimated the oxygen content of Mars's atmosphere to be about half that of the Earth's. Mount Wilson observations made in 1925 seemed also to indicate water and oxygen, though in much smaller quantities; but in 1933 Adams, using a Mount Wilson spectrograph powerful enough to separate distinctly by the Doppler effect any Martian bands from their telluric counterparts, concluded that the oxygen on Mars, if there is any, does not exceed in density a tenth of one per cent of that on the Earth at sea level, and that the amount of Martian water vapor too must be very small. Russell suggests that the red color of Mars is due to iron oxide, and that in the process of its formation the oxygen of the planet's atmosphere has been depleted.

As the intensity of solar radiation at the distance of Mars is only four-ninths that received by the Earth, it is to be expected that the temperature of the planet would be low, and this has been considered a serious drawback to the belief that the polar caps and other white areas which melt are frozen water. However, ice and snow evaporate at temperatures far below zero when the air above them is very dry, as the Martian atmosphere appears to be.

Direct evidence in regard to the temperature of the planet's surface was obtained in 1924 when measures of its radiation were made with the thermocouple (page 341) at Flagstaff by Lampland and Coblentz, and at Mount Wilson by Pettit and Nicholson. At both observatories it was found that the temperature rose several degrees above the Centigrade zero near the center of the illuminated disk, but was usually low near the limb. At Flagstaff it was found that the temperature of the south polar cap was

$-100°$ C. about two months before the southern summer solstice, and that it gradually increased to about $0°$ C. a few days after the solstice; and that the temperature of the east limb (where the surface had just emerged from darkness into morning sunlight) was as low as $-85°$ C., giving evidence of an enormous diurnal fluctuation. It appeared also that the bright areas were at a lower temperature than the dark areas.

**Mars as the Abode of Life.** Life as we know it on the Earth, both animal and vegetable, depends on a number of special conditions, among which are a favorable temperature and a supply of water and oxygen. Oxygen and water appear to be very scarce on Mars, and the temperature far from salubrious; yet the seasonal changes of the dark areas are best interpreted as due to vegetation, and where vegetation flourishes, at least on the Earth, animal life is likely to exist also.

The greatest defender of the theory of the habitability of Mars was Lowell. He believed the reddish-ocher areas which compose most of the planet's surface to be deserts and the "seas" to be wooded or grass-covered plains. He pointed out that the canals intersect one another at large angles, often three or more crossing at the same point, quite differently from rivers; that to be seen at all they must be at least several miles wide; and, in particular, that they are of too straight and geometric a character (though their straightness is disputed by some observers) to be natural lines such as rivers or cracks. He therefore interpreted them as strips of vegetation along the sides of artificial lines of irrigation which have been built by intelligent beings in order to make the best use of the limited supply of water. He explained the wave of deepening color which sweeps from pole to equator in the Martian summer by the quickening of vegetation in the dark areas and along the canals by water from the melting polar cap.

Lowell's theory encounters no really insurmountable obstacle unless it be the apparent absence of Martian oxygen; but there are few astronomers who accept it without misgivings.

**Deimos and Phobos.** Mars has two tiny satellites, each probably less than twenty miles in diameter, which were discovered by Asaph Hall at Washington in 1877. They revolve in circular orbits in the plane of the planet's equator. The nearer one, **Phobos**, is only 5800 miles from the center of the planet and therefore less than 4000 miles from its surface. Although, like the Moon, it revolves in the direction of its planet's rotation, its period is only $7^h\,40^m$, about a quarter of Mars's day, and so it moves faster than the surface and, to a Martian observer, must rise in the west

and set in the east. The other satellite, **Deimos,** revolves at a distance of 14,600 miles in a period of $30^h\,18^m$, which is so little greater than the Martian day that the satellite remains above the horizon more than sixty hours or two of its months; so that it must go twice through all its changes of phase between rising and setting.

**The Asteroids.** Between the orbits of Mars and Jupiter revolve many bodies known as **asteroids,** or **minor planets.** Most of them are tiny telescopic specks which can be distinguished from faint stars only by their motion. For the four which were first discovered and which are the largest and brightest, Table 10 gives the diameters measured by Barnard and the albedos computed from these diameters by Russell.

Table 10

THE LARGEST ASTERIODS

| Name | Diameter | Albedo |
|---|---|---|
| Ceres................ | 488 miles | 0.06 |
| Pallas................ | 304 " | 0.07 |
| Vesta................ | 248 " | 0.26 |
| Juno................ | 118 " | 0.12 |

Vesta, although not the largest, is the brightest asteroid, and when nearest the Earth is sometimes visible to the naked eye. Most of the others are probably less than fifty miles in diameter. According to Crommelin, the combined volume of all the asteroids known is less than one-twentieth that of the Moon.

**Discovery of the Asteroids.** About the end of the eighteenth century, after the discovery of Uranus (page 286), whose distance fitted nicely into Bode's law (page 224), it was suspected that an undiscovered planet revolved in the space corresponding to the fifth term of Bode's series and a number of astronomers planned to search for it. On the first of January, 1801, Piazzi, who was observing on the island of Sicily and was not one of the organized searchers, found a little planet which he named Ceres. After observing it a short time, Piazzi became ill, and before he recovered or the news of his discovery had reached other astronomers the Earth had moved so far in its orbit as to leave Ceres in a position unfavorable for rediscovery. The elements of the orbit of the little planet were unknown and there was great danger that it could never be identified among the multitude of stars; but the German mathematician Gauss, then twenty-four years old, discovered a method of determining the elements of the orbit of a planet from three observations, calculated the orbit of Ceres, and predicted its apparent position in which it was rediscovered on the last day of the same year.

Ceres was so small that the German astronomer Olbers thought that other planets like it might exist, and the search was continued. Pallas was found by Olbers in 1802, Juno by Harding in 1804, and Vesta by Olbers in 1807. No more asteroids were then discovered until 1845, when Hencke found Astraea; but beginning with 1847 at least one has been found every year. Since 1891 most of the discoveries have been made by photography, a field in which Wolf of Heidelberg and Palisa of Vienna have led. More than 1500 asteroids have been catalogued, and from counts of their faint trails on Mount Wilson photographs Baade has estimated that there are at least 30,000 which, at opposition, are brighter than the nineteenth magnitude.

**Orbits of the Asteroids.** The mean distances of the known asteroids range from 1.29 (that of Hermes) to more than 5 astronomical units; their periods from less than two to more than twelve years; the inclinations of their orbit planes from 0 up to 48°; and their eccentricities from 0 to 0.66; but most of the orbits are fairly round, lie rather near the plane of the ecliptic, and have semimajor axes averaging about 2.8, the number in Bode's series between Mars and Jupiter.

Full discussions of the orbits of the 1091 asteroids discovered up to the middle of 1929 are contained in the monumental Volume 19 of the *Publications* of the Lick Observatory, compiled at the Students' Observatory of the University of California. Elements and ephemerides of all known planets have been published annually in *Kleine Planeten*, issued by the Astronomisches Rechen-Institut (later called the Coppernicus-Institut) at Berlin, Germany.

It was noticed by Kirkwood of Indiana in 1866 (when only eighty-eight asteroids were known), and has been abundantly verified since their numbers have so greatly increased, that no asteroids have periods which are

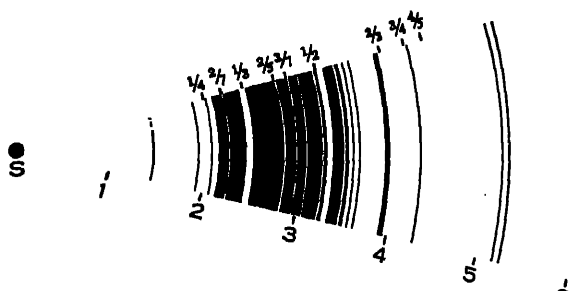



Fig. 188. *Kirkwood's Gaps in the Asteroid Belt.*

simple fractions of that of Jupiter. When the mean distances of the asteroids are plotted as in Figure 188, conspicuous gaps are found at the distances at which, if an asteroid did revolve, its period would be ½, ⅓, ⅖, etc., of Jupiter's period. Although the fact has not been demonstrated mathematically, it is probable that the perturbations of Jupiter have cleared

the asteroids from **Kirkwood's gaps** or else prevented their ever entering them.

**Nomenclature; Exceptional Asteroids.** At first, the names for asteroids were chosen from Greek mythology; but as their number increased, other mythologies were invoked, the works of Shakespeare, Wagner, and lesser writers were pressed into service, and many strange new names were invented. All are supposedly feminine except those of a few asteroids which are unconventional in behavior. Among these are the **Trojans,** each of which fulfills, with Jupiter and the Sun, the triangular case of the problem of three bodies (page 245), and which bear the names of Homeric heroes such as Hektor, Achilles, Aeneas, and Troilus. Twelve Trojans are now known, five on the west side of Jupiter and seven on the east side.

**Eros,** which comes within the orbit of Mars, is of particular interest for the use that has been made of it in determining the length of the astronomical unit, and others of this group may sometime be similarly utilized. At a close approach to the Earth, the geocentric parallax is large, and for this reason and also because the little asteroid presents a point-image, its distance in Earth-radii and in miles can be accurately determined. From the elements of its orbit its distance in astronomical units can be computed, and from the two values of the distance the number of miles in the astronomical unit is readily found. The latest work of this kind is the thorough study of Spencer Jones which resulted in the value of the Sun's distance, 93,005,000 miles, mentioned on page 106.

Eros is notable also for considerable variations in brightness which suggest a surface of diversified reflecting power and a rotation in about 5 hours with equator greatly inclined to the plane of the Earth's orbit. In 1931, when it approached within about 16,000,000 miles, it was reported by some observers to have shown an elongated disk.

**Adonis, Apollo,** and **Hermes** have perihelia within the orbit of Venus but are so small (probably less than a mile in diameter) that they can be observed only at their rare approaches to the Earth, and their orbits are not very reliably determined. In February, 1938, Hermes came within 485,000 miles, appeared for a brief time of the eighth magnitude, and swept by at an apparent speed of 5° per hour. At its closest point, the orbit of Hermes is only 220,000 miles from the orbit of the Earth, so the little planet may sometime approach closer than any other heavenly body, not excepting even the Moon. Perhaps it is a body of the same nature as the great meteorites of Arizona and Siberia (page 294).

At the other extreme of asteroid orbits is the orbit of **Hidalgo,** whose perihelion lies just outside Mars's orbit, and whose aphelion is just inside Saturn's.

**The Zodiacal Light and the Gegenschein.** The region of the zodiac on either side of the Sun is illuminated by a faint, soft light of triangular or lenticular shape which somewhat resembles the Milky Way but lacks the sparkle of the latter's millions of distant stars. This **zodiacal light** is most easily seen in our latitudes on spring evenings or autumn mornings when the portion of the ecliptic above the horizon is most nearly vertical. Its spectrum, as observed by Fath at the Lick Observatory in 1909, seems

identical with that of sunlight, and there is little doubt that the illumination is caused by the reflection of sunlight by a belt of small bodies revolving in orbits which lie mostly within that of Venus, but some of which may extend beyond the Earth. Elvey finds the light to be yellow, though slightly variable in color, and infers that the reflecting particles are larger than the particles in our air which, by scattering sunlight, produce the blue of the sky.

Far more difficult to see, and quite imperceptible when there is the least haze or artificial light, is the **Gegenschein**, a dim, oval spot of light which lies exactly opposite the Sun, sharing its annual motion as does the zodiacal light. Moulton's explanation of the Gegenschein is mentioned on page 245.

**The Giant Planets.** The four planets next beyond the asteroids are so much larger than those nearer the Sun that they may well be called the **giant planets.** The principal numerical facts regarding them are given in Table 11.

Table 11

THE GIANT PLANETS

| | Jupiter | Saturn | Uranus | Neptune |
|---|---|---|---|---|
| Mean distance from Sun, astronomic units | 5.20 | 9.54 | 19.19 | 30.07 |
| Sidereal period, years | 11.86 | 29.46 | 84.02 | 164.79 |
| Mean synodic period, days | 399 | 378 | 369 | 367.5 |
| Eccentricity of orbit | 0.048 | 0.055 | 0.047 | 0.009 |
| Inclination of orbit plane | 1°.3 | 2°.5 | 0°.8 | 1°.8 |
| Apparent diameter | 32″ to 50″ | 14″ to 20″ | 3″.8 | 2″.5 |
| Equatorial diameter in miles | 88,700 | 75,060 | 30,900 | 32,900 |
| Oblateness | 0.06 | 0.11 | 0.09 | ? |
| Mass, Earth = 1 | 318 | 95 | 15 | 17 |
| Mean density, water = 1 | 1.32 | 0.72 | 1.22 | 1.11 |
| Surface gravity, $g = 1$ | 2.65 | 1.18 | 0.90 | 0.89 |
| Mean albedo | 0.44 | 0.42 | 0.45 | 0.52 |
| Mean rotation period | $9^h 55^m$ | $10^h 14^m$ | $10^h 45^m \pm$ | $16^h \pm$ |
| Inclination of equator to orbit | 3°.1 | 26°.8 | 82° | 35° |

No distinct phases are perceptible on any of these outer planets because the angle *Sun-planet-Earth* is always small.

On each giant planet, the telescope reveals light and dark belts arranged parallel to the equator, which are very conspicuous on Jupiter (Figure 189), faint on Saturn, and almost imperceptible on Uranus and Neptune. Great changes are observed in the details of the markings on

Jupiter and Saturn, and the rotation periods determined from spots in different latitudes are different, as in the case of the Sun.

Contrary to opinions generally held at the beginning of the twentieth century, the visible surfaces of the giant planets are now known to be very cold: radiometer measures made by Lampland and Coblentz in 1924 indicate a temperature of $-130°$ C. for Jupiter, $-150°$ for Saturn, and $-170°$ for Uranus.

Fig. 189. *The Planet Jupiter, Photographed with 36-Inch Refractor of the Lick Observatory by H. M. Jeffers, 1939 October 13.*

The spectra of all four giant planets contain dark bands in the region extending from H$\beta$ into the infra-red, the intensity of which increases from Jupiter to Neptune. The green color of Uranus and Neptune is due largely to this heavy absorption of the red portion of their light. Discovered mainly by Slipher in 1907, these bands have since 1930 been identified by Wildt, by Slipher and Adel, and by Dunham with two hydrogen compounds which are well known on the Earth: **ammonia**, the strong-smelling substance much used in ice factories and for various domestic purposes, and **methane**, known terrestrially as a constituent of natural gas and as the poisonous "fire damp" which occurs in mines. Probably the belts seen on the giant planets are clouds composed of solid crystals or liquid drops of ammonia and other compounds, floating in a cold atmosphere of methane gas.

**Jupiter.** To the unaided eye Jupiter appears as a star five or six times as bright as Sirius. It is the largest of the planets, larger than all the others combined, with a diameter 11 times, a volume 1300 times, and a mass over 300 times the Earth's. Its rotation is so rapid that it may be noticed in less than an hour by observation at the telescope under good conditions, and has produced a very noticeable equatorial bulge. With even a very small

Fig. 190. *Spectrogram of Jupiter by V. M. Slipher, Lowell Observatory. The spectrograph slit was parallel to the planet's equator, and the rapid rotation has produced a perceptible inclination in the spectral lines (Doppler effect).*

telescope the belts are easily seen, and with a large instrument and good seeing the markings show a wealth of detail. The colors are delicate and beautiful, consisting mainly of red, yellow, tan, and brown tints, but excellent conditions are required to show them, since a little atmospheric disturbance blurs the details.

The chief markings are of a durable, but not permanent, character. The most famous one is the great red spot which has been observed in the Jovian southern hemisphere since 1857 and was "startlingly conspicuous" from 1878 to 1881. In its prime it was about 30,000 miles long and 7000 wide. It has disappeared and reappeared at times, but in its absence its place has been marked by the **great red spot hollow.** It cannot be permanently attached to a solid nucleus, for its rotational speed, which is usually slower than that of its surroundings, is perceptibly but temporarily accelerated when the spot is overtaken by the **south tropical disturbance,** another salient feature which has been known since 1901. Other markings sometimes change their appearance much more rapidly. The visible surface appears to be composed of clouds, but they must be clouds not of water but of some substance which vaporizes at a much lower temperature—probably ammonia.

**The Satellites of Jupiter.** Jupiter's system of eleven satellites is described in Table 12. The four large satellites, which were discovered by Galileo, are numbered in the order of their distance from Jupiter; the others in the order of discovery. The names of the Galilean satellites were given them by Simon Marius, who claimed priority of discovery; as his claim was believed fraudulent, the names have never been extensively used, and the fainter satellites have never been named.

Table 12

THE SATELLITES OF JUPITER

| Name | Discovery | Distance, Miles | Period | Diameter, Miles | Density, Water = 1 | Stellar Magnitude |
|---|---|---|---|---|---|---|
| Jupiter V | Barnard, 1892 | 112,500 | $11^h\ 57^m$ | small | ......... | 13 |
| Io | Galileo, 1610 | 261,000 | $1^d\ 18^h$ | 2450 | 2.7 | 5 |
| Europa | Galileo, 1610 | 415,000 | $3^d\ 13^h$ | 2050 | 2.6 | 6 |
| Ganymede | Galileo, 1610 | 664,000 | $7^d\ 4^h$ | 3560 | 1.5 | 5 |
| Callisto | Galileo, 1610 | 1,167,000 | $16^d\ 17^h$ | 3350 | 1.0 | 6 |
| Jupiter VI | Perrine, 1905 | 7,300,000 ± | $266^d$ ± | v. small | ......... | 14 |
| Jupiter VII | Perrine, 1905 | 7,500,000 ± | $277^d$ ± | v. small | ......... | 16 |
| Jupiter VIII | Melotte, 1908 | 14,600,000 ± | $740^d$ ± | v. small | ......... | 18 |
| Jupiter IX | Nicholson, 1914 | 14,750,000 ± | $758^d$ ± | v. small | ......... | 18 |
| Jupiter X | Nicholson, 1938 | 7,300,000 ± | $260^d$ ± | v. small | ......... | 16 |
| Jupiter XI | Nicholson, 1938 | 14,000,000 ± | $692^d$ ± | v. small | ......... | 17 |

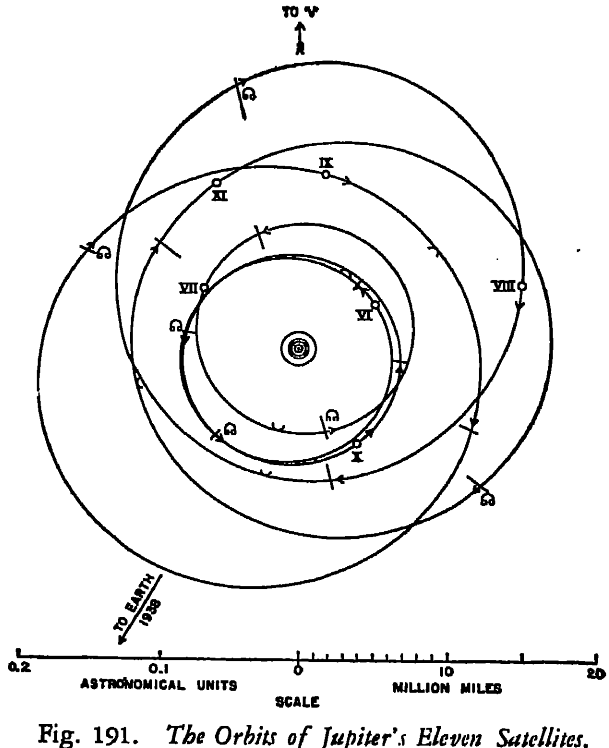



Fig. 191. *The Orbits of Jupiter's Eleven Satellites. (Drawn by Seth B. Nicholson.)*

The seven small satellites are very faint, and all but the inmost were discovered by photography. Jupiter VIII was discovered at Greenwich; the six others at the Lick and Mount Wilson observatories. The satellites'

distances from Jupiter divide them naturally into three groups (Figure 191), the first group consisting of the Galilean satellites and V; the second, of VI, VII, and X; and the third, of VIII, IX, and XI. The three members of the outermost group revolve in the retrograde direction. Jupiter's attraction on the outer satellites is so weak that their motion is greatly perturbed by the Sun and their orbits are non-reëntrant, being represented only approximately by the closed curves of Figure 191. The planes of the six outer orbits are highly inclined to the plane of Jupiter's equator, whereas those of the inner five sensibly coincide with it.

**Phenomena of the Galilean Satellites.** The four great satellites, which are so bright as to be visible in the smallest telescope—or even to the naked eye were it not for their proximity to the planet—present most

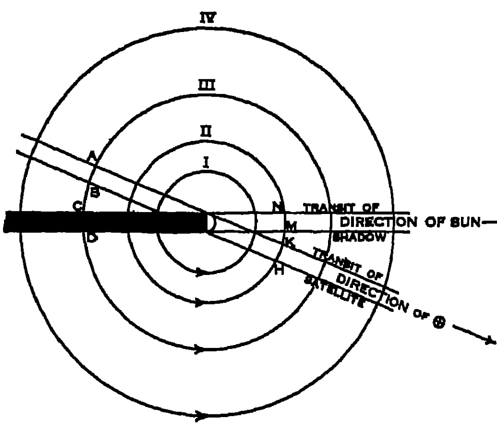



Fig. 192. *Phenomena of Jupiter's Galilean Satellites: Jupiter at East Quadrature.*

interesting phenomena. The plane of their orbits (and that of Jupiter's equator) so nearly coincides with that of the planet's orbit, and also with that of the ecliptic, that their orbits are presented almost edgewise to both the Sun and the Earth. The satellites, therefore, appear always in a nearly straight line passing through the planet, and all but Callisto are eclipsed at every revolution. Callisto escapes eclipse when Jupiter is far from his equinoxes.

When Jupiter is not at opposition, four distinct phenomena present themselves; these may be understood from a consideration of the orbit of Ganymede in Figure 192. When the satellite arrives at $A$, it is occulted behind the planet; it emerges from occultation at $B$ to disappear in eclipse

at *C*, reappearing at *D*. On arriving at *H* it passes in front of Jupiter and a **transit** of the satellite occurs, during which it may be seen against the planet as a dot of nearly the same brilliancy as the planet itself. From *M* to *N* the satellite's shadow falls upon the planet and may be easily seen as a black dot (**transit of the shadow**). At opposition, eclipses take place simultaneously with occultations, and the shadows are almost completely concealed behind the satellites which cast them (Figure 193).

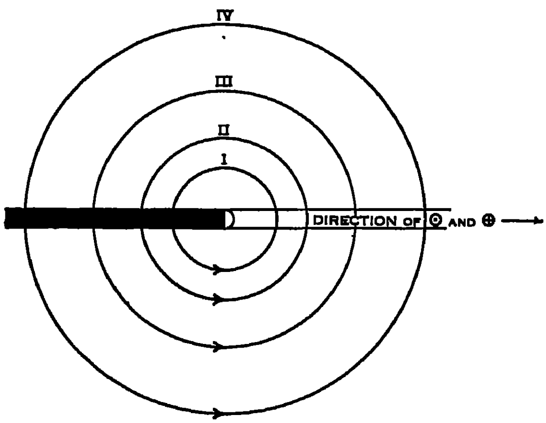



Fig. 193. *Phenomena of Jupiter's Galilean Satellites: Jupiter at Opposition.*

Callisto has a much lower albedo than the other large satellites, one of which, Europa, is about as white as snow. When in transit across Jupiter, Callisto appears dark. All four, especially Callisto, are variable in brightness in periods coinciding with their revolution periods, probably because one side is brighter than the other and because the satellites behave like the Moon and always keep the same face toward Jupiter.

**Saturn and Its Rings.** To the naked eye, Saturn appears as a star of the first magnitude, but not eminent in any way above several other stars. By a good telescope, it is revealed as a delicately banded globe poised within a shining ring—a spectacle which, once seen, can never be forgotten. So far as is known, no other body in the universe is like it (Figures 194, 195).

Galileo's telescope was not quite good enough to show the details of this planetary spectacle, but he perceived that Saturn was not like an ordinary star, and sent a mysterious anagram to Kepler, later transposing it into the sentence *Ultimum planetam tergeminum observavi* ("I have observed the farthest planet triple"). Subsequent observers drew Saturn as triple or as having a pair of ansae, or "ears," of

various types; and it was not until 1655 that Huyghens perceived the planet's true form which he announced, first in an anagram and later in its translation, *Annulo cingitur, tenui, plano, nusquam cohaerente, ad eclipticam inclinato* ("It is surrounded by a ring, thin, flat, nowhere touching, inclined to the ecliptic"). In 1675 D. Cassini found that the ring was divided in two by the space since known as Cassini's division. In 1850 G. P. Bond of Harvard discovered, within the two bright rings, a faint, semi-transparent crepe ring.

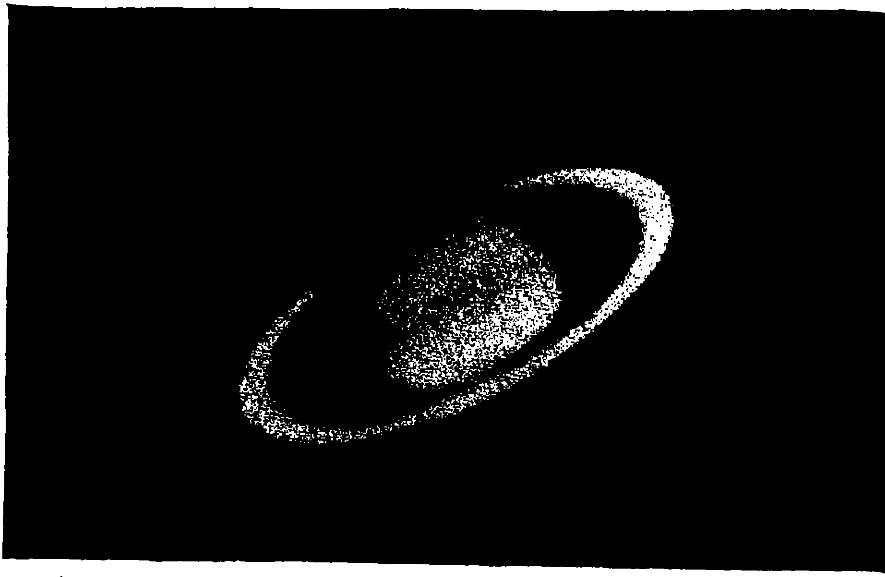


Fig. 194. *Saturn and Its Rings, Photographed by E. C. Slipher, October, 1941.*

The rings lie in the plane of the planet's equator, which is inclined about 27° to the plane of its orbit. Twice in the sidereal period of nearly thirty years this plane passes through the Sun just as does the plane of the Earth's equator in March and September (Figure 83); the Sun then shines only on the thin edge of the ring, and at about the same time the ring is presented edgewise to the Earth. At such times it disappears in small telescopes, but in large ones it seems a fine needle thrust through the ball. About seven and one-half years later the Saturnian solstice occurs, and the Earth and Sun have an elevation of about 27° above the plane of the ring, which then appears as an ellipse with a minor axis nearly half the major axis. When not presented edgewise to the Sun, the ring casts a shadow on Saturn which may usually be seen as a dark band outlining the edge of the ring. At the same time the shadow of the ball extends across the ring and may be seen, except when directly behind the planet, as a black band outlining one limb of the globe against the ring. The ring

reflects so much sunlight when most widely displayed that, to the unaided eye, the planet then appears more than three times as bright as when the ring is placed edgewise.

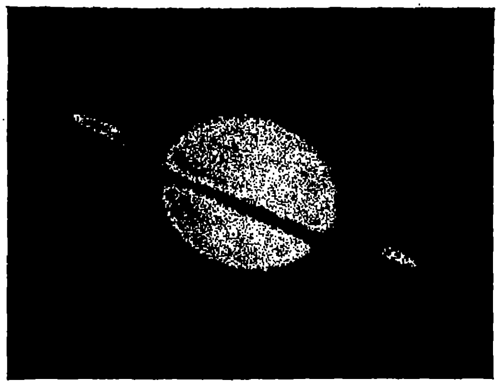


Fig. 195. *Saturn and Its Rings, photographed by E.C. Slipher, May, 1922, when the plane of the rings was passing nearly through the Earth and the Sun.*

Saturn's oblateness is greater than that of any other known planet, and its equatorial bulge is conspicuous; but its rotation period is slightly longer, and its diameter considerably less, than Jupiter's. Its mean density is very low, much less than that of water and less than that of any known solid except the rare element lithium; and yet it has been shown by the motion of the satellites that the inner core of the planet is denser than the outer layers, which must be very light indeed. Both the ball and the rings, and also some of the satellites, have high albedos, and the English astronomer Hepburn has suggested that they may be composed of ice or loosely packed snow.

**Dimensions and Constitution of the Rings.** The dimensions of the rings are given in Table 13. The middle ring, *B*, which is much the widest,

Table 13

THE RINGS OF SATURN

| | | |
|---|---|---|
| Radius of outer limit of ring system | | 86,300 miles |
| Width of outer ring (A) | | 11,100 " |
| Width of Cassini's division | | 2,200 " |
| Width of Ring B | | 18,000 " |
| Width of crepe ring (C) | | 11,000 " |
| Distance from inner edge of Ring C to surface of Saturn | | 6,000 " |
| Thickness of ring | less than | 100 " |

is also much the brightest. None of the rings is quite opaque; the outline of the ball is easily seen through the crepe ring and is visible, though less easily, through Ring *A*; and stars have been seen even through Ring *B*. Moreover, when the Earth and Sun are on opposite sides of the plane of the rings, as happens for a short time every fifteen years, the rings do not completely disappear but are visible in large telescopes by sunlight that has passed through them.

It was shown theoretically by Laplace that, if the rings were solid, they must collapse like an overloaded bridge under the force of Saturn's attraction, and in 1857 by Clerk-Maxwell that they could be neither solid nor fluid and must therefore consist of a swarm of independent bodies like meteors.

In 1895 Keeler verified the meteoric constitution of the rings by a beautiful application of the spectroscope and of Kepler's laws. He photographed

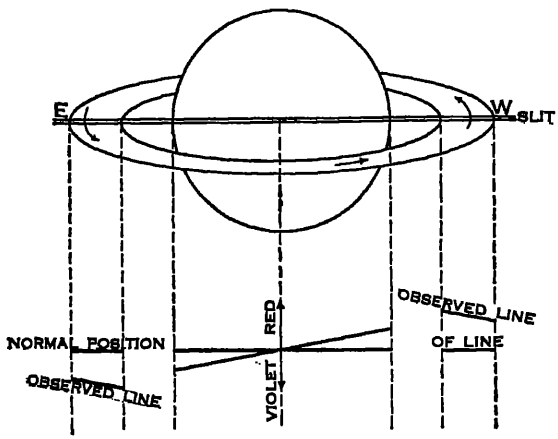



Fig. 196. *Spectroscopic Proof of the Nature of Saturn's Rings.*

the spectrum of Saturn by forming an image in such a way that the slit of the spectrograph cut through both ansae and through the center of the disk, as in Figure 196. Each spectral line was thus divided in three parts, consisting of the image of an east-west strip of the planet in the middle and of a strip of the ring at each end. The middle portion was inclined because of the rotation of the planet (page 262), the west end being displaced toward the red; but the end portions, though shifted bodily, were inclined the other way (Figure 197), showing that the inner particles of the ring were moving faster than the outer. The speeds observed at different distances from Saturn conform to Kepler's third law.

Cassini's division lies at a distance from Saturn where, if a particle revolved, its period would be half that of Mimas, the nearest satellite, and commensurable with the periods of three others, and is thus analogous to Kirkwood's gaps in the asteroid belt. More delicate divisions have been

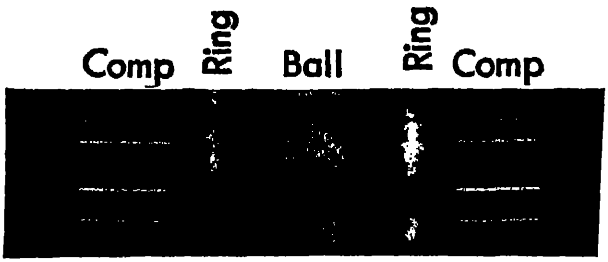



Fig. 197. *Portion of a Spectrogram of the Saturnian System, by V. M. Slipher.*

detected, which Lowell found to lie where periods would be nearly but not quite commensurable with the periods of Mimas or Enceladus. He showed that the slight discrepancies may be due to a lack of homogeneity of the internal constitution of Saturn.

Table 14

THE SATELLITES OF SATURN

| Name | Discovery | Distance, Miles | Period | Diameter, Miles |
|---|---|---|---|---|
| Mimas | W. Herschel, 1789 | 117,000 | $22^{h}.6$ | 600 ± |
| Enceladus | W. Herschel, 1789 | 157,000 | $1^{d}\ 9^{h}$ | 800 ± |
| Tethys | J. Cassini, 1684 | 186,000 | 1 21 | 1200 ± |
| Dione | J. Cassini, 1684 | 238,000 | 2 18 | 1100 ± |
| Rhea | J. Cassini, 1672 | 332,000 | 4 12 | 1500 ± |
| Titan | Huyghens, 1655 | 771,000 | 15 22 | 3000 ± |
| Hyperion | G. P. Bond, 1848 | 934,000 | 21 7 | 500 ± |
| Japetus | J. Cassini, 1671 | 2,225,000 | $79^{d}$ | 2000 ± |
| Phoebe | W. H. Pickering, 1898 | 8,000,000 | $546^{d}$ | 200 ± |

**The Satellites of Saturn.** Saturn has nine satellites which are described in Table 14. They are not so conspicuous as the Galilean satellites of Jupiter, being more distant and mostly smaller; but several of them, particularly Titan, may usually be seen with a moderate-sized telescope. The orbits of the five inner ones are circular and lie in the plane of the planet's equator and of the rings. Titan and Hyperion also revolve nearly in that plane, but the orbit of Japetus is inclined about 10°, while Phoebe revolves

*backward* almost in the plane of Saturn's orbit. Eclipses and occultations can occur only when the rings are placed nearly edgewise to the Sun and the Earth.

Kuiper reported in 1943 that the spectrum of Titan, like the spectra of the giant planets, shows atmospheric bands of methane and probably of ammonia.

**Uranus and Its Satellites.** The discovery of Uranus by William Herschel in 1781 caused great excitement, for the discovery of a new planet was then a thing unheard of. Herschel was knighted and was provided by the king of England with means for building great telescopes with which he subsequently did much important work.

The discovery did not take place in quite the way which might be inferred from Keats's lines, written years later:

> "Then felt I like some watcher of the skies
> When a new planet swims into his ken."

Herschel was "sweeping" the sky in the constellation Gemini with a seven-inch reflector of his own construction when he perceived an object which he distinguished from the stars by its disk. He thought it was a comet and so reported it, and the fact that it was a planet did not become known until nearly a year later, when Lexell, in Russia, computed its orbit, which he found to be nearly circular and to lie beyond Saturn. Herschel wished to call the planet "Georgium Sidus" in honor of King George III, and many other astronomers called it Herschel; but the name Uranus, proposed by Bode, was finally adopted.

Uranus is barely perceptible to the naked eye on a moonless night. In the telescope it presents a little apple-green disk. It is only under the best conditions that vague belts can be perceived upon it, and there are no markings from which a rotation period can be found. Slipher has determined the period spectroscopically as about ten and two-thirds hours, and L. Campbell, at Harvard, has found that the planet's brightness varies in this period, suggesting that it is unequally bright in different longitudes. The rotation appears to take place in the plane of the satellite orbits, which is inclined at the extraordinary angle of 82° to the plane of the planet's orbit, and the motion, like that of the satellites, is retrograde.

The satellite system is described in Table 15.

**Neptune and Its Satellite.** The discovery of Neptune has already been discussed (page 249). This distant body appears of the eighth stellar magnitude, quite invisible to the unaided eye. In the telescope it shows a little greenish disk without any markings except, under the finest condi-

Table 15

THE SATELLITES OF URANUS

| Name | Discovery | Distance, Miles | Period | Diameter (?), Miles |
|---|---|---|---|---|
| Ariel | Lassell, 1851 | 120,000 | $2^d\ 12^h.5$ | 500 |
| Umbriel | Lassell, 1851 | 167,000 | 4 3.5 | 400 |
| Titania | W. Herschel, 1787 | 273,000 | 8 17 | 1000 |
| Oberon | W. Herschel, 1787 | 365,000 | 13 11 | 800 |

tions, vague belts. Its rotation, determined spectroscopically by Moore and Menzel in 1928, is direct and has a period of about sixteen hours.

Neptune has one satellite, discovered by Lassell in 1846. Though the diameter of this satellite is estimated at 2,000 miles, it is so faint as to be very difficult to see. It revolves at a distance of 222,000 miles in five days twenty-one hours, in a plane inclined about 35° to the ecliptic, and in the *retrograde* direction—a circumstance which makes the direct rotation of the planet somewhat surprising. Flammarion has suggested for Neptune's satellite the appropriate name of Triton.

**Pluto.** Of Pluto little is known except that it is more than a hundred times fainter than Neptune, that it presents no perceptible disk, and that it is yellowish in color. In diameter and mass it is almost certainly smaller than the Earth, and it seems to belong in the class of the smaller planets or even of the asteroids, rather than in that of the giant planets where its distance from the Sun would place it.

**Illustration of Dimensions in the Solar System.** In order to obtain a clear idea of the relative dimensions and distances of the planets, it is useful to imagine the model of the Solar System described by Sir John Herschel in his *Outlines of Astronomy* and widely quoted elsewhere, particularly by Young in his excellent textbooks.

"Choose any well-leveled field. On it place a globe, two feet in diameter; this will represent the Sun; Mercury will be represented by a grain of mustard-seed, on the circumference of a circle 164 feet in diameter for its orbit; Venus a pea, on a circle 284 feet in diameter; the Earth also a pea, on a circle of 430 feet; Mars a rather large pin's head, on a circle of 654 feet; Juno, Ceres, Vesta, and Pallas, grains of sand, in orbits of from 1000 to 1200 feet; Jupiter a moderate-sized orange, in a circle nearly half a mile across; Saturn a small orange, on a circle of four-fifths of a mile; Uranus a full-sized cherry, or small plum, upon the circumference of a circle more than a mile and a half, and Neptune a good-sized plum on a circle about two miles and a half in diameter." Pluto, on the same scale, would be represented by a pea or smaller object on an ellipse three and a quarter miles in length, with the globe which represents the Sun one and a quarter miles from its perihelion.

The reader may readily apply these distances to familiar objects in his own neighborhood.

## EXERCISES

1. Could we ever see an occultation of Mars by Jupiter? By Venus?

2. As seen from Saturn, what would be the apparent diameter of the Earth when in transit upon the disk of the Sun? What would be the Earth's greatest elongation?

*Ans.* 2″; 6°

3. When Mercury is seen as a black dot in transit upon the Sun, is its apparent motion across the disk in the same direction as that of the Moon during a solar eclipse?

4. Suppose that Venus's rotation period is 224 of our days, its revolution period being 225. What would be the length of the solar day on Venus?

*Ans.* 50,400 of our days, or about 138 years

5. In that case, what region of the planet would be best adapted to habitation?

*Ans.* That of the great circle dividing the bright half of the planet from the dark. As this would revolve around the planet once in about 138 years, the population would perhaps be migratory.

6. Why are not eclipsed satellites of Jupiter dimly visible, like the Moon during a total lunar eclipse (page 149)?

7. When Jupiter is at east quadrature (evening star), do the satellites precede or follow their shadows as they transit across the planet's disk?

# CHAPTER 13

## METEORS AND COMETS

**General Description of Meteors and Comets.** A few minutes' watch of any part of the sky on a clear, moonless night is sufficient to detect a number of the sudden, brief flashes which are popularly called **falling stars.** Occasionally one of these appears as bright as Jupiter or Venus, and perhaps leaves behind it a phosphorescent **train** which may float slowly over the sky, remaining visible for minutes. Very rarely a brilliant flash occurs, accompanied by a loud noise, and a heavy body falls to the ground. Such a body, after its fall, is called a **meteorite,** but while moving through space the bodies which cause the flashes are called by astronomers **meteors.**

The word meteor, in its older and more general sense, means any natural object or phenomenon seen high in the air — a flash of lightning, a hailstorm, and a rainbow are all "meteors" in this sense — and the science of **meteorology** deals with phenomena of the weather but has nothing to do with meteors in the astronomical sense. Until near the end of the eighteenth century, the study of meteors was not considered to belong properly to astronomy at all; their flashes were believed to be atmospheric phenomena, and the falling of stones from the sky was regarded as a miracle or discredited as impossible.

A well-developed **comet** (ἀστήρ κομήτης, long-haired star) consists of a small bright **nucleus,** which appears star-like except with high magnification, surrounded by a diffuse, misty envelope or **coma,** which is greatly extended on the side opposite the Sun to form the **tail.** The tails of some comets attain an apparent length of many degrees and a real length of many millions of miles. Many comets, however, lack both nuclei and tails and are visible only in the telescope, appearing merely as little hazy patches of light which are distinguishable from faint nebulae only by their motion. Comets appear usually at unpredicted times and positions, cross the sky with a motion which is perceptible from night to night, and fade from view

after a few weeks or months. On the average about four or five comets are seen each year; the greatest number yet recorded for any year is eleven, in 1925.

Bright comets have throughout human history attracted a great deal of attention because of their sudden and extraordinary appearance, and have been regarded by the ignorant and the superstitious as omens of "famine, pestilence, and war." Like meteors, they were supposed to be situated in the Earth's atmosphere until Tycho Brahe proved that the parallax of a great comet which appeared in 1577 was less than that of the Moon and that therefore the comet could not be "sublunary."

Fig. 198. *Finsler's Comet of 1937, Photographed with 18-Inch Schmidt Camera on Palomar Mountain by Josef J. Johnson. The bright stars are Mizar and Alcor.*

The study of comets shows that although, like the planets, they move under the control of the Sun's gravitation, they are contrasted with the planets in almost every other respect. The planets are compact, opaque, and nearly spherical bodies moving all in the same direction in orbits which are regularly spaced, almost circular, and nearly in the same plane. The

comets move on highly eccentric orbits, most of which are either parabolas, or ellipses of such size and eccentricity that their observed portions appear parabolic. The planes of their orbits are inclined to the ecliptic at angles ranging from 0 to 90°, and a large portion of the comets move in the retrograde direction. The comets themselves (Figure 198) are bodies of strange form, immense size, and insignificant mass which change shape and dimensions greatly and rapidly, which shine partly by reflected sunlight and partly by light of their own, and which exhibit evidence of repulsive as well as of attractive forces.

The study of meteors proves that they are tiny solid bodies which move around the Sun like the comets but are never seen by man until they enter the Earth's atmosphere and become luminous by the transformation of the energy of their rapid motion into heat. It is known certainly that many meteors, possibly all, are connected with comets, and many are probably in fact the remains of comets which have disintegrated.

**Determination of the Path and Speed of a Meteor Through the Air.** For the observation of meteors the telescope is of little use because of its narrow field. Observations are best made with the naked eye by plotting the apparent path on a star map, or with a wide-field camera which records photographically the trails of bright meteors among the stars. The time of the flash should be noted and the duration of visibility estimated or timed with a stop watch.

Observation of a meteor from a single place is not sufficient to determine its real position, but if the same meteor is seen simultaneously by two observers stationed a few miles apart a difference of apparent place results from parallax, and from the two observations the actual height and location of the meteor may be found by triangulation.

The plot of the apparent path on the star map (or the photographic trail) gives the right ascension and declination of each point on the meteor's visible path. From the recorded time and the right ascension, the hour angle may be found, and the hour angle and declination may be transformed into altitude and azimuth, either by trigonometric calculation or by the use of a celestial globe. Neglecting the curvature of the Earth, let $A$ and $B$ (Figure 199) be the two observers, $M$ a point of the meteor's path, say the point of first appearance, and $P$ the point of the Earth's surface vertically beneath $M$. In the horizontal triangle $ABP$, the side $AB$ is the distance between the observers, and each of the adjacent angles $PAB$ and $PBA$ is the difference of azimuth of the meteor and one observer as seen by the other. These three quan-

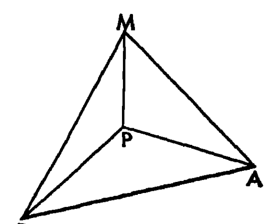



Fig. 199. *Determining the Height and Distance of a Meteor.*

tities being known, the triangle may be solved and the sides *AP* and *BP* computed. In the vertical right triangles *APM* and *BPM* the side *PM* is the vertical height of the meteor and the angles *MAP* and *MBP* are its apparent altitudes at the two stations; hence, in either triangle, the base and its adjacent angle are known and the height can be computed. From the distances *AP* and *BP* the point *P* may be directly located on a terrestrial map. These data for both the beginning and the end of the path, and the time during which the meteor was visible being known, its entire path and the velocity with which it traveled may be determined.

Meteors seldom appear at heights exceeding one hundred miles, and most of them disappear before descending lower than a height of thirty miles. They enter the atmosphere at various speeds, always many miles a second (relative to the Earth), but are greatly retarded within the air; hence those which strike the ground do so at moderate speeds and in most cases are not broken by the fall. According to Opik, the majority of *sporadic* meteors (those which do not belong to a swarm) have heliocentric velocities exceeding 26 mi./sec., the parabolic velocity (page 242) due to the Sun's attraction at the Earth's distance, and therefore must come to us from interstellar space on hyperbolic orbits.

**Source of the Light of Meteors; Their Size.** The luminosity of meteors is readily explained by the high temperature which must be produced by the conversion of their kinetic energy into heat. The kinetic energy of a body of mass *m* grams moving with a velocity of *v* centimeters a second is $\frac{1}{2}mv^2$ ergs. In free space, the velocity of a meteor is many hundreds of thousands of centimeters per second, vastly greater than that of any terrestrial projectile, and the energy is proportional to the *square* of the velocity. When the meteor encounters the atmosphere, its motion is checked and its kinetic energy is transformed into heat. From the observed brightness and velocity of meteors their mass can be estimated; thus the ordinary shooting star proves to be a mere grain with a mass less than that of an ordinary pinhead. The temperature produced by the meteor's violent rush through the air is enormous—several thousand degrees—and this explains the surprising brilliancy of the light, and also the fact that none of these tiny meteors are ever seen to reach the ground; they are reduced to dust by the attrition of the air and are vaporized briefly by the intense heat.

**Meteorites.** If the body has originally a mass of ten pounds or more, it may shine as brightly as the full Moon and a portion may reach the ground. This portion is then called a meteorite. The temperature of a freshly fallen meteorite is ordinarily neither very high nor very low; in its interplanetary path the meteorite was kept at a moderate temperature by

sunshine, and its passage through the air was too brief to permit the heat then generated to penetrate it very far. A meteorite of a few pounds' mass does not strike the ground very hard, for its speed has been greatly reduced by air resistance. However, if the mass were thousands of tons the impact would be terrific and would probably produce a violent explosion, disrupting both the meteorite and a portion of the Earth's crust.

Thousands of meteorites, many of which were actually seen to fall, are preserved in museums. They are classified roughly as stone and iron meteorites. Of meteorites contained in museums, the irons are more numerous, but this is because they are more easily identified; many more stones than irons have been seen to fall. The largest meteorite in any collection is an iron weighing about 36 tons which was brought by Peary from Greenland to the Museum of Natural History in New York City. A block of meteoric iron which lies near Grootfontein, South West Africa, is estimated by Luyten to weigh not less than 50 tons.

Fig. 200. *Meteor Crater, Arizona, Photographed by Clyde Fisher from an Airplane at Altitude of 2000 Feet, 1932 July 17.*

Near Cañon Diablo, in northern Arizona, is a crater known as Coon Butte, about three-fourths of a mile in diameter and with walls about 600 feet high, of which the floor is lower than the surrounding plain, as in the lunar craters. On the plain

around the crater great quantities of meteoric iron have been found, the largest pieces weighing several hundred pounds each; competent authorities are of the opinion that the crater was made by the impact of an enormous meteorite, of which these are fragments (Figure 200).

What may have been a still larger meteorite or group of meteorites is reported to have fallen on June 30, 1908, in a sparsely inhabited region of northern Siberia, and to have destroyed forests over an area several miles in radius.

Iron meteorites are nearly pure iron with usually a small admixture of nickel. Very often the iron is crystallized in a peculiar fashion so that when cut, polished, and etched it exhibits characteristic markings which are known as Widmannstätten figures (Figure 201). Stone meteorites

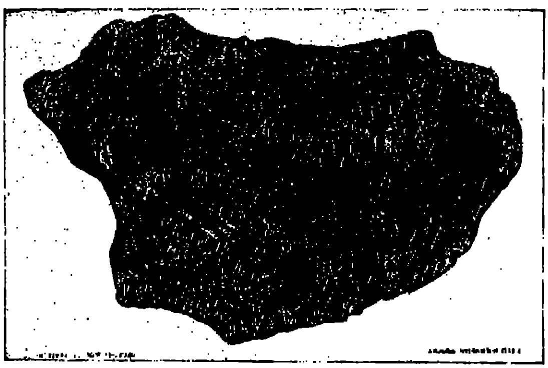


Fig. 201. *Etched Surface of a Meteorite, Showing Widmannstätten Figures.*

also contain some iron, and about thirty other elements have been identified in them, but none not already known in terrestrial substances. Many meteorites contain large quantities of occluded hydrogen, helium, and other gases.

If discovered soon after its fall, before the surface has become weathered, a meteorite is usually found to be covered with a thin black crust formed by the melting of its surface during its passage through the air.

**Number of Meteors.** Although an observer may, from a single point of view, see half the celestial sphere, he can see only a small fraction of the meteors in it, because of the relative shallowness of the air (Figure 202). Moreover, as every telescopic observer knows, there are many meteors too faint to be perceived by the naked eye. These facts being taken into account, it has been estimated from the

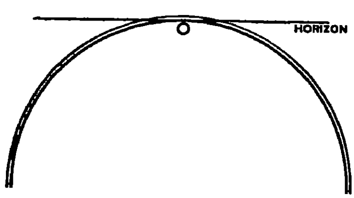



Fig. 202. *We Can See Half the Celestial Sphere But not Half the Meteors.*

number of meteors actually seen that the total number which enter the air daily must be many millions. Their dust, settling through the air, is slowly—but very slowly—adding to the mass of the Earth.

It may be noticed, by anyone who takes the trouble to watch the sky all night, that the meteors seen after midnight are about twice as numerous as those seen before. This is because, in the latter half of the night, we are riding on the front side of the Earth as it moves along its orbit (Figure 203) and receive meteors from all directions, whereas in the earlier half we see none of those which the Earth meets "head on." For the same reason, the meteors of the morning hours have a greater apparent velocity and are bluer because of the greater intensity of heat generated.

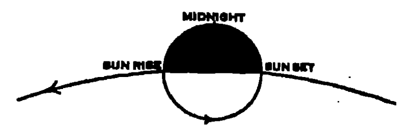



Fig. 203. *Why Meteors Are More Numerous After Midnight.*

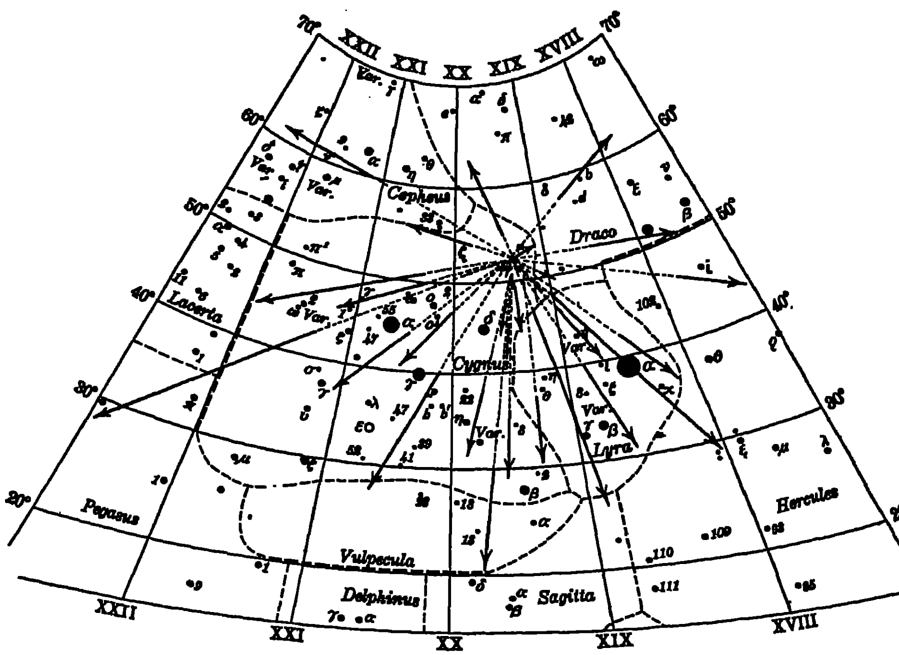



Fig. 204. *The Cygnids of 1922 August 15–26, Observed by R. M. Dole.* (*From* Popular Astronomy.)

**Meteor Swarms and Showers.** When the apparent paths of all the meteors observed on a given night are plotted on a star map, it is often found that many of them radiate from a single small spot in the sky; that is, if the lines representing the observed paths are extended backward, many of them intersect at nearly the same point (Figure 204). This point

is called the **radiant** of those particular meteors. The meteors seem to fly in all directions from the radiant, and it is not unusual for a "falling star" to move apparently upward instead of downward. Those seen nearest the radiant have the shortest apparent paths; a meteor exactly at the radiant seems stationary, simply appearing and vanishing without apparent motion.

The radiant has the same place among the stars for observers at different stations, and therefore must be infinitely distant, while the apparent divergence of the meteor paths is an effect of perspective, the meteors concerned moving, in fact, along parallel lines as in Figure 205. Meteors

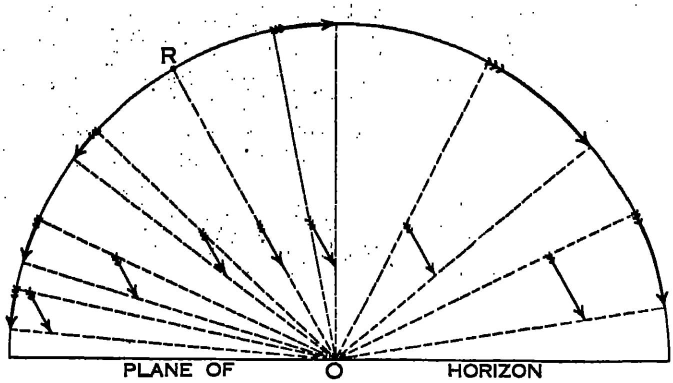



Fig. 205. *The Radiant of a Meteor Shower Is an Effect of Perspective.*

which move in this way are said to belong to a **meteor swarm** which is named for the constellation in which the radiant appears; thus, the swarm of **Perseids** has a radiant in Perseus, the **Leonids** one in Leo, etc. Sometimes the Earth encounters a swarm so dense that thousands of meteors are seen in a single night, as happened with the Leonids in 1833 and 1866, the Andromids in 1872, and the Draconids in 1933. A display of many meteors from the same swarm is called a **meteoric shower** (Figure 206).

The words "swarm" and "dense" are far from applicable if taken with their usual connotation. Even when meteors appear on the same path at the rate of four or five a second, they must still be several miles apart, and it is only by hyperbole that pinheads or dust specks several miles apart can be said to constitute a dense swarm.

If the swarm is large, the Earth may require several days to cross it, so that a shower may last several nights; but since the Earth can encounter a given swarm at only one place in its orbit, the shower cannot be repeated oftener than once a year.

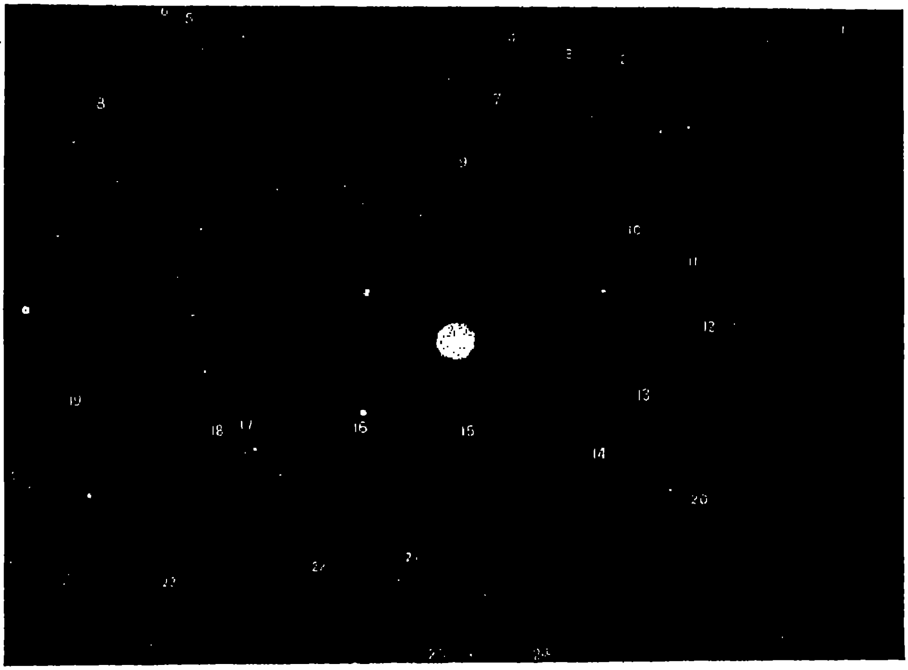


Fig. 206. *Meteors of the Draconid Shower of 1933 October 9, Photographed by F. Quénisset at Observatoire Flammarion, Juvisy, France. The great star in the center is Vega, and above it and at left is $\epsilon$ Lyrae. Note the divergence of the trails from the radiant.*

**Orbits of Meteor Swarms.** The position of the radiant is determined by the direction and magnitude of the relative velocity of the meteors and the observer, and this depends both upon the orbital and diurnal motions of the Earth and upon the orbital motion of the meteors. The motions of the Earth can be readily allowed for, and from the resulting motion of the meteors the elements of their orbits can be computed. Meteor swarms have thus been found to move around the Sun in elliptic orbits, and many of these have been proved to be identical with the orbits of known comets, thus establishing a close connection between the two classes of bodies.

The first such identification was made by Schiaparelli in 1866, between the Perseids and Tuttle's comet of 1862. A famous case is that of Biela's comet and the Andromids. The comet, discovered in 1826 by Biela, was shown to move on an elliptic orbit with a period of about 6.6 years, and was identified with a comet seen as early as 1772 by Montaigne and in 1805 by Pons. There was nothing especially remarkable about it until 1846, when it divided in two. When next seen in 1852 the two components had moved farther apart, and they have not been seen since; but in several years when the comet was due to appear, particularly in 1872, there have been notable showers of Andromids. Here the meteors seem to be products of the

comet's disintegration; in some other cases the comet seems to be accompanied by the meteors, being the most conspicuous body of the aggregation.

Table 16

METEOR SWARMS

| Name | Approximate Date of Shower | Period | Appearance of Meteors | Associated Comet |
|---|---|---|---|---|
| Lyrids ............... | April 20 | 415 years | Swift | 1861 I |
| Eta Aquarids........ | May 4 | 76 | Swift, few | Halley's |
| Perseids............. | August 10 | 120 | Swift, bright, with trains | 1862 III |
| Draconids............ | October 9 | 6.6 | Faint, brief shower | 1900 III |
| Orionids.............. | October 22 | 76 | Swift, few | Halley's |
| Leonids............... | November 14–15 | 33.3 | Very swift, blue | 1866 I |
| Andromids........... | November 17–27 | 6.6 | Slow, with trains | Biela's |
| Geminids............. | December 11–12 | 1.8 | Swift, white, short | |

The most important meteor swarms are listed in Table 16. The Perseids are evidently distributed fairly equally in a ring which extends all the way around their orbit, for a noticeable shower occurs every year. The same may be said of the scanty Orionids and Eta Aquarids which, according to Olivier, are manifestations of a single swarm moving in the orbit of Halley's comet; their orbit is so placed that we encounter them both before and after their perihelion passage. While a few stragglers of the other swarms also are seen each year, the main shower generally occurs only once in the meteors' period, showing that these swarms are more compact.

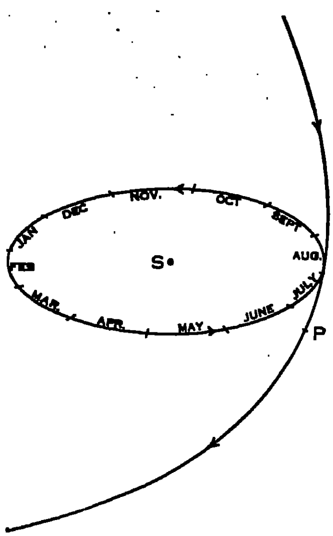



Fig. 207. *Orbits of the Earth and the Perseid Meteors, Projected on the Plane of the Meteor Orbit.*

**Discovery and Designation of Comets.** Large comets which come near the Sun and the Earth are spectacular objects, some of which have been visible in broad daylight and readily command attention. Records exist of about four hundred which appeared before the invention of the telescope. The number discovered since that event is greater, but most of these later comets have been faint. In most cases neither the time nor the place of their first appearance can be pre-

dicted, and they are usually found by searchers with small, wide-field telescopes, or, in recent decades, upon celestial photographs.

Fig. 208. *Donati's Comet of 1858, Drawn by G. P. Bond.* (Annals of Harvard College Observatory, *Volume III.*)

The news of such discoveries is communicated to the principal observatories by telegraph and cable through central stations at Harvard and Copenhagen. When a comet is found, its right ascension and declination at a designated time on the night of discovery are telegraphed to the central station, which relays the news to the subscribing observatories. Measurements of the comet's position are then made and communicated by various observers, and as soon as positions on three nights are made known a preliminary orbit is computed and the elements and a short ephemeris are sent in. This prompt procedure has probably saved many comets from being "lost."

In the preliminary computation the orbit is assumed to be parabolic, for this assumption simplifies the work and is usually accurate enough at least to provide a "finding ephemeris" good for several days. When more observations have accumulated, they are used for correcting the preliminary elements to obtain a more accurate orbit. For the determination of a parabolic orbit the method developed by Olbers in the early part of the nineteenth century is probably the most convenient; but for an elliptic orbit, and especially for modifying a preliminary parabolic orbit to agree with later observations, the method developed by Leuschner from the older method of Laplace is often shorter and better. Much of the computation of the orbits of

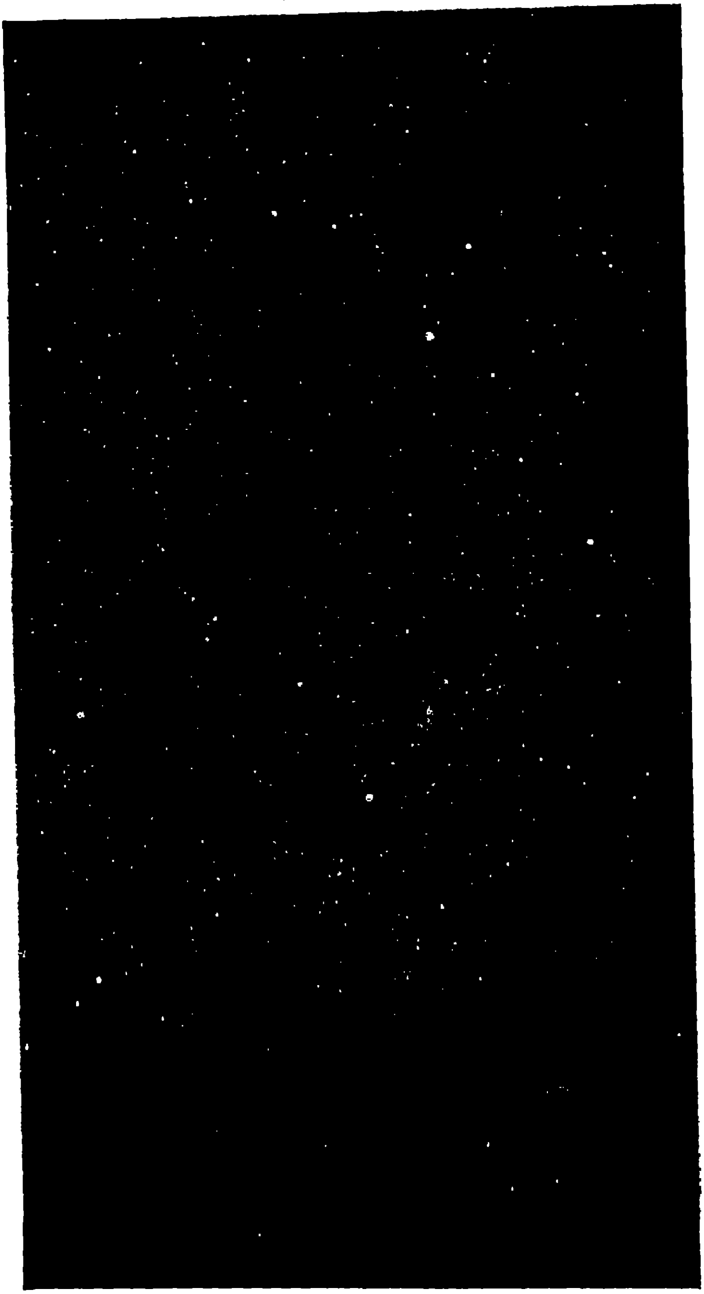


Fig. 209. *Tail of the Great Comet a 1910, Photographed by Lampland at Lowell Observatory. Length about 20° or 40,000,000 miles. The head was concealed by the pine forest west of the Observatory.*

comets and asteroids is done at the Students' Observatory of the University of California, of which Leuschner was the first Director.

Comets are temporarily designated by a letter and year number in the order of their discovery; later, when the elements of the orbit have been well determined, the letter is superseded by a Roman numeral following the year number, the order being that of perihelion passage. A comet is often known also by the name of its discoverer or of a person who has made an important research upon it. Thus Morehouse's comet, Comet *c* 1908, and Comet 1908 III are identical.

**The Orbits of Comets.** After Tycho had proved that comets are more distant than the Moon (page 290), Kepler made observations of two comets of his time, and came to the conclusion that they moved freely through the planetary orbits, with a motion not far from rectilinear. Hevelius ("a noble emulator of Tycho Brahe," as Halley calls him), supposing that Kepler was right, "complained that his calculations did not agree perfectly with the matter of fact in the heavens," and

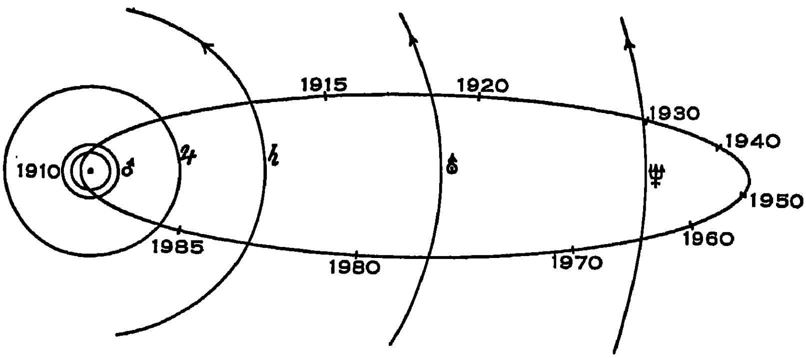



Fig. 210. *The Orbit of Halley's Comet.*

became aware that the path of a comet was curved toward the Sun. In 1680 appeared a great comet whose orbit was shown by Doerfel and by Newton to be a parabola.

Later, by using the gravitational principles developed by Newton, and "by a prodigious deal of calculation," Halley determined parabolic orbits for twenty-four bright comets which had appeared from 1337 to 1698. Noticing that the orbits of the comets of 1531, 1607, and 1682 were very similar, he concluded that these comets were the same, and that the orbit was really a long ellipse with a period of about seventy-five years. He accordingly predicted the return of this comet in 1758 and, although he did not live to see it, his prediction was fulfilled; the comet, which has ever since been famous as Halley's comet, was seen again promptly in 1835 and 1910. By taking account of planetary perturbations, Cowell and Crommelin have been able to determine its path with such accuracy as to identify it with recorded comets at every appearance save one since 240 B.C. Figure 210 is a diagram of its orbit with the distances of the planets shown for comparison. The actual orbit is inclined about 17° to the plane of the ecliptic.

About three-fourths of all the cometary orbits which have ever been computed appear to be parabolic. If they are strictly so, the mathematical significance is that these comets fall from rest at an infinite distance from the Sun, pass around it once, and recede again to infinity. Since, however, the parabola is the limiting case between the ellipse and the hyperbola, it is highly probable that the orbits which seem parabolic are either slightly elliptic or slightly hyperbolic; and since no distinctly hyperbolic orbits have been found, astronomers are mostly of the opinion that the orbits of these comets are really long, eccentric ellipses with periods of many centuries or millenniums.

Some of these comets have very close perihelia; for instance, the great comets of 1668, 1843, 1880, 1882, and 1887, the orbits of which were similar to one another, passed within a few hundred thousand miles of the Sun's surface and must have gone right through the corona. Their periods are at least several centuries, but they swung through 180° of their orbits near perihelion in a few hours, moving at a speed of 300 miles a second or over. Near aphelion, where such comets spend most of their time, their motion is quite slow. The majority of observed comets have perihelia within the orbit of the Earth, and all but one[1] approach more closely to the Sun than the orbit of Jupiter; but it is very likely that many comets have greater perihelion distances and escape detection for this reason, for, in general, comets are very inconspicuous objects until they are well within the orbit of Mars.

Many comets are known definitely to move in elliptic orbits with periods less than a century. Most of these are faint, Halley's comet being the only "great comet" among them. The shortest known period is that of Encke's comet, about 3.3 years. About fifty periodic comets have their aphelia at about the distance of Jupiter from the Sun, and are said to belong to Jupiter's comet family. Saturn and Uranus have families of two comets each, much less certainly established, and Neptune has a family of seven, one of which is Halley's comet. Halley's comet, the comet of 1827 (which also belongs to Neptune's family), Tempel's comet of the Leonid meteors,

[1] The exception is Comet 1925 II, discovered in 1927 by Schwassmann and Wachmann, the orbit of which lies wholly between those of Jupiter and Saturn. It is the only comet that has been observed at its aphelion, which it passed in 1933. Its period is about 16 years and its orbital eccentricity is 0.14—less than that of Mercury. But for its hazy appearance and the fact that its diameter and brightness have varied capriciously—the latter a hundredfold in a few days—it might well have been classed as an asteroid. Still more asteroid-like in its motion is a little comet discovered by Miss Oterma in 1943, the orbit of which lies between those of Mars and Jupiter.

and Tuttle's comet of the Perseids all have retrograde motion; all the other short-period comets move directly. This is in contrast to the case of the parabolic comets, about as many of which have retrograde motion as direct.

The existence of so numerous a family of comets as that of Jupiter could not well have come about by chance. According to the **capture theory**, these comets moved originally in parabolic orbits which were changed to ellipses by the perturbative action of Jupiter. This would happen if Jupiter and the comet approached each other in such a way that Jupiter's attraction caused the comet to lose about half of its velocity. It was shown by Chandler by computations on the orbit of Comet 1889 V that in 1886 it had passed within the orbit of Jupiter's closest satellite, and that the action of Jupiter was such as to have reduced the comet's period from twenty-seven to six years. Barnard observed in 1889 that this comet was double, and that the two parts were separating at such a rate that they must have been together in 1886, at the time of the encounter. However, an approach to Jupiter is as likely to result in acceleration as in retardation of a comet, and acceleration of parabolic velocity would change the orbit to a hyperbola. Crommelin shows on the theory of probability that only one comet in half a million would pass close enough to Jupiter to have its velocity reduced by half, and he is inclined to agree with a theory proposed by Proctor, who suggested that the Jupiter comets originated in the planet, having been emitted by volcanic or analogous action.

**Dimensions, Mass, and Density of Comets.** Comets are the bulkiest objects in the Solar System. The nucleus, when one is present, may have any diameter up to ten thousand miles; the coma, a diameter from ten thousand to several hundred thousand miles; and the tail, a length of many million miles. The tail of the great comet of 1843 was said to be more than five hundred million miles long—longer than the radius of the orbit of Jupiter.

The mass of comets, on the other hand, is so small that it has never been detected at all, for comets have no perceptible perturbative action on other bodies. Even when Comet 1889 V went through Jupiter's satellite system it produced no appreciable derangement of the satellites. It is fairly certain that no comet's mass exceeds one one-hundred-thousandth the mass of the Earth.

The mean density of a comet must accordingly be exceedingly low—certainly thousands of times lower than that of air at sea level. It is therefore not surprising that comets are transparent. Faint stars are often seen through the heads of comets thousands of miles in diameter. This low mean density does not signify, however, that a comet is a tenuous gas throughout; it is more likely that the head, at least, contains much solid matter in the form of small bodies like meteors, separated by distances

which are large compared to the diameters of the bodies, probably with highly rarefied gas between.


Fig. 211. *Spectrum of the Head of Cunningham's Comet,* c *1940, Photographed at the McDonald Observatory by Polidore Swings.*

**The Spectra of Comets.** The spectrum of a comet which is faint and distant from the Sun appears to be continuous and is probably due wholly to reflected sunlight. As the comet nears the Sun, bright bands make their appearance and increase in intensity as the distance diminishes. The brightest bands are due to molecular carbon ($C_2$) and are identical with those of the Swan spectrum seen in the blue light of a Bunsen flame. They have sharply defined edges or "heads" at λλ4737, 5165, and 5635, and each is composed of a multitude of fine lines which grow fainter toward the violet (Figure 211). Other bands have been identified with nitrogen, carbon monoxide, cyanogen, and certain molecules which, under terrestrial conditions, are not chemically stable. The ionized molecules of nitrogen and carbon monoxide predominate in the tail (Figure 212).

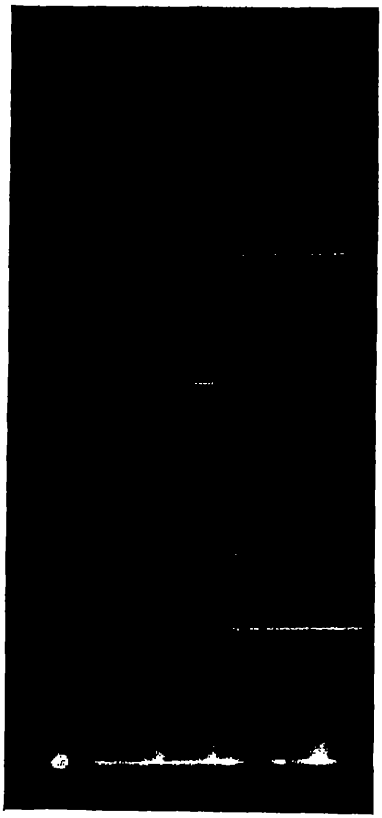


Fig. 212. *Objective-Prism Spectrogram of Halley's Comet, by V. M. Slipher, 1910.*

The emission bands in comet spectra appear when the comet is at a heliocentric distance far too great for them to be caused directly by the Sun's heat, and the variation of their intensity with distance is also out of accordance with such a cause. Evidently, the process is one of fluorescence, the energy of the Sun's radiation being absorbed by the cometary material and reëmitted wholly in the particular wave lengths of the comet spectra. The problem has been treated theoretically by Zanstra, Swings, and others.

Great comets that approach very near the Sun show bright D lines of sodium (Figure 213); this is the only *atom* that has been recorded with certainty in comets (though bright lines of iron and other metals were suspected to have appeared briefly in the spectrum of the great comet of 1882). The great comets of 1910 and 1927 shone with a beautiful golden color because of their copious emission of sodium light. In the case of the latter comet, this color so enhanced its contrast with the blue sky that the comet was easily observed and even photographed at the Lowell Observatory at midday within a few degrees of the Sun.

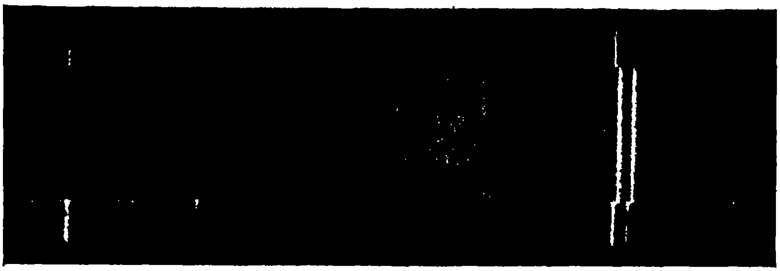


Fig. 213. *Slit Spectrogram of the Great Daylight Comet of 1927, by V. M. Slipher.*

The objective-prism spectrogram of Halley's comet (Figure 212) was made with the edge of the prism placed parallel to the comet's tail, so that the tail itself may be said to have served as a slit. Each image of the double-tailed comet is thus a bright spectral line or band. The tailless image of the head at the left end of the spectrum is due to cyanogen, which evidently did not extend into the tail. The bright horizontal streaks are the spectra of stars. The spectrogram of the great comet of 1927 (Figure 213) was made in full daylight, early in the afternoon of December 19, the day after perihelion passage. The comet was then some 20,000,000 miles from the Sun, but so nearly in the Sun's direction that it would have been lost in the bright sky except for its yellow color and great brilliancy (several times the brilliancy of Venus). Most of the broad central spectrum shown in the photograph is due to sunlight reflected from the sky, but the bright central D lines belong to the comet. Their displacement relative to the corresponding comparison lines is a Doppler-Fizeau effect due to the comet's recession at the rate of 90 km./sec.

**The Nature of a Comet's Tail.** The tail of a comet does not trail behind it all the way, but extends always in a direction nearly opposite that of the Sun, lagging a little behind the prolongation of the radius vector (Figure 214). It cannot be a fixed appendage, for if so, the tail of

the comet of 1843, for example, must have traveled with a velocity comparable with that of light as the comet swept around the Sun. Photographic studies of numerous comets since 1890 have shown condensations in the tail which moved away from the head with velocities of several miles a second, thus proving that the tail is a flowing stream of matter. It is evident that the Sun exerts upon this matter a repulsion which is stronger than its gravitational attraction.

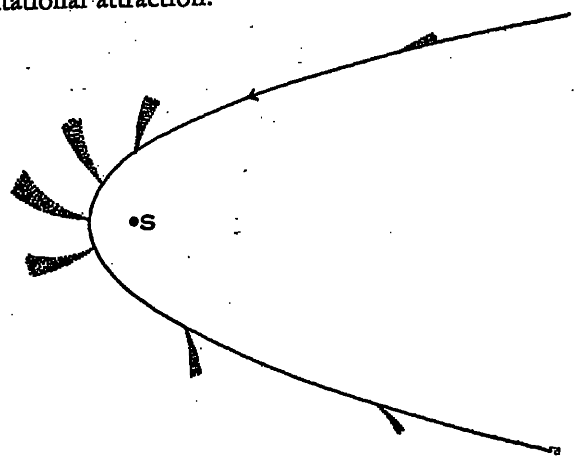



Fig. 214. *Positions of the Tail of a Comet.*

Probably this repulsion is due to the pressure of the Sun's radiation (page 172), and the particles of the tail must be very small. Radiation pressure, however, is not sufficient to explain all the motions of the matter within a comet. The nuclei of some great comets—for example, Donati's comet of 1858 (Figures 208, 215) and Morehouse's of 1908—have been surrounded at times by a number of envelopes of approximately parabolic form which were observed to dilate as if composed of matter expelled in all directions from the nucleus, the particles sent toward the Sun being driven back by the Sun's repulsion. The pressure due to the radiation of the nucleus is certainly not powerful enough to expel these envelopes, and it is more likely that the repulsive force here is electrical, the particles being ionized by the action of the Sun.

**The Disintegration of Comets.** The material which flows outward in the tail of a comet can never rejoin the head. Being repelled by the Sun, each particle must travel on a hyperbolic orbit which is convex to the Sun and has the Sun in its outer focus, while the main body continues in its elliptic or parabolic path which is concave to the Sun. The tail-forming material of a comet must therefore waste away, although there is no certain observational evidence that the tails grow smaller or fainter at successive returns.

Fig. 215. *Head of Donati's Comet, Drawn by G. P. Bond at 15-Inch Refractor.* (Annals of Harvard College Observatory, *Volume III.*)

In addition to this wastage, a comet that comes very near to the Sun or to a large planet is subjected to enormous disruptive tidal forces which must be very effective in separating its loosely connected parts. Biela's comet and the fifth comet of 1889 are known to have divided in two; the great comet of 1882, after its close perihelion passage, was attended by a number of smaller cometary bodies and its nucleus was divided into five. There is evidence also that comets have been disarranged by encounters with invisible obstacles, probably meteor swarms, as was the case of Brooks's comet of 1893, photographed by Barnard. This disintegration of comets affords a plausible explanation of the origin of meteor swarms.

**Encounters of the Earth with Comets.** A direct collision of the Earth with the head of a comet is not at all impossible, since many comets cross its orbit. The necessary condition is that the comet and the Earth arrive at the node at the same time. It has been computed from the theory of probability that such a collision is likely to happen about once in 15,000,000 years. Passages of the Earth through the tails of comets should be much more frequent since the tails are of such immense size.

On May 18, 1910, the head of Halley's comet crossed the plane of the Earth's orbit directly between the Earth and the Sun, thus performing a

transit like the transits of Venus and Mercury and changing from a morning to an evening object. However, it was not seen against the Sun. Its distance from the Earth was about 15,000,000 miles, and the tail is known to have been considerably longer than this and to have extended nearly in the Earth's direction. On the mornings of May 17 and 18 the tail, though faint, was a magnificent object which extended like the beam of a searchlight across the sky a distance of about 120°. A broader and fainter secondary tail involved a long arc of the ecliptic, and there is little doubt that the Earth passed through this secondary tail soon after the transit of the head. Nobody was inconvenienced, although certain newspapers had caused some apprehension by describing the possible dire effects of the poisonous gases of the tail. It is, in fact, inconceivable that so rare a gas could do any harm. The only effect observed near that time which might be attributed to the comet was a slight iridescence of the sky noticed by Barnard and others during daylight on May 19. A similar encounter with a comet took place on June 30, 1861. An aurora was observed, but there is no special reason for connecting it with the comet.

## EXERCISES

1. The Earth passes through about 80,000,000,000,000 cubic miles of space every twenty-four hours. Verify this statement. If it sweeps up 80,000,000 meteors per day, how much space, on the average, is allotted to each meteor?

*Ans.* 1,000,000 cubic miles

2. Suppose the Sun's repulsive force upon a particle in a comet's tail just equals the Sun's gravitational attraction. What path does the particle pursue relative to the Sun?

*Ans.* A straight line which is tangent to the comet's orbit at the point where the particle left the head

3. In a very eccentric orbit such as that of the comet of 1882, the aphelion distance is practically twice the mean distance. What is the greatest distance from the Sun attained by such a comet if its period is 1000 years?

*Ans.* 200 astronomical units

# CHAPTER 14

## THEORIES OF THE EVOLUTION OF THE SOLAR SYSTEM

**Regularities of the Planetary System.** So far as the law of gravitation is concerned, a planet might revolve in any direction in any plane passing through the Sun, and in a conic section of any size and of any form from an ellipse of zero eccentricity (circle) to an infinitely eccentric hyperbola (straight line). The character of its orbit depends only on the speed, direction, and distance from the Sun at which the planet was launched. If, then, the world were a "fortuitous concourse of atoms," we should expect a great variety and even a great confusion among the planetary orbits. It is true that there would be no hyperbolic or parabolic orbits of bodies belonging permanently to the system; but the elliptic orbits would be of a great range of size and eccentricity, their planes would be inclined to each other at a great variety of angles, and about as many planets would revolve in one direction as in the other.

Instead of the random arrangement which would thus exist if the bodies of the Solar System had been brought together by chance, we observe a neat and orderly system which is characterized by the following regularities:

1. The system is relatively compact, the distance of the nearest star being thousands of times that of the farthest planet.
2. The orbits of all the principal planets and of the great majority of the asteroids lie nearly in the same plane.
3. All the planets and asteroids revolve around the Sun in the same direction.
4. The orbits of all the principal planets except Pluto and of most of the asteroids are nearly circular.
5. The Sun rotates in the direction of the planets' revolution.
6. The plane of the Sun's equator lies near the planes of the planets' orbits.

7. There is a fair degree of regularity in the distances of the planets, expressed approximately by Bode's law.

8. The principal satellites of several of the planets form, with their respective primaries, systems which possess regularities similar to those of the planetary system itself.

Men of science attribute this remarkably simple and orderly state of the planetary system to the working of some process of evolution.

**The Meaning of Evolution.** The word **evolution** means an unrolling or unfolding. As used in modern science, it means usually a passage from unorganized simplicity to organized complexity. An example is the development of the semi-fluid, relatively homogeneous contents of an egg into a highly organized and complex animal.

To the ancients, the world consisted of a stationary Earth and a rotating firmament which carried stars fixed on its surface, and Sun, Moon, and planets which followed definite, reëntrant curves. Such simple motions as were observed among the heavenly bodies repeated themselves indefinitely. Even after Copernicus's description of the Solar System was accepted, the motions of the heavenly bodies were seen to be so orderly and the system of the world seemed so permanent that men long assumed that the universe had been created in its present perfect state at some epoch in the not inconceivably remote past and that it would continue in this state until its final cataclysmic destruction at the hand of its Creator. However, the smaller works of nature with which we are familiar come to maturity not by sudden creation but by a process of growth; and many of the facts that have been set forth in the preceding pages lead astronomers to believe that in the universe at large the order is that of gradual, ceaseless change—of evolution.

A theory of evolution is not a theory of creation. To say that stars have evolved from a nebula or that a nebula has originated in the close approach of two stars merely postulates the existence of an earlier state of the material in question which is different from the present state. The hypotheses which will be mentioned in the following pages make no attempt to explain the ultimate origin of the material.

**Growth of the Theory of Evolution.** Thomas Wright's theory of the Milky Way (page 444) turned the attention of the brilliant German philosopher Kant to the subject of the origin of celestial systems, and in 1755 he published a book in which he advocated the theory that the Solar System had evolved from a nebula. Kant's work, however, attracted little attention and had no important influence on science.

In 1796 the great French mathematical astronomer Laplace published a work on general astronomy entitled *Exposition du Système du Monde*, at the end of which, in the last of seven brief appended notes, he outlined a hypothesis of the evolution of the Solar System which became famous as the Laplace **Nebular Hypothesis.**

Laplace's hypothesis is in many ways similar to Kant's, but there is no reason to believe that it was inspired by Kant, for Laplace states that the only previous attempt to account for the origin of the planets with which he was acquainted was that of Buffon, who had suggested their genesis in a collision of a comet with the Sun—a suggestion which Laplace disposes of in three paragraphs. The greater influence of Laplace's hypothesis is doubtless due in part to the great prestige of his mathematical work and in part to its having appeared just after the French Revolution, when his country was prepared for the introduction of new ideas.

Though presented by its author, as he said, "with that diffidence which everything ought to inspire which is not the result of observation or of calculation," the Nebular Hypothesis had an incalculable effect upon the scientific, philosophic, and religious thought of the following century. It furnished geologists with an account of the pre-geologic history of the Earth and encouraged them to view the changes which the Earth had undergone as continuous rather than cataclysmic. Biologists came to view the different forms of life, which are made evident by fossil remains in the strata of the Earth's crust, as forming a continuous chain from the lowest (simplest) animal or plant to the highest yet evolved—which may be man. These ideas were given definite form in Charles Darwin's book *Origin of Species*, published in 1858, which stirred up a storm of opposition on theological grounds that persisted into the twentieth century.

During the nineteenth century, as new information accumulated, the Nebular Hypothesis was modified or added to by a number of investigators. In 1900 the astronomer Moulton and the geologist Chamberlin, both of the University of Chicago, offered as a substitute for the Laplace hypothesis the **Planetesimal Hypothesis,** according to which the Solar System originated in a spiral nebula which itself was engendered by the close approach of two stars. More recently, the Englishmen Jeans and Jeffreys have proposed what they call the **tidal theory,** which resembles the planetesimal theory in postulating a close approach of two stars, but differs in the subsequent details of the development of the planets and satellites. The planetesimal and tidal theories are sometimes referred to as hypotheses of **dynamic encounter.**

The student should bear in mind that none of these theories rest on any such secure foundation of observation or calculation as do most of the results of astronomical research. The changes in the universe for which it is their purpose to account took place so many millions of millions of years ago that it is doubtful that any such theory can ever be securely established. It is undeniable that no theory of inorganic evolution exists which is accepted by any large group of astronomers today; but the importance and fascination of the subject are so great, and such earnest attention has been given it by leading thinkers, that the student should not be left without some knowledge of what has been attempted.

**The Laplace Nebular Hypothesis.** Laplace reasoned that, to impart to the planets the motions which they now possess, the Sun must once have been surrounded by an atmosphere which, because of its excessive heat, extended to a distance equal to that of the farthest planet. The body would then have resembled some of the nebulae which appear as bright nuclei surrounded by nebulosity. He imagined that, previous to this state, the nucleus was less luminous and the nebulosity more diffuse, and so, reasoning backward, he arrived at a state wherein the nebulosity was so diffused as to be invisible. He suggested that every star had evolved by the condensation of such a nebula under the mutual gravitation of its parts, and that double stars and groups like the Pleiades had been the product of condensation around two or more nuclei.

As the nebula contracted, any rotatory motion which it possessed must have increased in swiftness, and so the centrifugal force at the equator must have increased. Laplace conjectured that the nebula would flatten into an oblate spheroid and that, as the centrifugal force grew still greater, there would come a time when it equaled gravity, and a ring of matter would then be left in equilibrium at the equator while the main body continued to shrink inside it.

The ring thus formed would probably be denser or thicker at one point than at others, and the material here would gather to itself, by gravitational attraction, the material in the remainder of the ring and form a planet. The main body would meanwhile go on contracting and might abandon other rings to form other planets, the final globe being the Sun.

Any planet thus formed would probably be given a rotation by the oblique impact of some of the particles of its ring; it would contract as it cooled, and might abandon rings of its own, which might form satellites in a miniature system or might remain in the form of rings such as those actually seen around Saturn.

The Laplace hypothesis is in agreement with many of the observed facts of the Solar System, but there are some that it fails to explain and there are also serious theoretical difficulties. The origin of the Moon, with its great mass and its high inclination, and perhaps also that of the inner satellite of Mars, which revolves more rapidly than its planet rotates, are better explained on the tidal theory of G. H. Darwin (page 256). The retrograde motion of the tiny outer satellites of Jupiter and Saturn cannot be accounted for by the Laplace theory. No nebulae such as Laplace postulated are known, but that fact is not strong evidence against the hypothesis, for if such a nebula were at a distance of 200 light-years or greater, and were no larger than Neptune's orbit, its disk would be less than 1″ in diameter and it might easily have escaped notice.

Probably the most serious difficulty is the moment of momentum or, as it is now more often called, the angular momentum. This quantity may be defined as follows: Suppose a particle of mass $m$ moving with velocity $v$ along a line from which the perpendicular distance of a given point or center is $p$. The **angular momentum** of the particle with respect to the point is the product $mvp$. A rotating body or system may be considered as made up of particles. The angular momentum of the entire system is the sum of the moments of all its particles taken with respect to the axis of rotation, and is represented by the summation $\Sigma mvp$. The angular momentum of the present Solar System can easily be computed, and so can that of the hypothetical nebula at the time when it abandoned the ring which developed into Neptune. It is a principle of dynamics that in a system which is not acted on by outside forces the total angular momentum is constant. If, for a given particle of mass $m$, $p$ diminishes, $v$ must increase; this is in accord with Laplace's theory that the nebula rotated faster as it contracted. But the angular momentum of the present system is less than 1/200 that of the hypothetical nebula before Neptune was born, and, moreover, the planet Jupiter now has more than 95 per cent of the total amount of angular momentum, which is inconsistent with an earlier uniform distribution. Pluto has but little bearing on the question because of his small mass.

**The Planetesimal Hypothesis.** According to the hypothesis of Chamberlin and Moulton, the Solar System owes its present state to the close approach of two stars which swept toward each other with a speed greater than the parabolic velocity (page 242), so that the relative orbit was a hyperbola and their association was brief. We are concerned only with what happened to one of the stars, which is now the Sun and which we shall call $S$; the other may be called $S'$. In the vicinity of $S$ there may have been a quantity of scattered material, some of it ejected from the Sun, and $S$ was, perhaps more than at present, in a condition to eject more on slight provocation. As $S'$ approached, it raised enormous tides on $S$ and so encouraged the ejection of material in two opposite directions, toward and away from $S'$; then as $S'$ swept rapidly through perihelion, its attraction for this material acquired a component at right angles to the line $SS'$ and imparted angular momentum to the particles.

To make this clear, let $S_1'$ (Figure 216) be an early position of $S'$ on its hyperbolic relative orbit, and let $P$ and $P'$ be two particles ejected from $S$ in directions toward and away from $S_1'$. These particles have departed almost radially from $S$, and if left to themselves would fall directly back like a bullet shot vertically into the air. But $S'$ continues swiftly in its orbit and soon occupies the position $S_2'$. Its attraction upon $P$ and $P'$ will then be in the directions $PA$ and $P'A'$ respectively, and its effect upon the motion of these particles relative to $S$ may be determined by the kind of reasoning which we employed in discussing perturbations (page 245). Thus, draw $PC$ and $P'C'$ equal and parallel to $SB$, the acceleration produced by $S'$ in $S$. Complete the parallelograms $PCAD$ and $P'C'A'D'$ upon the diagonals $PA$ and $P'A'$, taken of such length as to represent the accelerations produced by $S'$ in $P$ and $P'$. The sides $PD$ and $P'D'$ will then represent the perturbative action, and since $PA > SB > P'A'$, $PD$ must be directed toward a point between $S$ and $S'$ while $P'D'$ is directed to a point beyond $S$. That is, the action of $S'$ in its new position is to give to the matter earlier ejected from $S$ an angular momentum in the direction of the orbital motion of $S'$. $S'$ moved rapidly away from the scene of action, and is now indistinguishable from other stars if, indeed, it is visible at all; and $P$ and $P'$ were left revolving in elliptic orbits under the attraction of the Sun.

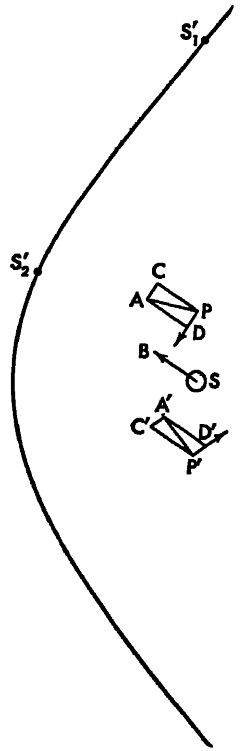


Fig. 216. *Tidal Effects of an Encounter of Two Stars.*

As $S'$ swung around the Sun, material was ejected from $S$ at frequent intervals or continuously, and all the particles were acted upon in the way described above but in different directions and by different amounts. In Figure 217 the curves $S_1$ and $S_1'$ represent the paths followed by the particles $P$ and $P'$, and the curves $S_2$ and $S_2'$, $S_3$ and $S_3'$ . . . represent the paths followed by particles which were ejected later. The dotted ellipse is the orbit finally pursued by particle No. 3 after $S'$ had receded so far that its influence was unimportant. The heavy, dotted spiral arms represent the locus of all the particles at an epoch closely following the encounter of the two stars. The result of this encounter, according to the theory,

was thus to produce from the debris ejected from the Sun a double-armed spiral of a form (but not size) which is common among the extra-galactic nebulae (Chapter 21).

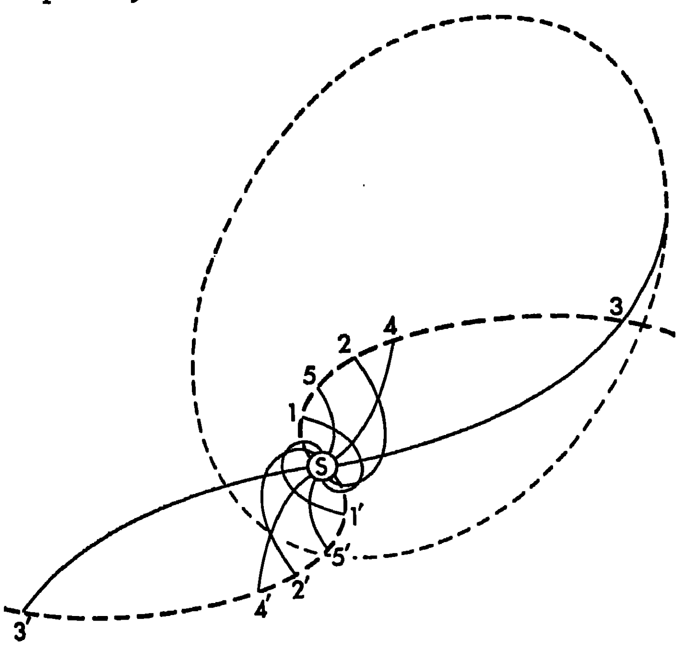



Fig. 217. *Formation of a Spiral According to Chamberlin and Moulton.*

The motion of the particles was not along the spiral arms but at large angles with them, and the orbits of the numerous particles crossed each other, also at large angles. Collisions were inevitable; and as the particles collided, their attraction held them together to form **planetesimals** which grew by accretion in further collisions until they formed the planets as they now exist. It may be shown that one effect of the collisions would be to decrease the eccentricity of the orbits of the nuclei, and thus is explained the present approximate circularity of the planetary orbits. Not all the planetesimals were swept up, and the process is still going on (at a greatly diminished rate), as evidenced by the meteors which daily enter the atmosphere of the Earth.

The angular momentum imparted to some of the material was insufficient to make it clear the Sun at the next perihelion passage, but in falling back it struck slantingly, thus giving the Sun itself an angular momentum in the direction of the motion of $S'$. This persists in the Sun's rotation and equatorial acceleration. The fact that the Sun's equator is somewhat in-

clined to the orbits of the planets is explained by supposing that $S$ had already a rotation about a different axis, which was modified but not completely obliterated by the new angular momentum.

The planetesimal hypothesis attributes the satellites to secondary nuclei associated with the primary nuclei from the beginning or captured by them later; views the comets as survivors of an earlier system of planetesimals, or as matter ejected from the visiting star, or as matter ejected from the Sun on other occasions and prevented from falling directly back by the perturbations of other stars; and gives plausible explanations of the asteroids, the zodiacal light, and the rotation of the planets.

It should be emphasized that, though the hypothetical result of the encounter of $S$ and $S'$ was a spiral aggregation similar in shape to many known spiral nebulae, it was not even remotely similar to them in size. If any spirals of the type described by Moulton and Chamberlin exist, they are, like any possible Laplacian nebulae, too small to be recognized as such.

**The Tidal Hypothesis.** According to Jeffreys and Jeans, the tidal action of the visiting star $S'$ caused two long filaments of gas to be shot out on opposite sides of the Sun, and the perturbing action of $S'$ upon the particles of this gas started them revolving in eccentric ellipses and may have caused the filaments to curve into the form of the letter $S$, similar to the arms of the spiral nebula NGC 7479 (Figure 308). (Jeans finds it possible that only one filament was ejected, and the subsequent theory works just as well with only one.) The filament was thickest and densest near its middle, that part having been formed when $S$ and $S'$ were nearest each other. Condensations formed in the ejected gas, and the smaller condensations soon dissipated into surrounding space while the larger increased by attracting to themselves the material around them, and the filament broke up into detached masses which later condensed into planets. Jupiter and Saturn were formed from the middle of the filament and are therefore the largest. The orbits of the planets were rounded up, as in the planetesimal hypothesis, by the resistance of the scattered material through which the planets moved. At its first perihelion passage, each gaseous planet was affected by the tidal action of the Sun as the Sun had been acted upon by the star $S'$, with similar results—the planets gave birth to satellites as the Sun had brought forth the planets. The particular case of the Earth-Moon system, however, is admitted to be best explained by the tidal hypothesis of Darwin (page 256).

Spitzer has pointed out the fact, strongly opposed to the tidal and

Laplace hypotheses alike, that if gas is released from the gravitative control of a dominant mass such as the Sun it must expand instead of condensing into a planetary body.

**Age of the Solar System and of the Earth's Crust.** There is no doubt that the Solar System is exceedingly old. Jeffreys has based an estimate of its age on the eccentricity of the orbit of Mercury, assuming that this orbit was rounded up by the resistance of scattered material, the remaining relic of which is the material of the zodiacal light. He reasons that the time needed for this rounding up is about the same as that required for the disappearance of the resisting medium; for, if the latter interval were much the shorter, the medium would have gone before the eccentricity was much reduced, while if it were much the longer the eccentricity would now be practically zero instead of its actual value of 0.2. He concludes that the time elapsed since the encounter of the two primeval stars is between 1000 millions and 10,000 millions of years.

Various methods have been employed for estimating the age of the Earth's crust. The earliest of these was suggested by Halley in 1715 and is based on the saltness of the sea.

The water carried to the sea by rivers contains a minute quantity of salt. The total amount of water in the sea remains practically constant because it evaporates as fast as it is delivered. The vapor thus produced condenses in the form of rain, to be again carried to the sea by the rivers. During the evaporation, however, the salt is left behind, so that the salinity of the oceans must be slowly increasing. If we knew the rate at which the salt is carried by all the rivers, the present saltiness of the ocean would permit a calculation of its age. Of course, our knowledge of this rate is very uncertain, but modern data, according to Jeans, suggest that the oceans must be many hundreds of millions of years old.

Probably the most reliable method is that of Russell, which is based on the study of **radioactivity**, the marvelous process by which the heavy elements uranium and thorium slowly and spontaneously disintegrate into elements of lower atomic number. The final product of this disintegration is lead. Uranium lead is an isotope that can be distinguished from ordinary lead by its atomic weight, which is exactly 206, whereas that of ordinary lead is 207.18. In their natural places in the rocks, neither of these substances—uranium and uranium lead—is ever found without the other; and so it is reasonable to assume that all the uranium lead has been produced from the uranium since the rocks were formed. The rate at which uranium is transformed into lead is known: one per cent of any given quantity of uranium is thus transformed in 66,000,000 years. The ratio

of the amount of uranium lead to the amount of uranium found in ancient rocks indicates an age of the Earth's crust of over 1300 millions of years, which is of the same order of magnitude as Jeffreys's estimate of the age of the Solar System.

**Bibliography.** Those who wish to pursue the subject of this chapter in detail may find material in the following books:

F. R. Moulton, *An Introduction to Astronomy*. New York, The Macmillan Company, 1916 (second edition).

F. R. Moulton, *Astronomy*. New York, The Macmillan Company, 1931.

T. C. Chamberlin, *The Two Solar Families*. The University of Chicago Press, 1928.

J. H. Jeans, *Astronomy and Cosmogony*. Cambridge University Press, 1929 (second edition).

J. H. Jeans, *The Universe Around Us*. Cambridge University Press, 1944 (fourth edition).

H. Jeffreys, *The Earth*. Cambridge University Press, 1929 (second edition).

H. N. Russell, *The Solar System and Its Origin*. The Macmillan Company, 1935.

# CHAPTER 15

# THE STARS—THEIR CLASSIFICATION, DISTANCE, AND BRIGHTNESS

**Star Catalogues.** The stars are so numerous that progress in our knowledge of them depends largely upon statistical studies. For this purpose, it is necessary that the results of observations of many stars be gathered together in a single work. A **star catalogue** is a list of stars giving for each its apparent position (usually expressed as right ascension and declination), its magnitude, and such other particulars as may serve the special purpose of the compiler.

The earliest star catalogue is believed to have been made by Hipparchus about 150 B.C. No copy of it is now in existence, but the catalogue of Ptolemy, contained in the Almagest, is supposed to be derived from it. Ptolemy's catalogue contains 1030 stars and gives their magnitudes, their latitudes and longitudes, and their positions in the constellation figures. The latitude or longitude of some of the stars is in error by as much as a degree, yet the work is invaluable as a description of the ancient constellations. Other catalogues of about the same extent as Ptolemy's were made by the Persian Al-Sufi during the tenth century, by Ulugh Beg of Samarcand during the fifteenth, and by Tycho Brahe in 1580. A supplement to Tycho's catalogue, which extended it to the South Pole, was published by Halley in 1677. In 1601 Bayer, who introduced the Greek-letter designations of stars now so commonly used, published a set of maps of the constellations with a list of the stars in each constellation on the back. Hevelius, in 1690, published the first catalogue containing telescopic stars.

Modern star catalogues are very numerous and extensive. Most of them give the visual magnitudes of the stars, their right ascensions and declinations referred to a designated epoch, and values of the precession and sometimes of its rate of change; and some give other data, such as spectral type, photographic magnitude, etc.

One of the most important is that known as the *Bonn Durchmusterung* (abbreviation BD or DM) published by Argelander, which contains more than 324,000 stars down to magnitude 9.5 between the north pole and $-2°$

declination, arranged in zones corresponding to every degree of declination. The right ascensions are given to tenths of seconds of time and the declinations to tenths of minutes of arc, and all positions are referred to the equinox of 1855.0. This work is continued to declination $-22°$ in Schönfeld's *Southern Durchmusterung*.

The *General Catalogue of 33,342 Stars*, collated during many years at the Dudley Observatory (Albany, New York) and published by the Carnegie Institution in 1937, gives right ascensions to the thousandth of a second of time and declinations to the hundredth of a second of arc; also magnitudes, spectral types, and proper motions. The epoch of this catalogue is 1950.0. Its stars are distributed over the entire sky, from the north to the south pole.

The *Henry Draper Catalogue*, compiled by Miss Cannon of the Harvard Observatory and named in memory of an American pioneer in stellar spectroscopy, gives the spectral classification and the visual and photographic magnitudes of 225,000 stars in all parts of the sky, together with positions sufficiently accurate for their identification. A *General Catalogue of Double Stars*, published by Aitken of the Lick Observatory in 1932, contains all available information concerning 17,180 pairs north of declination $-30°$ and supersedes an earlier, similar catalogue by Burnham.

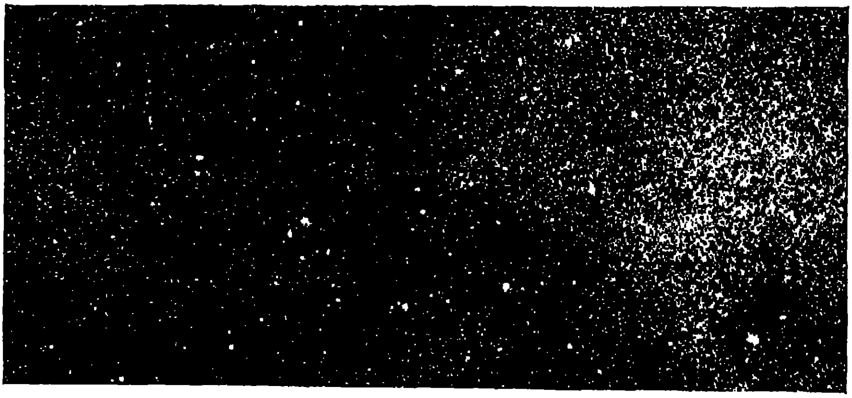


Fig. 218. *A Small Part of One of the Plates in Ross's* Atlas of the Milky Way.

**Photographs of the Sky.** More useful than star catalogues for some purposes, especially for the study of the fainter stars, are photographs of the sky, because each such photograph is a permanent record of the relative position and brightness of the stars within its boundaries at the time of observation. The greatest collection of originals of such photographs is

the one built up since 1885 by the staff of the Harvard College Observatory at its stations in Massachusetts, Peru, and South Africa, and stored at the headquarters of the Observatory in Cambridge, Massachusetts. Many other observatories also possess important accumulations. Photographic charts, consisting of prints from original negatives and made available to astronomers throughout the world, include the 206 *Franklin-Adams Charts* which cover the entire sky; the beautiful *Atlases of the Milky Way* by Barnard and by Ross; and the *Carte du Ciel*, a project begun in 1887 by eighteen coöperating observatories in various parts of the world and still unfinished. Connected with the last-named project is the *Astrographic Catalogue*, also unfinished, designed to give the positions of three or four million stars as measured on the original plates.

**Distant Suns.** Each star of the multitudes that adorn the sky is a hot, self-luminous ball of gas—a body of the same nature as the Sun. As seen from one of them, the Sun—or rather the whole Solar System—would itself appear as a star, in no way preëminent among the others; the planets and comets would be hopelessly lost in the light of their primary.

That the stars are suns is proved beyond doubt by their spectra; but long before the spectroscope was first used their sunlike nature was made evident by their great distances combined with their apparent brightness. In order to be seen at all at such distances, the stars must shine by their own light and must be comparable with the Sun in real brightness. The present chapter is devoted partly to the analysis of starlight by means of the spectroscope, partly to the determination of the distances of the stars, and partly to the classification of stars according to the intensity of their light.

**The Study of Star Light.** It is a noteworthy fact that our knowledge of the heavenly bodies has come to us entirely by way of a single sense—the sense of sight. We cannot touch them, or hear them, or taste or smell them. In the case of the stars, even the sense of sight is limited as compared with the bodies of the Solar System, for the latter are near enough to be observed in detail by means of the telescope, while the stars are so distant as to appear only as points. The stars, however, afford an advantage in being self-luminous; and the study of the quality and quantity of their light reveals many facts concerning their nature.

**Instruments of Stellar Spectroscopy.** On account of the faintness of the stars, a spectroscope to be used for their study must be attached to an astronomical telescope, the objective of which serves to concentrate the light into a small, bright image.

Probably the simplest form of stellar spectroscope is the **ocular spectroscope,** which is simply a small prism, or more usually a direct-vision combination of prisms (page 158), that may be attached to the eyepiece of a telescope. The one shown in Figure 219, a common form, is only a little more than an inch long. No slit is necessary for this instrument; the diminutive image of a star, formed by the telescope objective, takes the place of the slit, the eyepiece forms the collimator, and no view telescope is needed since the pencil of light that emerges from the prism is already small enough to enter the pupil of the eye. The resulting spectrum is as narrow as the image of the star—a mere line of colored light—and in order that dark lines may be seen it is necessary to broaden the spectrum by means of a small concave cylindric lens placed with the axis of the cylinder at right angles to the refracting edge of the prism. This, of course, diminishes the intensity of the spectrum, and so cannot be applied to very faint stars. If the spectrum contains bright lines, these can be well seen without the cylindric lens as bright dots which are monochromatic images of the star. The only advantage of the ocular spectroscope over the types next to be discussed lies in the smallness of the prism, which renders the instrument light and inexpensive.

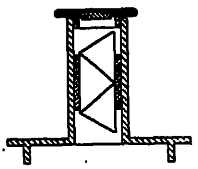


Fig. 219. *The Ocular Spectroscope.*

The oldest form of star spectroscope is the **objective prism,** which was first used in 1823 by Fraunhofer. It is simply a prism placed in front of the objective of an astronomical telescope, the latter taking the place of the view telescope of an ordinary spectroscope. No slit or collimator is needed, since the star is only a point of light and is so distant that its rays are already parallel. When the instrument is used visually, as Fraunhofer used it, the spectrum is broadened by a cylindric lens placed over the eyepiece. Much more commonly it is used photographically, in which case the refracting edge of the prism is placed parallel to the equator (the telescope being mounted equatorially) and the driving clock is given a rate differing slightly from the sidereal rate, so that the spectrum is trailed upon the plate in a direction parallel to its lines. This broadens the photograph by any desired amount. When used with a photographic objective covering a large field, this type of spectrograph is extremely useful, for it records the spectra of many stars together on the same plate so that they may be conveniently and rapidly classified (Figure 220).

For accurately determining the wave length of lines or the displacement due to radial velocity, a complete **star spectrograph,** with a slit, is used in connection with an astronomical telescope. This instrument is similar to the prism spectrograph described in Chapter 8, but it must be constructed with great rigidity to prevent flexure of its parts and guarded against changes of temperature during the long exposure necessary for

photographing the spectra of faint stars, and it must also be provided with a device for guiding the telescope so as to keep the star's light within the slit. The slit spectrograph can be applied to only one star at a time, but it possesses the great advantage of providing an artificial comparison spectrum beside the star spectrum, which is essential for accurate measurements.

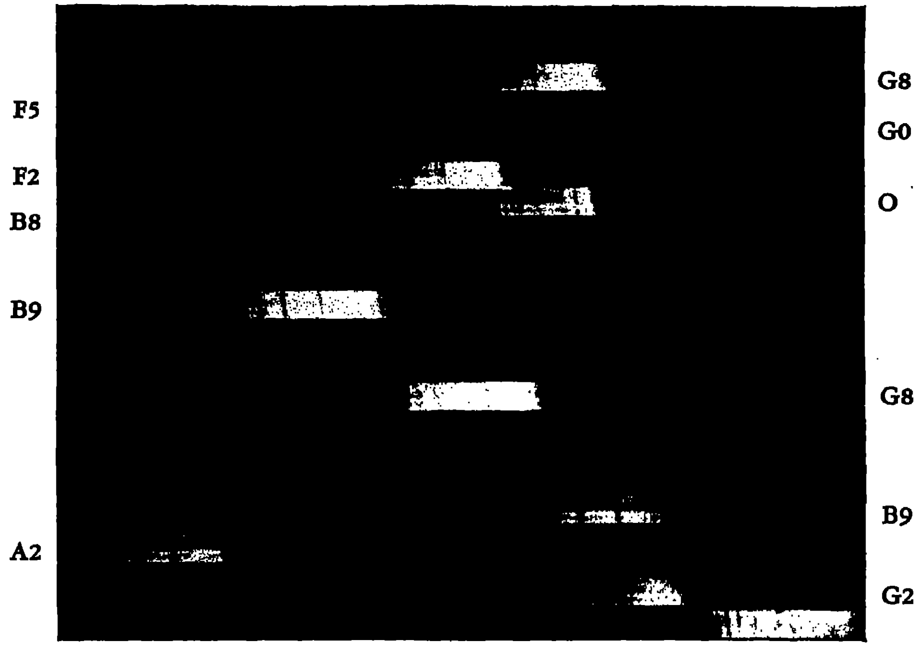



Fig. 220. *Spectra of Telescopic Stars in Cassiopeia, photographed by J. J. Nassau with the 24-Inch Schmidt camera and objective prism of the Warner & Swasey Observatory, 1944.*

In Figure 221 is shown the star spectrograph of the Lick Observatory, and in Figure 222 a spectrogram of the star Arcturus made with this instrument, with the titanium spark as comparison, enlarged to four times its original dimensions. Such spectrographs are used mainly with very large telescopes, and are made in a great variety of sizes and dispersive powers.

The arrangement of the optical parts of a star spectrograph and of the telescope to which it is attached is shown diagrammatically in Figure 223. $T$ is the objective of the astronomical telescope, $S$ the slit, $C$ the collimator, $P$ the prism (two or more prisms are sometimes used in a train), and $O$ the

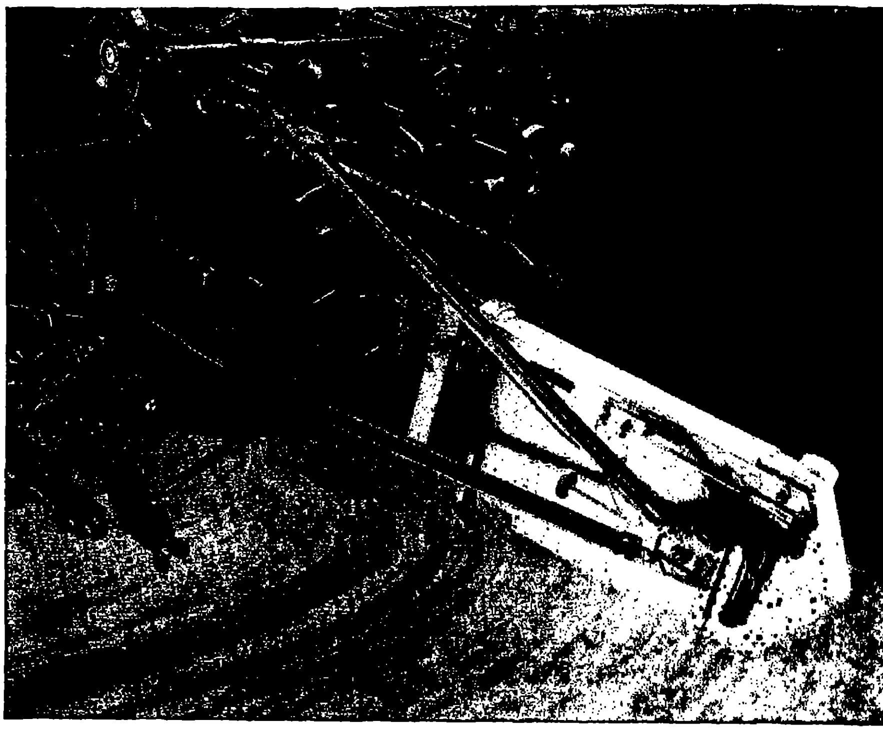


Fig. 221. *Lower End of the 36-Inch Lick Refractor with Star Spectrograph Attached. The optical parts of the spectrograph are enclosed in the white case.*

objective of the camera which focuses the spectrum upon the photographic plate at *F*. The objective *T* forms no part of the spectrograph, but performs the important duty of gathering a large amount of star light into a small image. In order that a maximum amount of the star's light may pass through the slit, it is necessary that the slit be kept accurately in the focal plane of the objective *T*. The slit might at first thought seem to be unnecessary, since the star image is so small; but without the slit jaws the wandering of the star image due to "bad seeing" and imperfect guiding would, during an exposure of any length, blur the lines of the spectrum. Moreover, the slit is necessary for photographing the comparison spectrum.

Fig. 222. *Spectrogram of Arcturus made with the equipment shown in Figure 221 (enlarged four times). Spark spectrum of titanium for comparison.*

A change of temperature during an exposure would result in a change of the refractive index of the prism and in contraction or expansion of the parts of the spectrograph, which would cause the spectrum to move on the plate and blur the spectral lines; therefore the instrument is ordinarily inclosed in a felt-lined case which is provided with an electric thermostat that holds the temperature constant within a small fraction of a degree. For "guiding" the telescope, the jaws of the slit are made of polished metal and are so placed as to reflect into a small observing telescope placed just above them any part of the star's light that does not enter the slit. It is the observer's duty so to control the main telescope as to keep this unused light at a minimum throughout the exposure of the plate.

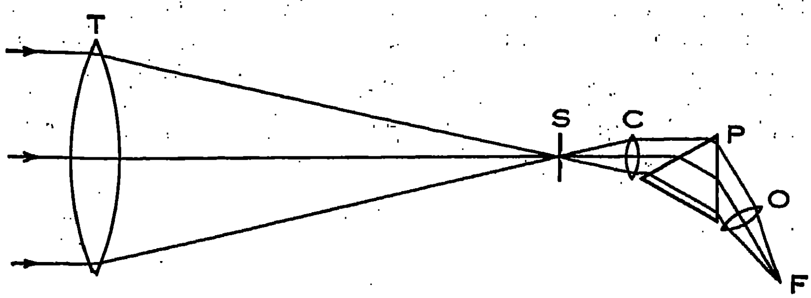



Fig. 223. *The Stellar Spectrograph.*

Light for the comparison spectrum is produced by an electric arc or spark between metallic terminals, or by a vacuum tube. The source of light is placed beside the slit, and its light is reflected into the spectrograph by diminutive prisms placed over the ends of the slit. The star's light falls upon the middle of the slit, between the prisms, and in the resulting photograph the star spectrum appears between two identical impressions of the comparison spectrum. Measurement of the relative position of star and comparison lines is made with a high degree of accuracy upon the finished photograph under a microscope.

**Historical Outline of Stellar Spectroscopy.** Fraunhofer observed the spectra of a few of the brightest stars and recognized that some of them, like Arcturus and Capella, were like the spectrum of the Sun, whereas others, such as Sirius and Castor, showed only a few dark lines. In 1864, soon after Kirchhoff and Bunsen had identified the spectra of many of the chemical elements, the study of stellar spectra was taken up extensively by Sir William Huggins in England and by Father Angelo Secchi in Italy. Huggins used the slit spectroscope in a detailed study of about a hundred stars, identifying in their spectra the lines of many known terrestrial elements, and so established the fact that these substances were widely distributed throughout the visible universe. Secchi, using an ocular spectroscope, examined the spectra of about four thousand stars in less detail, and divided them into four classes according to spectral type, a classification which is still found useful. In 1885, the photographic study of star spectra with the objective prism was begun by E. C. Pickering at the Harvard College Observatory, where it has been pursued

assiduously ever since. A classification based on this study was formed by Miss Antonia Maury, and has since been superseded by one due chiefly to Miss Annie Cannon, who was the authority for the classification of about a quarter of a million stars.

The Doppler-Fizeau principle was first applied to stars in 1866 by Huggins, who derived an approximation to the radial velocity of Sirius by visual measurements of the lines of its spectrum. In 1888 Vogel began making measurements of much greater accuracy by photography, and a few years later the photographic method was developed and applied extensively by Campbell at the Lick Observatory. Since 1900 the study of stellar spectra in its different branches has been carried on vigorously by a number of large observatories in America and Europe, and the value of this study has been greatly enhanced by the application of modern theories of the atom and of radiation.

**The Spectral Sequence.** Almost all of the many thousands of stars which have been studied fit into an orderly sequence, in which a number of properties, mostly pertaining to their spectra, progress by imperceptible gradations. Some of the more important of these properties are:

1. The color of the star, ranging from bluish-white through white, yellow, and orange to deep red.

2. The relative intensity of the violet and red portions of the spectrum. The violet grows relatively weaker down the sequence from blue stars to red.

3. The effective temperature which, as shown by various methods (page 373), ranges from 30,000° C. or more for the bluest stars down to about 3000° C. for the reddest.

4. The intensity of the dark hydrogen lines of the spectrum. Weak in the blue stars, these lines increase to a conspicuous maximum in white stars such as Sirius, and then fade down the sequence.

5. The intensity of the dark lines of ionized calcium, notably H and K, which reaches a maximum in the spectra of the orange stars.

6. The number of dark lines of the metals, which begins in the spectra of white stars and increases greatly through the remainder of the sequence.

**Significance of the Spectral Sequence.** For many years it was believed, quite naturally, that a star's position in the spectral sequence was an indication of its chemical composition. A star in whose spectrum only hydrogen lines were prominent was supposed to consist mainly of hydrogen, and one having conspicuous carbon bands was thought to be mostly carbon. It was further supposed that a star's chemical composition depended upon its stage of evolution, that is, upon its effective age; and much able thought and discussion were devoted to theories of stellar evolution.

It is now known that the properties that vary through the spectral sequence are an index simply of the degree of excitation or ionization of the atoms in the star's atmosphere where the spectral lines originate, and that this in turn depends upon the temperature and pressure in that atmosphere. It is therefore now believed that, in general, the stars do not differ greatly in chemical composition. As to stellar evolution, the attainment of a satisfactory theory seems further away in 1945 than it did in 1910.

**The Harvard Spectral Classification.** Of the many systems of classification of stellar spectra which have been proposed—by Secchi, Huggins, Vogel, Lockyer, and others—the only generally accepted survivor is the system evolved principally at the Harvard Observatory, and based on the appearance of the photographic region—that is, the part of the spectrum from green to ultra-violet or, roughly, from $H\beta$ to K. Intended to represent stages of stellar evolution, its classes were denoted by capital letters, arranged at first in alphabetic order; but as the study progressed and ideas were modified, the order of the letters was changed. The principal classes of the sequence, in their present order, are **O, B, A, F, G, K,** and **M.** A relatively small number of faint stars are placed in additional classes designated N, R, and S, which seem to branch off the sequence near K. Another small group, known as Wolf-Rayet stars and forming a part of the original class O, seems not to be closely related to the sequence. More than 90 per cent of all stars of known spectra fall into classes A, F, G, and K. Each class contains subdivisions which are denoted by numerals up to 9, as B8 or K2. The complete description of the classification, as published in the *Annals of the Harvard College Observatory*, covers nearly thirty quarto pages. Briefly outlined, the classification is as follows:

Class O. Blue-white stars showing in their spectra the lines of ionized helium, of doubly and triply ionized oxygen and nitrogen, and of hydrogen; also, the Wolf-Rayet stars discussed below.

Class B. Blue-white stars, sometimes called helium stars or Orion stars. The most prominent features of the spectra are the dark lines of neutral helium and of hydrogen; H and K are present but very weak. Examples are Rigel, the belt stars of Orion, and Spica.

Class A. White stars, with spectra dominated by strong hydrogen lines. Helium is absent, enhanced metallic lines are present but very weak, H and K are plainly perceptible. Examples are Sirius, Vega, and Fomalhaut.

Class F. Intermediate between classes A and G. The H and K lines

grow stronger than the hydrogen series, metallic lines increase in number and prominence. Examples, Procyon and Canopus.

Class G. Yellow stars, with solar-type spectra crowded with fine metallic lines and with strong H and K. Examples are the Sun, Capella, and $\alpha$ Centauri.

Class K. Orange stars, intermediate between classes G and M. H and K are very strong, neutral lines of metals are strong, hydrogen is relatively weak; brightness of continuous spectrum drops off rapidly in the violet. The sunspot type of spectrum. Example, Arcturus.

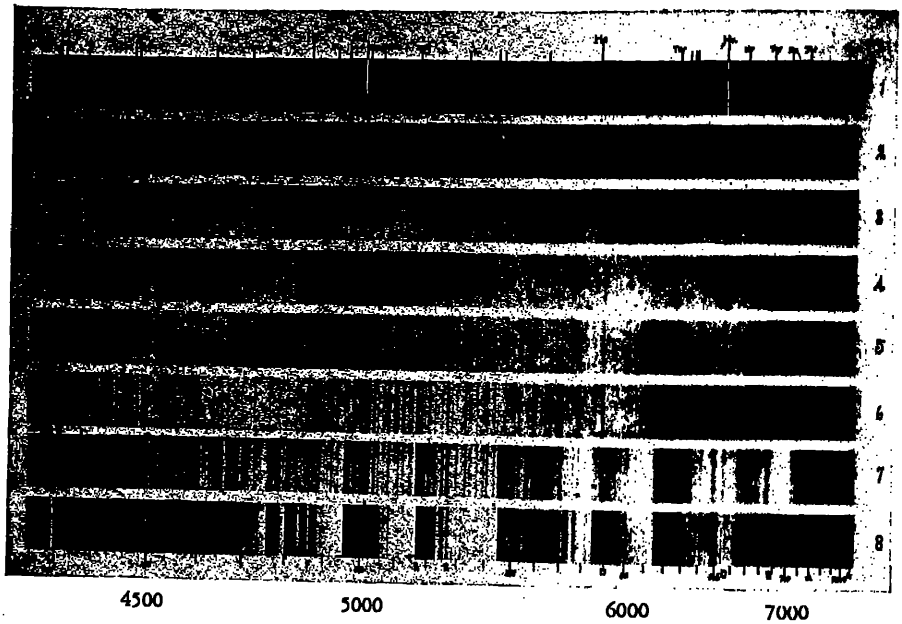



Fig. 224. *Nebular and Stellar Spectra (Blue to Red), Photographed by V. M. Slipher at Lowell Observatory. 1, Orion Nebula; 2, Rigel (B8); 3, Sirius (A0); 4, Procyon (F5); 5, the Sun (G0); 6, Arcturus (K0); 7, Betelgeuse (M2); 8, Mira (M6e).*

Class M. Red stars with spectra dominated by the dark bands of titanium oxide, each of which consists of many fine lines forming a "head" on the side toward the violet and fading gradually toward greater wave-lengths. Other lines are similar to those of class K. In the spectra of many M stars, bright hydrogen lines appear upon the continuous spectrum among the dark lines; these stars are denoted by Me (e for emission). All known Me stars are variable in brightness. Examples of M stars, Antares and Betelgeuse; of an Me star, Mira.

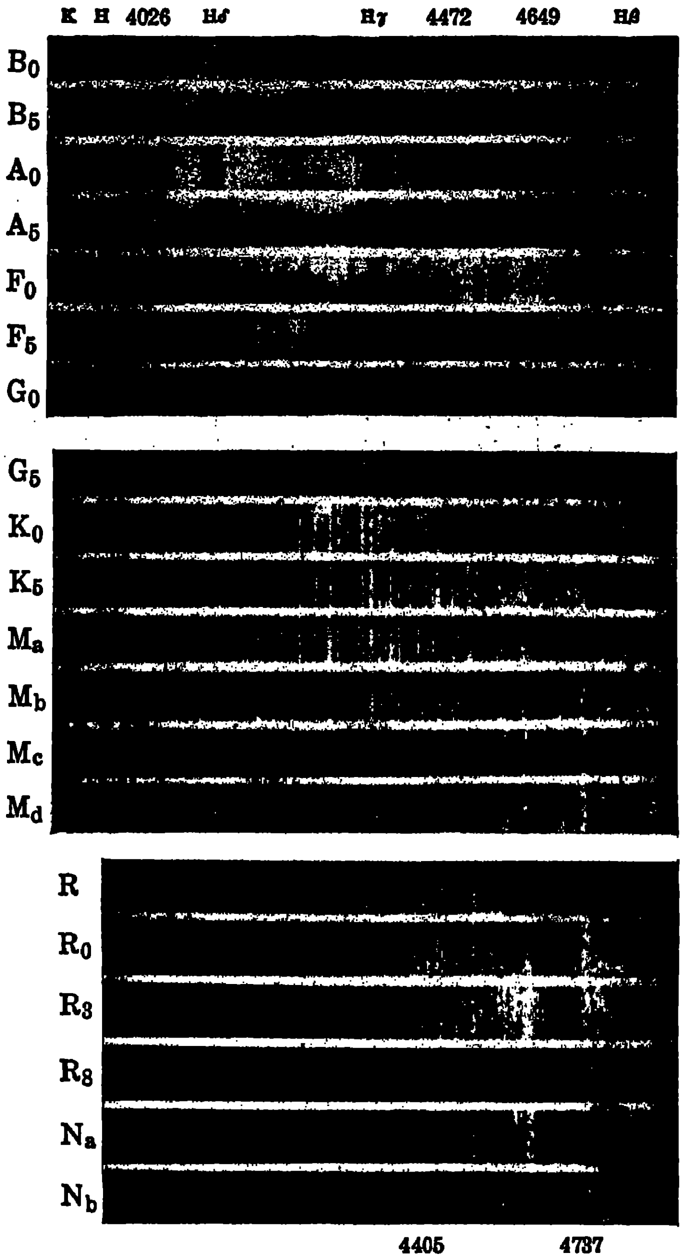



Fig. 225. *Stellar Spectra (Violet to Green), Photographed by Curtiss and Rufus at the University of Michigan.*

Class N. "Carbon stars," with spectra dominated by dark Swan bands, each composed of hundreds of fine lines forming a head on the redward side and fading toward violet. Continuous spectrum is very deficient in violet light and stars are very red. The characteristic features are most conspicuous in the visible and infra-red portions of the spectrum. A striking example, and one of the brightest, is the variable star Y Canum Venaticorum, magnitude 4.8 to 6.0, called "Superba" by Secchi because of its blood-red color.

Class R. Bands similar to those of Class N, but spectra extending farther into violet. Only a few such stars are known, and all are very faint.

Class S. Spectra like those of class M except that bands of zirconium oxide replace those of titanium oxide. The zirconium-oxide bands resemble the bands of class M in appearance, but are differently situated. Stars very few and faint.

Wolf-Rayet Stars (lower divisions of class O). Spectra distinguished from those of all other stars by broad, bright bands, many as yet unidentified, each fading out equally on both sides and not composed of lines, and all superposed on a faint continuous spectrum; also bright lines of hydrogen and ionized helium. Wolf-Rayet stars are the hottest stars known, estimates of their temperature being as high as 100,000°. None have been found outside the Milky Way and Magellanic clouds.

The characteristics of the spectral classes may be best understood from a study of the spectra themselves, and for this purpose representative spectra are reproduced in Figures 224–225. The progression of spectral

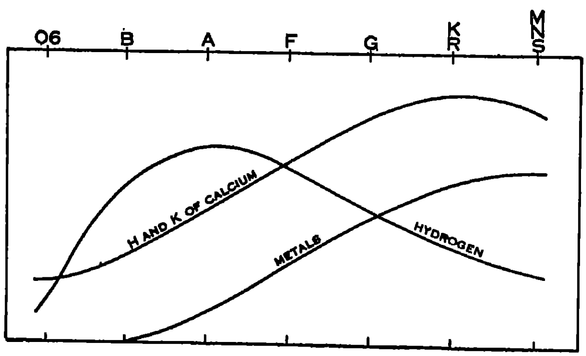



Fig. 226. *Characteristics of the Spectral Types.*

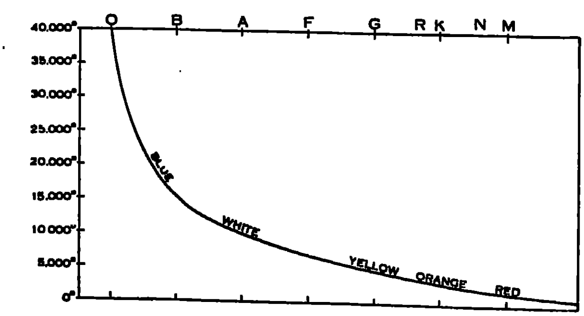



Fig. 227. *Correlation of Color and Temperature with Spectral Type.*

properties through the sequence is shown graphically in Figure 226, and the relation of temperature and color to spectral type in Figure 227. The properties of the principal spectral classes are summed up briefly in Table 17.

Table 17

THE PRINCIPAL STELLAR SPECTRAL TYPES

| Class | Distinguishing Feature | Example | Number of Stars Brighter than Mag. 6.25 | Color Index (page 340) | Effective Temperature |
|---|---|---|---|---|---|
| O | Ionized helium.................. | $\zeta$ Puppis | 20 | − 0.3 | >30,000° |
| B | Neutral helium.................. | $\epsilon$ Orionis | 696 | − 0.3 | 20,000 |
| A | Strong hydrogen lines............ | Sirius | 1885 | 0.0 | 11,000 |
| F | Intermediate class............... | Procyon | 720 | + 0.3 | 7,500 |
| G | Many metallic lines, strong H and K | The Sun | 609 | + 0.6 | 6,000 |
| K | Intermediate class............... | Arcturus | 1719 | + 1.0 | 4,200 |
| M | Titanium oxide bands........... | Antares | 457 | + 1.5 | 3,000 |
| N | Carbon bands.................. | 19 Piscium | 8 | + 2.5 | 3,000 |

**The Distances of the Stars Determined by Their Heliocentric Parallax.** The relation between the distance of a star and its heliocentric parallax was pointed out on page 99. The **heliocentric parallax** may be most simply defined as the angle subtended at the star by the semidiameter of the Earth's orbit, and this angle is inversely proportional to the star's distance. One second of arc is the angle subtended by any line at a distance 206,265 times its length. A star which has a parallax of one second is therefore at a distance of 206,265 astronomical units, one with a parallax of one-half a second is twice as far away, and, in general, a star with a parallax of $p$ seconds is at a distance of $206265/p$ astronomical units. A star's distance may therefore be determined by measuring its parallax, the method being in fact a triangulation (page 79) in which the base line is the radius of the Earth's orbit. Great as this base line is, it is very small compared with the distance of even the nearest star, and the determination of star distances is extremely difficult because of the slenderness of the triangle.

Success in detecting parallactic displacement in a star was first attained almost at the same time—about 1838—by three different observers: Bessel in Germany, who measured 61 Cygni; Struve in Russia, who chose Vega; and Henderson at the Cape of Good Hope, who observed $\alpha$ Centauri. Henderson measured the star's right ascension and declination with a meridian circle at intervals throughout the

year, and studied the variation of these coördinates; but the corrections for precession, nutation, and aberration are enormous as compared to the quantity which he sought, and moreover the instrumental corrections are mingled with the parallax because, like it, they have a yearly period. The alternative method, which was used by Bessel and Struve and is now adopted by all parallax observers, consists of measuring the apparent distance of the star under investigation from a number of comparison stars which are very near it in the sky. Since all these stars are affected by precession and the like in the same way, and since the great majority of stars which are likely to be used as comparison stars are so far away that their parallax is insensible, this "differential" method gives the parallactic displacement free from the troublesome corrections to which the "absolute" method of Henderson is subject.

At the bottom of Figure 228 is represented the orbit of the Earth. *A*, *B*, and *C* are neighboring stars which, seen upon the background of very

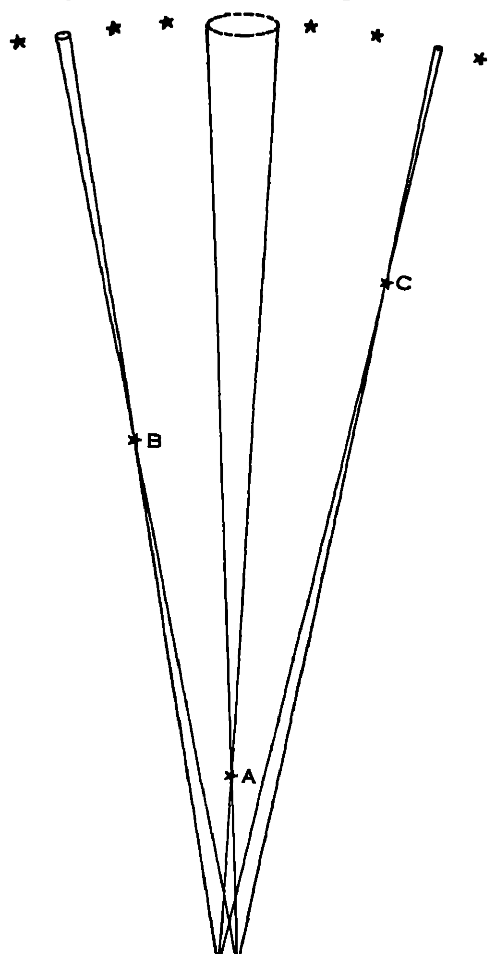



Fig. 228. *Heliocentric Parallax.*

distant stars at the top of the drawing, seem to reflect the Earth's motion in tiny parallactic ellipses. The nearest star, $A$, appears to move in the largest ellipse. The background stars are so remote that their parallactic ellipses dwindle to points. Of course the size of the Earth's orbit relative to the distance of the stars is grossly exaggerated in the drawing.

Throughout the nineteenth century parallax measurements were made either with the filar micrometer or with the *heliometer*, an instrument of great accuracy devised by Fraunhofer for measuring the diameter of the Sun. These methods were extremely laborious, and at the beginning of the twentieth century the distances of less than a hundred stars were known. Parallaxes are now determined almost entirely by measurements of photographs taken with long-focus telescopes, a method introduced by Schlesinger at the Yerkes Observatory about 1903. At the present time over three thousand parallaxes have been measured with considerable certainty.

**Units of Stellar Distance.** The Earth's mean distance from the Sun, which is convenient enough as an astronomical unit of distance in the Solar System, is much too small for use between the stars. A unit which brings vividly to mind the colossal distances involved is the **light-year**, the vast distance over which light, traveling 186,000 miles a second, passes in a year. It is equivalent to about 63,300 [1] astronomical units, and so bears about the same relation to the astronomical unit as the mile to the inch.[2] If any star had a parallax of exactly one second, its distance would be $206265 \div 63300 = 3.26$ light-years; but no known star has so large a parallax or so small a distance. The distance of any star whose parallax is $p$ seconds is $3.26/p$ light-years.

Table 18

APPROXIMATE EQUIVALENTS

| | Light-years | Parsecs | Ast. Units | Millions of Millions of Miles | Centimeters |
|---|---|---|---|---|---|
| One light-year = ........... | 1.00 | 0.31 | 63,300 | 6 | $10^{18}$ |
| One parsec = .............. | 3.26 | 1.00 | 206,265 | 20 | $3 \cdot 10^{18}$ |
| Distance of Proxima = ...... | 4.3 | 1.32 | 272,000 | 25 | $4 \cdot 10^{18}$ |

[1] The number of seconds in a year divided by the number of seconds required by light to travel from the Sun to the Earth.

[2] One mile = 63,360 inches.

For some purposes, a more convenient unit is the distance which a star would have if its parallax were 1″. As a name for this unit, English astronomers have invented the monstrous word **parsec.** A parsec is equal to 3.26 light-years. The distance of a star whose parallax is $p$ seconds is simply $1/p$ parsecs.

Table 19

THE NEAREST STARS

(Compiled by P. van de Kamp, 1944)

| Name and Const. | Vis. Mag. | Spec. | Par'x | $D_{ly}$ | Abs. Mag. | Luminosity | $\mu$ | Rad. Vel. |
|---|---|---|---|---|---|---|---|---|
| The Sun......... | −26.7 | G0 | | $\frac{1}{63300}$ | 5.0 | 1.000000 | | |
| Proxima Centauri . | 10.5 | M? | ″761 | 4.3 | 15.4 | 0.00007 | 3″.68 | |
| α Centauri....... | 0.3 | G4 | .761 | 4.3 | 4.7 | 1.3 | 3.68 | − 22 |
| | 1.7 | K1 | | | 6.1 | 0.36 | | |
| Barnard's * Oph.. | 9.7 | M5 | .530 | 6.1 | 13.3 | 0.00048 | 10.30 | −110 |
| Ll 21185 UMa... | 7.6 | M2 | .411 | 7.9 | 10.7 | 0.0052 | 4.78 | − 87 |
| Wolf 359 Leo.... | 13.5 | M5e | .408 | 8.0 | 16.6 | 0.00002 | 4.84 | − 90 |
| Sirius, α CMa.... | −1.6 | A0 | .381 | 8.6 | 1.3 | 30. | 1.32 | − 8 |
| | 7.1 | A5 | | | 10.0 | 0.10 | | |
| Ross 154 Sgr.... | 11 | M4e | .350 | 9.3 | 13.7 | 0.00033 | 0.67 | |
| Ross 248 And.... | 12.2 | M6 | .317 | 10.3 | 14.7 | 0.00013 | 1.58 | |
| Luy 789–6 Aqr... | 12.3 | M5e | .315 | 10.3 | 14.8 | 0.00012 | 3.27 | |
| ε Eridani......... | 3.8 | K0 | .305 | 10.7 | 6.2 | 0.33 | 0.97 | + 15 |
| Procyon, α CMi.. | 0.5 | F3 | .295 | 11.0 | 2.9 | 6.9 | 1.25 | − 3 |
| | 10.8 | A? | | | 13.2 | 0.00052 | | |
| 61 Cygni........ | 5.6 | K5 | .294 | 11.1 | 7.9 | 0.069 | 5.22 | − 63 |
| | 6.3 | K6 | | | 8.6 | 0.036 | | |
| Ross 128 Vir..... | 11.1 | M5 | .292 | 11.2 | 13.4 | 0.00044 | 1.40 | |
| ε Indi........... | 4.7 | K5 | .291 | 11.2 | 7.0 | 0.16 | 4.67 | − 40 |
| τ Ceti........... | 3.6 | K0 | .290 | 11.2 | 5.9 | 0.44 | 1.92 | − 16 |
| Σ 2398 Dra...... | 8.9 | M4 | .287 | 11.3 | 11.2 | 0.0033 | 2.29 | 0 |
| | 9.7 | M5 | | | 12.0 | 0.0016 | | |
| BD–12° 4523 Oph | 9.7 | M4 | .281 | 11.6 | 11.9 | 0.0017 | 1.24 | |
| Groom 34 And... | 8.1 | M1 | .278 | 11.7 | 10.3 | 0.0076 | 2.91 | + 8 |
| | 10.9 | M6 | | | 13.1 | 0.00058 | | |

**Stellar Distances.** Table 19 is a list of the eighteen nearest known stars (of which six are double), with their magnitudes, spectral types, heliocentric parallaxes, distances (in light-years), and other characteristics which will be discussed in later pages. Most of these stars are very faint and have no other designation than their numbers in star catalogues or the names of astronomers who have investigated them; such are Barnard's star and Wolf 359. Three, however—α Centauri, Sirius, and Procyon—are among the brightest stars in the sky. The first of these, α Centauri, a conspicuous double star in the southern hemisphere, is the nearest of all

naked-eye stars. A little more than two degrees away from it is Proxima, so named soon after its discovery, when it was believed to be even nearer, the nearest star of all. The distance of this group of stars is about 270,000 astronomical units, and the Solar System is thus seen to be a mere speck in the immensity of space, separated from its nearest neighbor by thousands of times its own diameter. The known stellar population within four parsecs of the Sun averages one star to about 520 cubic light-years, but it is likely that the actual star density is somewhat greater than this, for there may be a number of faint stellar neighbors yet undiscovered.

If the Sun is represented by a plum an inch in diameter, $\alpha$ Centauri will be a pair of plums a fraction of a mile apart and 500 miles away, and Proxima will be a grain of sand about 25 miles from the pair of plums. The most distant star in the table will on this scale be about 1500 miles away.

Heliocentric parallaxes of $0''.01$ or smaller cannot be very reliably measured, and so the distances of stars beyond 300 light-years cannot be satisfactorily determined in this way. Not one star in a thousand of those which can be observed with large telescopes is so near us as this, but the distances of many of these remote stars can be determined by indirect methods which will be discussed later (pages 351, 352, 370, 379, 400, 409, 411). The light by which we see most of the stars left them long before we were born, and that by which the faintest stars are photographed left them before the dawn of civilization or even before the appearance of mankind on the Earth.

**Star Magnitudes.** A star's **magnitude** may be defined as a number in an arbitrary scale that expresses the apparent brightness of the star as compared with that of other stars. In the scale used by astronomers the brightest stars are given the smallest numbers. Hipparchus and Ptolemy classified the stars visible to the unaided eye into six "magnitudes," describing about twenty of the brightest stars as of the first magnitude and the stars just visible to the naked eye as of the sixth. This classification is preserved in modern celestial photometry, although there seems no better reason for six magnitudes of lucid stars than for five, or seven, or another number. The word magnitude, it should be noticed, here refers to brightness and not to size. Two stars of the same size may differ greatly in magnitude if their temperatures or their distances from the Earth are very different.

Little attention seems to have been paid to the subject of the brightness of the stars from the time of Ptolemy until the nineteenth century, for

although the system of magnitudes was extended to the telescopic stars, there was no general agreement among different observers. In 1827 Sir John Herschel compared the light of faint stars, observed with a reflector of eighteen inches aperture, with that of bright stars observed with smaller apertures, and concluded that the average first-magnitude star sends us about one hundred times as much light as a star of the sixth magnitude. Pogson of Oxford, in 1854, by noting the smallest aperture with which certain stars could be seen and comparing the average "magnitudes" of these stars as estimated by various well-known observers, became convinced that each "magnitude" is about two and one-half times as bright as the next.

The relation between the number expressing a star's magnitude and the amount of light that the eye receives from the star is an example of the **psychophysical law** which was suggested to the German psychologist Fechner by the results of Herschel and Pogson. This law states that if the intensity of a sensation varies in arithmetical progression, then the intensity of the stimulus that produces it must vary in geometric progression. Thus, if one listens to three sounds, $a$, $b$, $c$, and judges the *difference* of loudness between $b$ and $c$ to be the same as that between $a$ and $b$, then in reality the *ratio* of the intensity of $c$ to that of $b$ must be the same as the ratio of intensity of $b$ to $a$. The law holds for each of the other senses as well as for those of sight and hearing, but is not rigorously true when applied to very feeble or very powerful stimuli.

The brightnesses of stars of the sixth, fifth, · · · first magnitude *appear* to be separated by a constant difference; measurement shows that they really bear a constant *ratio*, which Pogson found to be about 2.5. Let the exact value of this ratio be $\rho$. Then a fifth-magnitude star sends us $\rho$ times as much light as a sixth; a fourth-magnitude star sends us $\rho$ times as much as a fifth, or $\rho^2$ times as much as a sixth; a third-magnitude star $\rho^3$ times as much as a sixth, and so on. In general, if $m$ and $n$ represent the magnitudes of two stars, $m$ referring to the brighter, so that the difference of magnitude is $n - m$, then the ratio of their brightness is $\rho^{(n-m)}$; or, if the light sent to the Earth by the two stars is represented by $l_m$ and $l_n$, respectively,

$$\frac{l_m}{l_n} = \rho^{(n-m)}.$$

But Herschel had found that a difference of five magnitudes corresponds to a ratio of 100; and this being a convenient relation, it was adopted as exact in the **Pogson scale** of magnitudes, which is now universally used, so that $\rho^5 = 100$ and $\rho = 100^{\frac{1}{5}} = 2.512 \cdots$

The most convenient expression for the relation between magnitude and actual brightness is found by taking the logarithm[3] of each side of the above equation, thus:

$$\log \frac{l_m}{l_n} = (n - m) \log \rho.$$

But the logarithm of $100^{\frac{1}{5}}$ is exactly 0.4; hence,

$$\log \frac{l_m}{l_n} = 0.4(n - m)$$

or

$$n - m = 2.5 \log \frac{l_m}{l_n} = 2.5(\log l_m - \log l_n).$$

The following examples illustrate the use of this formula.

1. Find the difference of magnitude of two stars, one of which is twice as bright as the other. Here $l_m/l_n = 2$, $\log l_m/l_n = 0.30$, hence $n - m = 2.5 \times 0.30 = 0.75$; or the brighter star outshines the fainter by three-fourths of a magnitude.

2. How does the brightness of Sirius, magnitude $-1.58$, compare with that of $\beta$ Tauri, magnitude $+1.78$? Here $n - m = 3.36$, and so $\log l_m/l_n = 1.344$; hence, $l_m/l_n = 22.08$—that is, Sirius is 22.08 times as bright as $\beta$ Tauri.

**Fractional and Negative Magnitudes.** Since there are stars of practically all gradations of brightness from Sirius down to the faintest star that can be perceived, it is of course necessary to express the magnitudes of most of them by fractions. The scale of magnitudes is so adjusted that about half of Ptolemy's "first magnitude" stars are brighter and half are fainter than what is now the standard first-magnitude star, which is represented very nearly by Aldebaran and Altair. A star 2.512 times as bright as this is of the zero magnitude, and the magnitudes of the two stars Sirius and Canopus, which are brighter still, are negative. The magnitudes of the planets Venus and Jupiter and, at times, those of Mercury and Mars, must also be expressed by negative numbers; Venus becomes a little brighter than the $-4$th magnitude, which means that she is then more than one hundred times as bright as Aldebaran. The magnitude of the Sun, according to Russell, is $-26.72$.

**The Forty Brightest Stars.** The twenty brightest stars, with their visual magnitudes, spectral types, and distances in light-years, are listed in Table 20.

[3] The logarithm of a number is the exponent of the power to which 10 must be raised to produce the number. Thus, $\log 1 = 0$, $\log 10 = 1$, $\log 100 = 2$, $\log 1000 = 3$, etc. In general,

$$\log 10^a = a.$$

From this definition and the principles of elementary algebra, it follows that the logarithm of the product of two numbers is the sum of their logarithms; the logarithm of their quotient is the difference of their logarithms; and the logarithm of the $n$th power of a number is $n$ times the logarithm of the number. That is,

$$\log ab = \log a + \log b; \quad \log \frac{a}{b} = \log a - \log b; \quad \log a^n = n \log a.$$

The logarithms of numbers between 1 and 10, between 10 and 100, etc., have values between 0 and 1, between 1 and 2, etc. Tables of logarithms, such as the short one on page 344, give the decimal parts only.

Table 20

STARS BRIGHTER THAN VISUAL MAGNITUDE 1.50

| | Mag. | Sp. | Dist. in l.-y. | | Mag. | Sp. | Dist. in l.-y. |
|---|---|---|---|---|---|---|---|
| α Canis Maj. (Sirius) | − 1.58 | A0 | 8.6 | α Orionis (Betelgeuse) | 1.0 to 1.4 | M2 | 300 |
| α Argûs (Canopus) | − 0.86 | F0 | 100 | α Crucis | 1.05 | B1 | 220 |
| α Centauri | + 0.06 | G0 | 4.3 | α Tauri (Aldebaran) | 1.06 | K5 | 53 |
| α Lyrae (Vega) | 0.14 | A0 | 27 | α Virginis (Spica) | 1.21 | B2 | 120 |
| α Aurigae (Capella) | 0.21 | G0 | 42 | β Geminorum (Pollux) | 1.21 | K0 | 29 |
| α Bootis (Arcturus) | 0.24 | K0 | 33 | α Scorpii (Antares) | 1.22 | Ma | 250 |
| β Orionis (Rigel) | 0.34 | B8 | 540 | α Piscis Australis (Fomalhaut) | 1.29 | A3 | 23 |
| α Canis Min. (Procyon) | 0.48 | F5 | 11 | α Cygni (Deneb) | 1.33 | A2 | 400 |
| α Eridani (Achernar) | 0.60 | B5 | 70 | α Leonis (Regulus) | 1.34 | B8 | 67 |
| β Centauri | 0.86 | B1 | 190 | | | | |
| α Aquilae (Altair) | 0.89 | A5 | 15.7 | | | | |

The twenty stars which follow those of the above list in order of brightness are given in Table 21. These two tables contain all the stars which are brighter than visual magnitude 2.00.

Table 21

STARS OF VISUAL MAGNITUDE 1.50 TO 2.00

| | Mag. | Sp. | Dist. in l.-y. | | Mag. | Sp. | Dist. in l.-y. |
|---|---|---|---|---|---|---|---|
| β Crucis | 1.50 | B1 | 272 | β Argûs | 1.80 | A0 | ? |
| α Geminorum (Castor) | 1.58 | A0 | 47 | α Trianguli Australis | 1.88 | K2 | 130 |
| γ Crucis | 1.60 | Mb | 72 | α Persei | 1.90 | F5 | 130 |
| ε Canis Majoris | 1.63 | B1 | 326 | η Ursae Majoris | 1.91 | B3 | 130 |
| ε Ursae Majoris | 1.68 | A0 | 48 | ζ Orionis | 1.91 | B0 | 410 |
| γ Orionis (Bellatrix) | 1.70 | B2 | 250 | γ Geminorum | 1.93 | A0 | 93 |
| λ Scorpii | 1.71 | B2 | 204 | α Ursae Majoris | 1.95 | K0 | 109 |
| ε Argûs | 1.74 | K0 | 326 | ε Sagittarii | 1.95 | A0 | 163 |
| ε Orionis | 1.75 | B0 | 410 | δ Canis Majoris | 1.98 | F8 | 410 |
| β Tauri | 1.78 | B8 | 93 | β Canis Majoris | 1.99 | B1 | 360 |

**Magnitude of Faintest Star Visible in a Telescope of Given Aperture.** Observation shows that the faintest star visible in a telescope of *one-inch* aperture is a star of the ninth magnitude. Since the light-gathering power of a telescope varies as the square of the aperture, this star is $a^2$ times as bright as the faintest that can be seen in a telescope of $a$ inches aperture. Hence, by substituting 9 for $m$, and $a^2$ for $l_m/l_n$ in the equation connecting magnitude and brightness, we may obtain the magnitude, $n$, of the faintest star visible in a telescope of $a$ inches aperture:

$$\begin{aligned} n &= 9 + 2.5 \log a^2 \\ &= 9 + 5 \log a. \end{aligned}$$

For example, since the logarithm of 10 is 1, the *minimum visibile* of a ten-inch telescope is $9 + 5 = 14$. The logarithm of 100 being 2, the *minimum visibile* of a one-hundred-inch telescope is $9 + 10 = 19$. Table 22 gives the limiting magnitudes of stars that theoretically may be seen with telescopes of various apertures. Fainter stars may be *photographed* with the same instruments; thus some of the plates taken with the 100-inch telescope at Mount Wilson show stars down to the twenty-first magnitude.

Table 22

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Aperture in inches..... | 1 | 3 | 6 | 12 | 20 | 24 | 36 | 40 | 60 | 100 | 200 |
| Limiting magnitude.... | 9.0 | 11.4 | 12.9 | 14.4 | 15.5 | 16.0 | 16.8 | 17.0 | 17.9 | 19.0 | 20.3 |

**Enormous Range in Apparent Brightness of Stars.** Since a difference of five in the magnitudes of two stars corresponds to a hundredfold ratio of brightness, a first-magnitude star sends to the Earth 100 times as much light as a star of the sixth magnitude, 100 × 100 or 10,000 times as much as one of the eleventh, 100 × 10,000 or 1,000,000 times as much as one of the sixteenth, and 100 × 1,000,000 or 100,000,000 times as much as one of the twenty-first. It would require at least 3,000,000,000 of the faintest stars photographed with the Hooker telescope to give us as much light as does Sirius. On the other hand, since the Sun is more than twenty-five magnitudes brighter than Sirius, the light of $100^5$, or 10,000,000,000, stars as bright as Sirius would be no brighter than sunlight.

**Determination of Magnitude by the Method of Argelander.** Probably the simplest method of observing stars for magnitude is that known as the method of Argelander, in which there is found a sequence of stars consisting of the star to be measured and at least two other stars whose magnitudes are known. The observation may be made with or without a telescope, but the stars should be at about the same altitude and as near together in the sky as possible. It is best to find one comparison star just perceptibly brighter and one just perceptibly fainter than the star to be measured; in any case the proportion of brightness of the three stars is noted. From the known magnitudes of the comparison stars the desired magnitude can easily be determined by this method with an accuracy of a tenth of a magnitude.

**Visual Photometers.** For the more accurate measurement of the light of stars, a number of different instruments known as photometers are in use. One of the simplest is the **wedge photometer**, which consists essentially of a wedge of dark glass inserted in the focal plane of the telescope, or between the eyepiece and the observer's eye. The observation is made by pushing the wedge into the beam of light until the star is extinguished or made equal to an artificial "star" formed by a small lamp shining through a pinhole. The star to be measured and a comparison star of known magnitude are thus observed in turn, and readings are made on a

scale which gives the thickness of dark glass traversed by the light. It can be shown on elementary physical principles that the difference of magnitude between the comparison star and the star under measurement is proportional to the difference of the scale readings. Another important type is the **polarizing photometer** with which the great photometric star catalogues of the Harvard and Potsdam observatories were formed, and in which the apparent brightness of two stars (one of which may be artificial) is equalized by passing the light of one or both through a system of polarizing prisms.

**Photographic Photometry.** The use of star photographs for determinations of relative brightness was begun in 1857 by G. P. Bond at the Harvard Observatory, and the method has been extensively used at Harvard and other large observatories. It has the advantage that the resulting magnitudes are not affected by individual peculiarities of vision, and the great further advantage that it can reach much fainter stars than the visual methods.

The most commonly used method is the one developed by E. C. Pickering at Harvard and known as the method of sequences. A list of stars in some readily accessible region, such as the Pleiades or the vicinity of the north pole, is chosen so that their magnitudes form a graduated series, and these magnitudes are determined with great care. A photograph of these stars is then compared with a photograph of the stars to be measured, preferably taken on the same plate, and the unknown magnitudes are found by interpolation in the sequence. In making the comparisons, both the size and the density of the images are taken into account. The standard North Polar Sequence formed at the Harvard Observatory contains ninety-six stars, ranging from the second to the twenty-first magnitude, and located within two degrees of the 1900 position of the North Pole.

**Color Index.** Since the eye is most sensitive to yellow light and the ordinary photographic plate is most sensitive to blue, a red star appears fainter, relative to a white one, on a photograph than in the sky. This is very noticeable in the photograph of the constellation Orion reproduced in Figure 280, where anyone familiar with only the visual appearance of the constellation may have some difficulty in identifying the red star $\alpha$ Orionis because of its faintness, although to the eye it appears quite comparable to Rigel. Because of this difference of sensitiveness to different colors, it is necessary to express magnitudes determined photographically and those determined visually on different scales. The number found by subtracting the visual magnitude from the photographic is a measure of the redness of the star, and is called the **color index.** The color index, of course, varies with the spectral type, and by general agreement the color index of the white A0 stars is considered as zero and that of the K0 stars as unity.

**Photovisual Magnitudes.** By using special plates sensitive to the yellow and red, together with a suitable screen, a combination may be formed for making photographs in which the relative sensitiveness to light of different colors is about the same as that of the eye. Magnitudes determined from such photographs are known as **photovisual magnitudes.** The method has the advantages of the regular photographic method, and at the same time the results are expressed in terms of the visual scale. The approximate spectral type of stars too faint for observation with the spectrograph may be determined by comparing their photographic and photovisual magnitudes, which gives their color index.

**The Measurement of Total Radiation.** The most complete way to compare one star with another is to measure their total radiation, including ultra-violet, visible, and infra-red rays. Many attempts have been made to do this with different forms of **radiometers,** and the greatest success has been attained by the use of the **vacuum thermocouple** in the hands of Coblentz and Lampland at the Lowell Observatory and of Nicholson and Pettit at Mount Wilson. The thermocouple consists of a junction of two metals such as bismuth and platinum. If the free extremities of the two pieces of metal are connected through a galvanometer, an electric current may be detected in the circuit when radiation falls upon the junction, and the strength of this current is proportional to the intensity of the radiation, which may therefore be measured by reading the galvanometer deflection. In the instrument as used for the study of the stars, the couple is made very small—the receiver upon which the light falls is only a small fraction of a millimeter in diameter—and is enclosed in a vacuum chamber to prevent loss of heat by conduction in the surrounding air; the receiver is placed in the focal plane of a large telescope. The telescope must be a reflector because the glass lenses of a refractor absorb radiation of certain wave lengths more strongly than that of others. As used with the 100-inch Hooker telescope, the instrument is sufficiently sensitive to detect the heat of a candle 120 miles away.

The difference of the "radiometric" and visual magnitudes of a star is called the star's **heat index.** The heat index of A0 type stars, like their color index, is taken as zero. That of the red stars, whose radiation is relatively more intense in the infra-red, amounts in some cases to several magnitudes.

**The Photoelectric Photometer.** A very sensitive instrument for measuring radiation electrically is the photoelectric photometer, which is reliable to about one-hundredth of a magnitude. Its development in this country has been led by Stebbins, who at first (about 1908) made use of the properties of selenium, an element whose electrical resistance varies

with the intensity of the light to which it is exposed. For this he soon substituted the **photoelectric cell**, a device which, in its less sensitive forms, is widely used in industry and the arts, for example in the projection of sound films in moving-picture theaters. It consists of a small evacuated glass chamber within which is exposed a surface of one of the alkali metals, sodium, potassium, or caesium. These metals possess the property of emitting electrons copiously when acted upon by radiation, and the electron stream constitutes a current, the fluctuations of which may be amplified as in radio sets and made to actuate a galvanometer or other electrical device. Stebbins now uses, in conjunction with large telescopes, a very sensitive photocell in which the metallic surface, of caesium oxide, responds to radiation of all wave lengths from 3500 to 12000 angstroms. The electron current is amplified by a factor of about a million, and this amplified current is measured or recorded with a sensitive galvanometer. Because the caesium cell responds to the radiation from its immediate surroundings at ordinary temperatures, producing a so-called "dark current," it is embedded, when in use, in dry ice (solid carbon dioxide), of which the temperature is about $-80°$ C.

Thermocouples and photocells are employed in various forms of **microphotometers**, which can measure the amount of light transmitted through any small portion of a photographic negative. The microphotometer is used in determining star magnitudes from photographs and in recording the relative intensity and width of the lines in photographed spectra.

**Absolute Magnitude and Luminosity.** The apparent brightness of a star depends upon both its intrinsic brightness and its distance, so that if two of these are known the third may be calculated. The distance is conveniently expressed by the heliocentric parallax $p$; the apparent brightness by the apparent magnitude $m$; and the intrinsic brightness by the **absolute magnitude** $M$, defined as the apparent magnitude which the star would have if removed to a distance of ten parsecs. The difference $m - M$ is called the **distance modulus.**

Let $l_p$ be the intensity of light which we receive from a star whose parallax is $p$ seconds, and $l_t$ the intensity of the light which we should receive from the same star if it were situated at a distance of 10 parsecs—i.e., if its parallax were 0.″1. Since the intensity of a star's light is inversely proportional to the square of its distance, and its parallax is inversely proportional to the distance itself, we have

$$\frac{l_p}{l_t} = p^2 \div (0.1)^2 = 100\,p^2.$$

But the difference of magnitude of two stars (or of the same star at different distances) is equal to 2.5 times the logarithm of the ratio of their brightness—that is (page 337, third equation),

$$\begin{aligned} M - m &= 2.5 \log 100\, p^2 \\ &= 2.5\,(\log 100 + 2 \log p) \\ &= 2.5(2 + 2 \log p) \end{aligned}$$

or, very simply,

$$M = m + 5 + 5 \log p.$$

To avoid the pitfalls of negative logarithms, it may be preferable for the student to compute absolute magnitude from the distance instead of from the parallax. Representing the distance in parsecs by $D_{ps}$ and in light-years by $D_{ly}$, we can easily transform the above equation to

$$\begin{aligned} M &= m + 5 - 5 \log D_{ps} \\ &= m + 7.566 - 5 \log D_{ly}. \end{aligned}$$

It is of great interest to compare the intrinsic brightness of the stars about us with that of the Sun. The **luminosity** of a star is defined as the ratio of the amount of light which we should receive from it to the amount which we should receive from the Sun if both star and Sun were removed to the same distance from us, say ten parsecs. The luminosity being represented by $L$ and the *absolute* magnitudes of Sun and star by $S$ and $M$, respectively, the fundamental equation gives

$$\log L = 0.4(S - M).$$

The absolute magnitude, $S$, of the Sun, is computed as follows: If $x$ is the number of astronomical units in ten parsecs, the Sun would, if removed to that distance, be only $1/x^2$ as bright as now; hence, if $s$ and $S$ are the apparent and absolute magnitudes of the Sun,

$$S - s = 2.5 \log x^2.$$

The value of $x$ is 2,062,650, and that of $s$ (page 337) is $-26.72$. Computing by logarithms from these values, we find easily that $S = +4.85$; hence, the luminosity of any star of absolute magnitude $M$ is given by

$$\begin{aligned} \log L &= 0.4(4.85 - M) \\ &= 1.940 - 0.4\,M. \end{aligned}$$

For example, the apparent magnitude of Vega is 0.14 and, according to the parallax of 0″.123 given by Schlesinger, its distance is 26.5 light-years. We have then $\log D_{ly} = 1.423$, so that $M = 0.14 + 7.57 - 7.11 = 0.60$; and $\log L = 1.940 - 0.240 = 1.700$; hence $L = 50$. That is, the candle power of Vega is fifty times that of the Sun, and if placed at the standard distance of ten parsecs Vega would have an apparent magnitude of 0.60.

**Determination of Distance from Absolute Magnitude.** It is often possible to infer the absolute magnitude of a star, or of the average of a group of stars, from other considerations. The formulae in the preceding section may then be reversed to find the parallax and distance:

$$\begin{aligned} \log p &= \tfrac{1}{5}(M - m) - 1 \\ \log D_{ps} &= \tfrac{1}{5}(m - M) + 1 \\ \log D_{ly} &= \tfrac{1}{5}(m - M) + 1.513. \end{aligned}$$

These relations are of great value in determining the distances of objects that are too remote for direct measurements of parallax.

## EXERCISES

1. Show that, in the spectrograph illustrated in Figure 223, in order that neither star light shall be wasted nor any part of the prism remain unused, the collimator $C$ must have the same ratio of aperture to focal length as the telescope objective $T$.

2. Verify the distances given for the first five stars in Table 19.

3. How does the brightness of Venus, when her magnitude is $-4$, compare with that of a sixth-magnitude star?

*Ans.* Venus is 10,000 times brighter

4. How does the brightness of Venus, magnitude $-4$, compare with that of a star of the 16th magnitude? With that of a star of the 21st magnitude?

5. If the distance of an eighth-magnitude star were reduced to 1/10 its present value, what would be the magnitude of the star?

*Ans.* 3

6. What is the photographic magnitude of a K-type star of visual magnitude 2?

*Ans.* 3

NOTE: For the following exercises, a table of logarithms or a slide-rule is needed. The three-place logarithms in Table 23 are of ample accuracy.

7. How does the brightness of Aldebaran compare with that of $\beta$ Canis Majoris (Tables 20 and 21)?

*Ans.* Aldebaran is 2.4 times brighter

8. What is the magnitude of a double star whose components are each of the fourth magnitude?

*Ans.* 3.25

9. What is the magnitude of a double star whose components are of magnitudes 2 and 3?

*Ans.* 1.64

10. What is the magnitude of the faintest star visible in a 15-inch telescope?

*Ans.* 14.9

Table 23

LOGARITHMS

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 000 | 041 | 079 | 114 | 146 | 176 | 204 | 230 | 255 | 279 |
| 2 | 301 | 322 | 342 | 362 | 380 | 398 | 415 | 431 | 447 | 462 |
| 3 | 477 | 491 | 505 | 519 | 531 | 544 | 556 | 568 | 580 | 591 |
| 4 | 602 | 613 | 623 | 633 | 643 | 653 | 663 | 672 | 681 | 690 |
| 5 | 699 | 708 | 716 | 724 | 732 | 740 | 748 | 756 | 763 | 771 |
| 6 | 778 | 785 | 792 | 799 | 806 | 813 | 820 | 826 | 833 | 839 |
| 7 | 845 | 851 | 857 | 863 | 869 | 875 | 881 | 886 | 892 | 898 |
| 8 | 903 | 908 | 914 | 919 | 924 | 929 | 934 | 940 | 944 | 949 |
| 9 | 954 | 959 | 964 | 968 | 973 | 978 | 982 | 987 | 991 | 996 |
| 10 | 000 | 004 | 009 | 013 | 017 | 021 | 025 | 029 | 033 | 037 |

# CHAPTER 16

## THE MOTIONS OF THE STARS

**roper Motion.** The stars, which are so often called "fixed stars" to inguish them from the planets, in reality possess rapid motions which evident in different ways. When observations of the right ascension declination of a star, carefully made on two or more different dates, compared, it is always found that the apparent position has changed. rge part of this change is due to precession, nutation, and aberration, ch, being common to all stars in a given small region of the sky, are ietimes called "common motions." These changes, and also the ıal parallactic displacement which is perceptible in the nearest stars, of course really due to the motion of the Earth. When all these effects been accounted for, however, there remain small and usually random ıges of apparent position which increase steadily with the passage of s, and which, being proper or peculiar to individual stars, are called **per motions.**

roper motion must necessarily be expressed in units of angular measnent. It is in all cases small—only a few seconds per century for most he stars in which it has been detected. It is determined by comparing catalogues of widely separated epochs or photographs made with

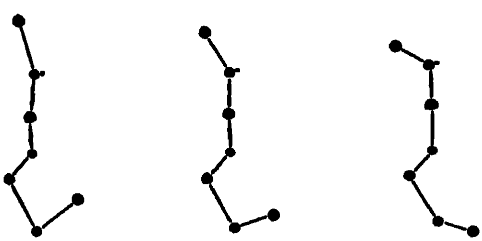



Fig. 229. *The Changing Dipper.*

the same instrument at intervals of several years. The greatest proper motion known is 10″.3 a year, that of a tenth-magnitude star in Ophiuchus which is usually referred to as "Barnard's proper-motion star," having been discovered photographically by Barnard at the Yerkes Observatory in 1916. The only bright stars with a proper motion of more than 2″ a year are $\alpha$ Centauri, 3″.7, and Arcturus, 2″.3. These motions are so slow that hundreds of years must elapse before the stars change their alignment with other stars by an amount noticeable to the unaided eye (Figure 229).

**Components of a Star's Motion.** Proper motion, being a change of *apparent* position (page 8), is only the projection of the star's motion upon the celestial sphere. In any of the three cases presented in Figure 230,

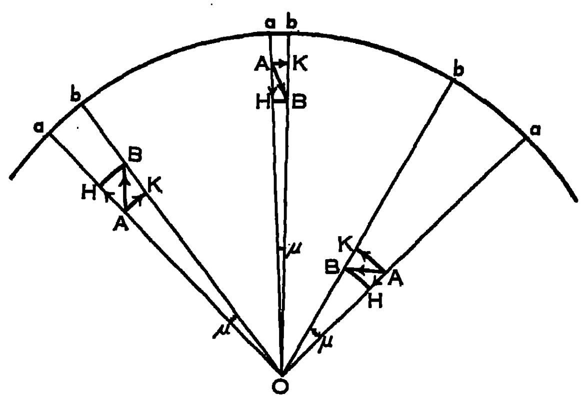



Fig. 230. *Components of a Star's Motion.*

let the star move in one year from $A$ to $B$, relative to the Solar System $O$. Then the proper motion will be the arc $ab$ of the celestial sphere, measured by the angle $AOB$ or $\mu$.

The distance $AB$, which may be called the star's **space motion**, cannot be determined directly, but only as the resultant of the **radial motion**, $AH$, and the **cross motion** (often called tangential motion), $AK$. These motions are best expressed as velocities, in kilometers per second.

The cross motion may be computed from the proper motion and the heliocentric parallax if these are known. If $D$ is the distance of the star from the Solar System expressed in astronomical units, and $\mu$ the annual proper motion in seconds of arc, then the cross motion in astronomical units per year is equal to $\mu D/206265$, or, since (page 331) $D = 206265/p$, simply to $\mu/p$. Let the cross motion in kilometers per second be $\chi$;

then[1]

$$x = \frac{4.74\,\mu}{p}.$$

The radial velocity is determined in kilometers per second by measurements of the Doppler-Fizeau displacement in the lines of the star's spectrum. To the observed radial velocity must be applied a correction, known as the reduction to the sun, for the motion of the observer; the result is the radial motion of the star referred to the center of the Solar System and represented by the line *AH* in the figure. With existing spectrographs the radial velocity of a star can be determined with an uncertainty of only about one kilometer per second if the lines of the spectrum are numerous and sharp, as they are in the case of the G-type or redder stars; the velocities of stars which have broad, diffuse lines are not so accurately known.

The reduction to the Sun is the projection of the observer's heliocentric velocity upon the line of sight to the star, and comprises two parts, one due to the earth's rotation and the other to its revolution. The former is zero for a star at the pole of rotation (and would be zero also for an observer at the Earth's pole, wherever the star might be); for an observer in latitude 0° and a star in declination 0°, it varies from $-0.47$ km./sec. at hour-angle $6^h$ to $+0.47$ km./sec. at hour-angle $18^h$. The annual part of the reduction to the Sun is always zero for a star at the pole of the ecliptic. On the ecliptic, it ranges from zero (for a star in opposition to the Sun) to a maximum of $\pm 30.27$ km./sec. for a star in a direction perpendicular to the Earth's line of apsides at the time of the Earth's perihelion passage when the observer's orbital motion is swiftest. For any star between the ecliptic and its pole, the annual part of the reduction for a point on the ecliptic in the same longitude must be multiplied by the cosine of the star's latitude.

In Figure 230, the angle $\mu$ is of course enormously exaggerated. *AKBH* is really always sensibly a rectangle, and so its diagonal may be computed by the Pythagorean principle; that is, the space motion of a star is the square root of the sum of the squares of its cross motion and its radial motion. From the foregoing discussion it appears that its determination depends upon *three distinct kinds of observation:* that of proper motion, by comparison of the apparent positions of the stars at dates many years apart; that of the distance of the star, most directly determined by the heliocentric parallax derived from observations of position about six months apart; and that of radial velocity, from an observation of the spectrum.

[1] 4.74 = number of kilometers in the astronomical unit ÷ number of seconds in a year, as the reader may easily verify.

Proper motions have been studied since 1718, when Halley detected the motions of Sirius, Arcturus, and Aldebaran (then called Palilicium) by comparing their observed places with the positions given in Ptolemy's catalogue. Distances have been known for some of the stars, as we have seen, since 1838. Radial velocities were first measured by Huggins in 1866, but his observations, being visual, were very inaccurate. Accurate photographic determinations of radial velocity date from the work of Vogel and Scheiner at Potsdam about 1890 and that of Campbell at the Lick Observatory in 1892.

**Magnitude of Stellar Velocities.** The great majority of known velocities of stars relative to the Solar System are *less than fifty kilometers a second*, and so are comparable to the orbital velocities of the planets. A few stars move much more rapidly than this, and about fifty are known to have speeds of more than 100 km./sec. Table 24 gives the velocities with respect to the Solar System of some of the most rapidly moving stars. Slightly higher velocities than these are found in the globular star clusters and Magellanic clouds, and much higher ones in the extra-galactic nebulae. As the great distances of these objects preclude accurate determinations of their cross motions, our information consists of radial velocities only.

The columns in Table 24, at the right of that which gives the star's declination, contain the following data: the annual proper motion, $\mu$; the position angle $\rho$ (page 354) of its direction; the heliocentric parallax; the radial velocity, $V$; the cross motion, $\chi$; and the space motion, $S$.

As early as 1903, Frost noted that the B stars moved more slowly than others, and in 1910 it was discovered by Campbell from radial velocities and by Boss from proper motions that the average velocities of stars increase in passing through the spectral series from B to M, being about 6 km./sec. for the B stars and about 17 for the M stars. As might be expected, the dwarf stars of each type have higher velocities than the giants of the same type.

**The Motion of the Solar System.** If all the stars were known to be fixed relative to one another, as by a framework, their apparent motions would necessarily be explained by a motion of the observer relative to the frame. Since the apparent displacement of a star produced by the Earth's *orbital* motion is in a closed curve, progressive changes such as proper motion and radial motion could be produced only by a progressive traveling of the whole Solar System.

Let the circle in Figure 231 represent the infinite celestial sphere, and let the Sun, carrying the Earth and the other planets with it, move from $S$ to $S'$ in a hundred years, while the stars remain at rest. The proper mo-

Table 24

STARS OF RAPID MOTION

| Name | Mag. | Spec. | $\alpha$1900 | $\delta$1900 | $\mu$ | $\rho$ | Parallax | V km./sec. | $\chi$ km./sec. | S km./sec. |
|---|---|---|---|---|---|---|---|---|---|---|
| Van Maanen's star[a] | 12.34 | F0 | $0^h$ 44 | + 4°.9 | 3".01 | 155° | 0".24 | + 240[a] | 57 | 247 |
| Helsingfors 956 | 7.8 | G0 | 1 3 | + 61.0 | 0.64 | 85.5 | 0.02 | − 325 | 152 | 359 |
| $\mu$ Cassiopeiae | 5.26 | G5 | 1 2 | + 54.2 | 3.76 | 114.5 | 0.13 | − 97 | 137 | 168 |
| e Eridani | 4.30 | G5 | 3 16 | − 43.2 | 3.16 | 76.4 | 0.16 | + 87 | 94 | 128 |
| BeB 1366 | 8.9 | F2 | 4 9 | + 22 | 0.54 | 124.9 | 0.01 | + 339 | 256 | 425 |
| $o_2$ Eridani | 4.48 | G5 | 4 11 | − 7.8 | 4.08 | 212.7 | 0.21 | − 42 | 92 | 101 |
| Kapteyn's star | 8.8 | M0 | 5 8 | − 45.0 | 8.76 | 130.5 | 0.27 | + 242 | 154 | 287 |
| Wolf 359 | 13.5 | M6e | 10 52 | + 7.6 | 4.84 | 232 | 0.41 | ....... | 56 | ....... |
| 22H Camelopardalis | 7.60 | M2 | 10 58 | + 36.6 | 4.78 | 186.8 | 0.41 | − 87 | 55 | 105 |
| B0 7899 | 8.9 | Ma | 11 0 | + 44 | 4.52 | 282.1 | 0.18 | + 65 | 119 | 136 |
| Innes's star | 12.5 | ....... | 11 12 | − 57.0 | 2.69 | 291 | 0.07 | ....... | 182 | ....... |
| Groombridge 1830 | 6.46 | G5 | 11 47 | + 38.4 | 7.05 | 145.3 | 0.10 | − 97 | 334 | 348 |
| Wolf 489 | 13 | ....... | 13 32 | + 4 | 3.94 | 252 | ....... | ....... | ....... | ....... |
| Proxima Centauri | 10.5 | M? | 14 23 | − 62.3 | 3.68 | 282.9 | 0.76 | ....... | 23 | ....... |
| $\alpha$ Centauri | 0.06 | G0, K5 | 14 33 | − 60.4 | 3.68 | 281.4 | 0.76 | − 22 | 23 | 32 |
| $OA_s$ 14320 | 9.9 | G5 | 15 5 | − 15.9 | 3.68 | 195.7 | 0.03 | + 307 | 583 | 660 |
| Barnard's star | 9.67 | M5 | 17 53 | + 4.4 | 10.30 | 355.9 | 0.53 | − 117 | 92 | 149 |
| 61 Cygni | 5.12 | K6 | 21 2 | + 38.3 | 5.27 | 52.1 | 0.30 | − 64 | 84 | 105 |
| $\epsilon$ Indi | 4.74 | K5 | 21 56 | − 57.2 | 4.70 | 123.4 | 0.29 | − 39 | 77 | 89 |
| Cordoba 31353 | 7.44 | Ma | 22 59 | − 36.3 | 6.90 | 79.3 | 0.27 | + 12 | 121 | 122 |
| Cordoba 32416 | 8.3 | Ma | 23 59 | − 37.8 | 6.11 | 112.7 | 0.22 | + 26 | 132 | 134 |

[a] See page 381.

tions of stars *B*, *C*, *D*, etc., would be in the directions of the arrows, away from the point *X* toward which the Sun is moving and toward the opposite point *N*. The size of these proper motions would depend upon the speed of the Sun's motion, the star's distance from us, and the star's angular distance from the point *X*, being zero for stars *A* and *E*. The radial velocity of star *A* would be negative and that of *E* positive, of a numerical value equal to the speed of the Sun; the radial velocities of other stars would be less, depending on their apparent distances from *X* or *N*.

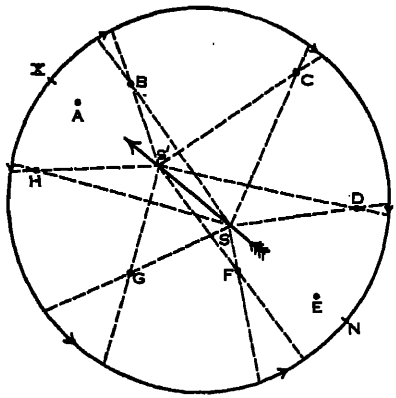



Fig. 231. *Motion of the Solar System.*

The fact that the stars are not at rest complicates matters, but by taking the average apparent motion (proper or radial) of a number of stars in each of many parts of the sky the solar motion may be determined with respect to the system of stars so chosen. The problem is analogous to that of the motion of a pedestrian through a swarm of gnats; the average of his velocities with respect to a number of the gnats should give his velocity with respect to the swarm, although the individuals of the swarm may be flying in many directions at different speeds.

As early as 1783 Sir William Herschel inferred from the proper motions of thirteen stars—all whose motions were then known—that the Solar System was traveling toward a point in the constellation Hercules; but not knowing the distances or radial velocities of any stars, he was unable to estimate the speed. Herschel applied the term **apex of the Sun's way** to the point *X* toward which the Sun is moving, and called the opposite point *N* the **antapex.**

Herschel and others improved his result as additional proper motions became known, and when distances also became available estimates of the speed of the Sun were made. During the twentieth century more accurate determinations of the solar motion have been made by using the radial velocities of stars. Boss has determined the position of the apex from the proper motions of the 6188 stars in his *Preliminary General Catalogue*, and Campbell and Moore of the Lick Observatory have determined both the direction and the speed from the radial velocities of more than 2000 stars in all parts of the sky.

Modern results agree that the Solar System is moving at a speed of **20 km./sec.** toward an apex near $\alpha = 18^h$, $\delta = +30°$, not far from the

direction of the star $\mu$ Herculis. The antapex is in the constellation Columba, about 15° south of Rigel. Expressed in other units, the speed of the Sun with respect to the stars around us is about twelve miles a second, four astronomical units a year, or one light-year in 16,000 years. Only ten million years ago—not very long, speaking astronomically or geologically—our system was about 600 light-years in the rear of its present position, in the general neighborhood of the bright stars of Orion.

It is important to remember that motion can be described only as *relative* to some point or points of reference. The motion of the Solar System described above is relative to the mean position in space of the stars used to determine it. These stars are our nearer neighbors, and are believed to belong mostly to what is called the local star cloud (page 448). Other considerations show that the whole group of stars, with many others, is being carried by the rotation of the Galaxy (page 448) at a much greater speed in a different direction.

**Motus Parallacticus and Motus Peculiaris.** The apparent motion of a star away from the solar apex or toward the antapex, which it possesses by virtue of the Sun's motion relative to the whole aggregation of stars, is called its *motus parallacticus;* the remaining apparent motion when this is eliminated from the proper motion is called the *motus peculiaris.* Dividing the *motus parallacticus* by the sine of the star's apparent distance from the solar apex gives the **secular parallax,** which may be defined as the apparent motion of the Solar System as seen from a point situated at the star's distance and in a direction at right angles to that of the solar motion. On the assumption that the peculiar motions of all the stars in a group are random, the secular parallax of the group may be calculated from the average of their proper motions; and the motion of the Solar System in a long interval, say a hundred years, affords a long base line which with the secular parallax gives the distance of the group. Although inapplicable to single stars, the method is valuable in statistical studies; it was from such studies that Kapteyn made his determination of the size of the discoid Galactic System (page 445).

**Moving Groups of Stars.** The study of stellar motions has disclosed a number of groups of stars in which the various members are moving through space with equal speed along parallel lines. In some cases the individuals of the group are not especially near one another, and their association would not be suspected from their positions in the sky. One of the most interesting examples is the **Taurus moving cluster,** discovered about 1915 by L. Boss and now known to include about a hundred stars, among them most of the bright stars of the Hyades.[2] Boss found that the proper motions of these stars converge toward a point of the sky a little east of $\alpha$ Orionis. Since, from the effect of perspective, apparent convergence results from actual parallelism, it is reasonable to suppose

[2] But Aldebaran, the brightest of the Hyades, is not a member.

that the space motions of these stars are parallel; and this is confirmed by the computed space motions of a number of the stars for which radial velocities have been obtained. Moreover, the space motions are equal within the probable value of the errors of observation, and it is highly probable that they agree within a small fraction of a kilometer per second; for if they did not, the cluster could not remain compact for many million years and it is unlikely that its age is not far greater than this.

If, in addition to the proper motions of the stars in a moving group, the radial velocity of one of their number can be obtained, then the cross motions, space motions, distances, and luminosities of all the stars of the group may be readily and accurately computed. To explain this, let the Solar System be at $O$ (Figure 232) and let the space motion of one of the stars of the group be from $A$ to $B$. Draw the line $OX$ parallel to $AB$; it will meet the celestial sphere at the "convergent" of the moving star-group—i.e., at the point toward which the proper motions converge, for parallel lines meet at the surface of the celestial sphere. The angle $AOX$ (which is the angular distance of the star from the convergent) equals the angle $CAB$ made by the direction of the star's motion with the line of sight. The radial velocity is represented by the side $AC$ of the rectangle $ACBD$, of which the space motion $AB$ is the diagonal and the cross motion is the side $AD$. Let the radial, space, and cross motions be $V$, $S$, and $\chi$, respectively, and let the angle $AOX$ or $CAB$ be $\theta$. Then

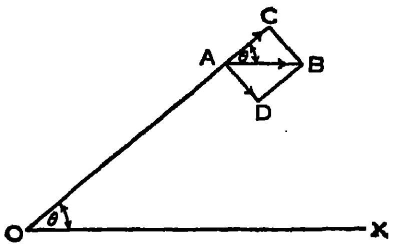


Fig. 232. *Finding the Distance of a Member of a Moving Star Group.*

$$S = \frac{V}{\cos\theta} \text{ and } \chi = S\sin\theta = V\tan\theta.$$

The heliocentric parallax, proper motion, and cross motion are connected, as we have seen (page 347), by the relation

$$\chi = \frac{4.74\,\mu}{p}$$

whence

$$p = \frac{4.74\,\mu}{\chi} = \frac{4.74\,\mu}{V\tan\theta}$$

from which the parallax, and hence the distance and luminosity, of the star may be found. Since $S$ is common to all the stars of the group, while $\mu$ and $\theta$ are known for each, a similar computation may be made for every star of the group.

It is known in this way that most of the stars of the Taurus moving cluster are contained in a roughly globular space about 35 light-years in diameter and 130 light-years distant, and that the brightest members are many times more luminous than the Sun. Their space velocity is about

46 km./sec. Presumably, there are among the stars of the group many interloping stars which do not partake of the group motion. About 800,000 years ago the group passed its perihelion at a distance of about 65 light-years. After about 65,000,000 years more, it will have receded so far as to appear as a sparse globular cluster about 20′ in diameter.

The Ursa Major group includes five of the seven bright stars of the Big Dipper, also Sirius, $\delta$ Leonis, $\alpha$ Coronae, $\beta$ Aurigae, and $\beta$ Eridani—stars in apparently widely separated parts of the sky. The Sun and many other stars are within the cluster as temporary interlopers. The group occupies a more or less spherical region about 500 light-years in diameter, and is divided by a gap about 50 light-years wide. It is moving at a speed of 29 km./sec. toward a point in the Milky Way southwest of Altair.

The stars of the Pleiades form a moving group, as Proctor pointed out many years ago. Other moving groups are in Perseus, Orion, and the region of Scorpius and Centaurus.

**Motions of Different Classes of Stars.** Strömberg has pointed out that if stars are classified according to their properties such as mass, luminosity, spectral type, velocity, and the like, it is found that the space motions of the different classes with respect to the Sun vary greatly. First, the individual motions of the stars in most classes are greater than the group motion and are prevailingly in two opposite directions, a fact which was detected a quarter century earlier by Kapteyn and attributed by him to two oppositely directed streams of stars. Second, the high-speed classes, such as the globular star clusters for which the group velocity is about 300 km./sec., all seem to be moving toward the same point of the sky, in the Milky Way near the border of Cygnus and Cepheus. These results are in harmony with the well-established theory of a rotation of the Galaxy (page 448), according to which the stars of this system are revolving around a galactic center or nucleus. The Kapteyn star streams flow in directions approximately toward and away from this nucleus and may be attributed to motion in highly eccentric orbits which, like that of some comets in the Solar System, is largely parallel to the line of apsides. The apex of the motion of the high-speed classes is in the galactic plane, in galactic longitude about 90° different from that of the nucleus, and in a direction about opposite that in which the Solar System is being carried by the galactic rotation; and so the apparent uniformity and rapidity of the motion of such objects as the globular clusters are explained by the motion of the Sun and its seemingly sluggish neighbors in the opposite direction.

**Binary Stars.** The term **double star** is sometimes applied to any two stars that lie close together on the celestial sphere. Ptolemy applied the Greek equivalent of the term to $\nu$ Sagittarii, which consists of a pair of fifth-magnitude stars about 14′ apart. Similarly, Mizar and Alcor, in the handle of the Great Dipper, which are separated by about 12′; $\alpha_1$ and $\alpha_2$

Capricorni, about 6′ apart; and $\epsilon$ Lyrae, in which the separation is 3½′, might be called double stars. In the stricter sense in which astronomers use the term, however, it is applied only to pairs in which the separation is not more than half a minute, or is much less if the stars are very faint.[3] A remarkably large number of stars are shown by a good telescope to be double, and many more have invisible companions which manifest their presence by their effect upon the motion or brightness of their primaries.

Until late in the eighteenth century it seems to have been generally assumed that the two components of a double star were in reality far apart and that they appeared close together merely through being almost in line with the Solar System, one behind the other. In 1789 Sir William Herschel began a careful study of double stars in the hope of detecting the parallactic displacement of the nearer component with respect to the more remote; but instead, after a number of years he found in a few pairs an orbital motion of one star around the other, showing that the two were really near together and subject to their mutual gravitation. To such a pair of physically connected stars Herschel gave the name of **binary star.** It is probable that the majority of the more than 20,000 known close visual double stars are binary, but in most cases the orbital motion is so slow that it cannot be detected until after many years' observation.

The presence of an invisible companion is detected in many cases by the spectroscope through the change of radial velocity produced in the visible star by orbital motion; in many others it is revealed by the change of brightness of the visible star when the companion passes before it. Stars whose duplicity is disclosed by the spectroscope are called **spectroscopic binaries;** those whose companions produce variations of brightness by occulting their primaries are called **eclipsing binaries** or (from the name of their prototype, $\beta$ Persei) **Algol variables.** This classification, being based on methods of observation, does not represent a difference in the real nature of the stars or of their motions. Many stars of each of the three classes belong also to another.

**The Study of Visual Double Stars.** The appearance of a double star is described by the magnitude, color, separation, and position angle of its components. The separation (denoted by $\rho$) is expressed in seconds of arc. The **position angle** (denoted by $\theta$) is defined as the angle be-

[3] Aitken adopts as a "working definition" of a double star, a pair whose apparent separation, $\rho$, and magnitude, $m$, are connected by the relation

$$\log \rho = 2.8 - 0.2\, m.$$

tween the great circle which bisects the two stars and the hour circle bisecting the brighter, and is measured from the north through the east.

The values of $\rho$ and $\theta$ are measured with the filar micrometer (page 48) as follows: At the beginning of the night's work a trail reading is taken upon the position circle of the micrometer by turning the telescope, with the driving clock stopped, to an equatorial star and placing the spider lines so that the star trails, in its diurnal motion, along one of the lines. The lines are then parallel to the equator, and 90° added to that reading of the position circle which corresponds to the exit point gives the north point or "zero reading." The telescope is then turned to the double star and one of the lines placed so as to bisect both components as at *AA*, Figure 233,

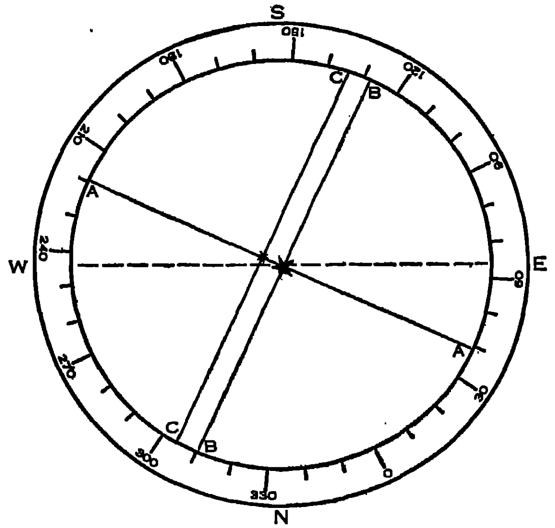



Fig. 233. *Measurement of Position-Angle and Separation.*

and the reading of the position circle in the direction of the fainter star is taken. This reading minus the zero reading is the position angle which, in the figure, is about 245°. After a number of settings are thus made and the mean taken, 90° is added to the mean and the micrometer is turned to this setting, which brings the spider lines perpendicular to the line joining the two stars. The separation is then measured by turning the micrometer screw until one line bisects each star, as at *BB*, *CC*, then interchanging the lines, placing the second line on the first star and the first line on the second; the difference between the readings of the screw, multiplied by the micrometer constant, gives *twice* the distance between the stars in seconds. This measurement also is of course repeated several times, and the results, both for position angle and separation, are carefully recorded. In the hands of a skilled observer, the whole operation requires only about ten minutes. Double-star astronomy is one great field in which visual observation has not been largely superseded by photography. Even in this field, as Strand has proved, photographic observations may be more accurate in the case of bright, wide doubles of which the components are not too different in brightness.

When an observation of this kind is repeated after the lapse of many years, if the star is a binary it will be found that $\rho$ or $\theta$, or more often both, have changed (Figure 234); and by means of successive repetitions—usually by successive generations of observers, for most binary periods are long—the orbital motion may be followed through a complete revolu-

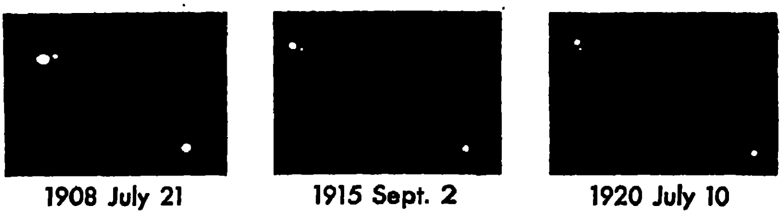



Fig. 234. *The Binary Krüger 60 and a Neighboring Star (as Seen in an Inverting Telescope). (Photographs by Barnard with 40-Inch Yerkes Refractor.)*

tion. To determine the relative orbit, a graphic method is used in which the brighter star is represented by a fixed point through which passes a fixed line representing the hour circle. A convenient scale of seconds of arc being chosen, the observed positions of the companion are then plotted according to position angle and separation. Except in a few cases where the motion is disturbed by a third body, the plotted points are found to lie, within limits set by the errors of observation, upon an ellipse (Figure 235).

While the bright component always lies within the ellipse, it does not, in general, appear at the focus. This is an effect of foreshortening; we usually see the binary not from a direction perpendicular to the plane of its orbit, but obliquely. The law of areas, however, is obeyed by the line joining the stars in the apparent as in the true orbit, since the projections of the areas on the plane tangent to the celestial sphere are proportional to the areas in the orbit. By mathematical analysis it is possible, from the apparent orbit, to compute all the elements except the *direction* of the inclination; from visual observations alone it cannot be determined whether, at a given point of the orbit, the companion is approaching or receding from the Earth; but this question also can be decided if even a single determination of the difference of radial velocity of the two stars can be secured. The value of the semimajor axis which results from the computation is necessarily expressed in seconds of arc; if the star's parallax is known, this may be transformed into astronomical units, and the sum of the masses may then be found (page 244).

Although thousands of visual binaries have been discovered, the orbits so far determined number only about 170; the periods of most are centuries long, and they have not yet been observed over a sufficient arc of their orbits.

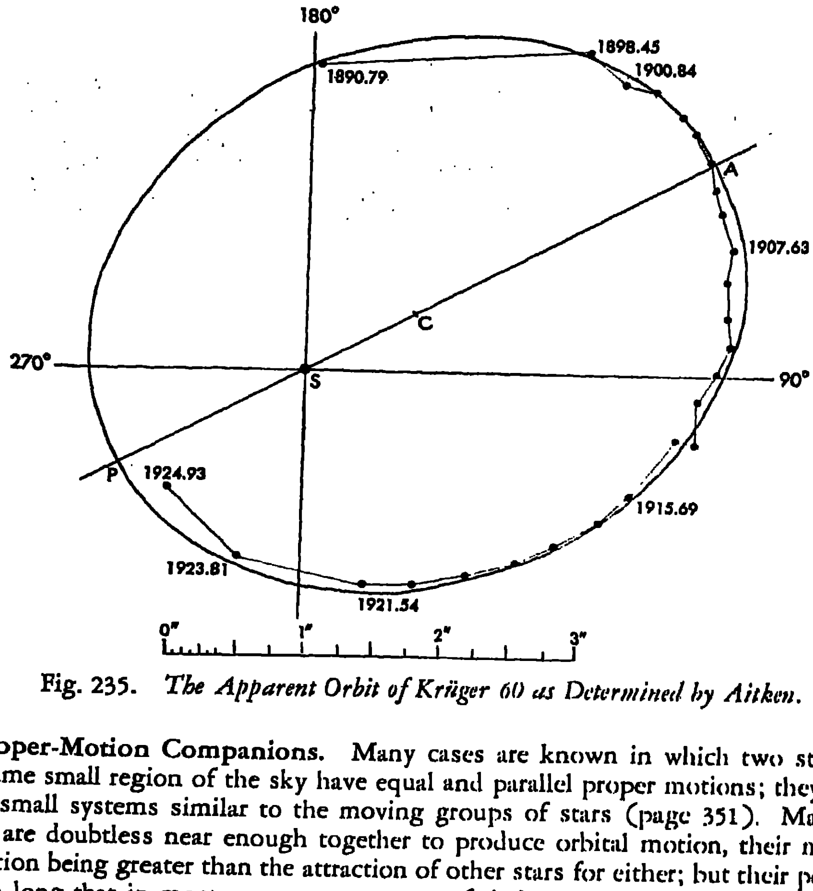



Fig. 235. *The Apparent Orbit of Krüger 60 as Determined by Aitken.*

**Proper-Motion Companions.** Many cases are known in which two stars in the same small region of the sky have equal and parallel proper motions; they thus form small systems similar to the moving groups of stars (page 351). Many of these are doubtless near enough together to produce orbital motion, their mutual attraction being greater than the attraction of other stars for either; but their periods are so long that in most cases no curvature of their paths has as yet been found. Besides such pairs of this kind as form close double stars, the most noteworthy case is that of Proxima and $\alpha$ Centauri, which have practically identical proper motions and parallaxes.

**The Study of Spectroscopic Binaries.** In the discussion of the problem of two bodies (page 240) it was noted that the center of mass of two mutually gravitating stars is unaffected by their motions and moves uniformly in a straight line, while the stars revolve in similar ellipses with the center of mass at their common focus. Figure 236 represents the orbits of two equally massive components of a binary whose center of mass is

at $C$. By definition, the straight line joining the two stars must always pass through $C$; hence, when one star is at $A_1$ the other is at $B_1$; when the first is at $A_2$ the second is at $B_2$; etc. Suppose the Solar System is in the

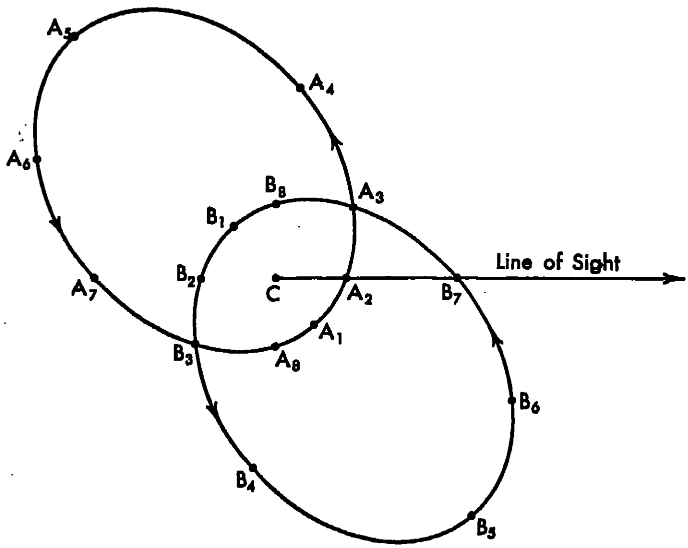



Fig. 236. *Orbits of the Components of a Spectroscopic Binary.*

direction of the right side of the page. From position 3 to position 6 star $A$ will, relatively to the center of mass, recede from the Solar System while $B$ approaches it; during the remainder of the revolution $B$ will recede and $A$ will approach.

The binary character of a star may be discovered spectroscopically in one of two ways, depending on the relative brightness of the components. (1) If one star is much brighter than the other, its spectrum alone will

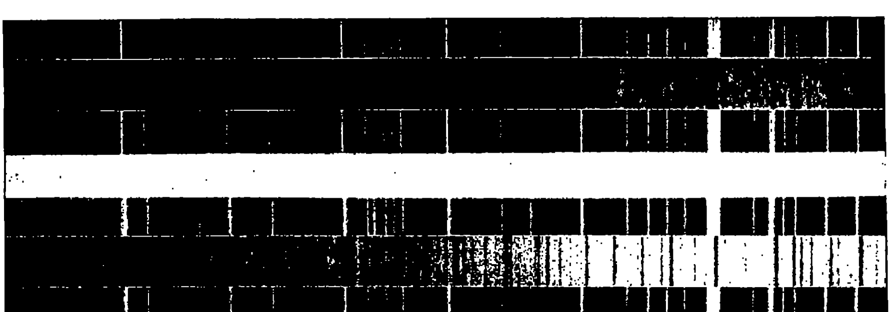


Fig. 237. *Spectrograms of $\mu$ Orionis (One Bright Component) from Yerkes Observatory.*

impress the photographic plate and its lines will be displaced alternately toward the red and toward the violet from the position corresponding to the velocity of the center of mass as the bright star alternately recedes and approaches (Figure 237). (2) If the two stars are about equally bright, they will be represented equally in the spectrum (Figure 238); then, when

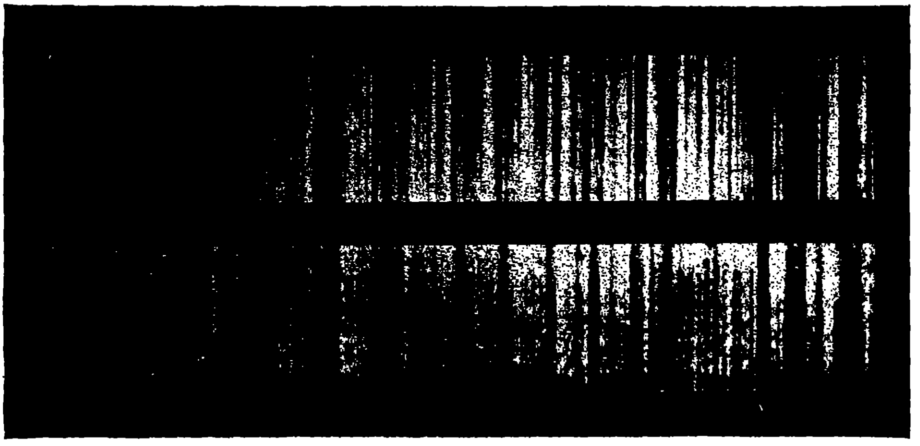


Fig. 238. *Spectrograms of Mizar (Lines Alternately Double and Single) from Yerkes Observatory.*

the lines of one are shifted to the red, the lines of the other will be shifted to the violet, as would occur at points 4, 5, and 8, whereas at points 3 and 6, where both stars are moving perpendicularly to the line of sight, the two spectra will be superposed. Thus, in case 1, all the lines of the spectrum are displaced together; in case 2, those lines which are common to both spectra appear alternately single and double.

Usually, of course, the plane of the orbit is inclined to the line of sight; the observed radial velocity, when reduced to the Sun, is then the projection, upon a plane containing the line of sight, of the resultant of the orbital motion and the motion of the center of mass. If the orbit plane is perpendicular to the line of sight, the orbital motion cannot be detected by the spectroscope.

The nature of the motion of the observed component is revealed by plotting the radial velocity, reduced to the Sun, as ordinates of a curve in which the abscissae represent time counted from a fixed epoch. This curve, called a **velocity curve**, is sinuous in form and repeats itself exactly if the motion is truly elliptical. If the orbit is circular, the velocity curve is a simple sine curve; otherwise, the ascending and descending portions

are not of equal steepness. In Figure 239 is shown the velocity curve of a binary of which the spectrum of only one component has been observed;

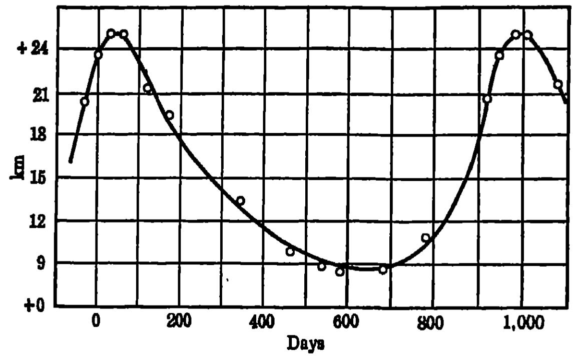



Fig. 239. *Velocity Curve of One Component of a Spectroscopic Binary (5 Tauri, by Harper, Dominion Astrophysical Observatory).*

in Figure 240 are shown the velocity curves of both components of a binary where the two are about equally bright and equally massive. The circles in either figure represent observations, and the height of the dotted line in Figure 240 represents the radial velocity of the center of mass of the pair of stars.

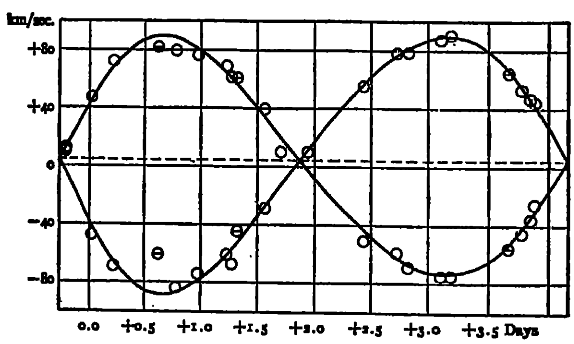



Fig. 240. *Velocity Curves of Both Components of a Spectroscopic Binary (Boss 373, by Sanford, Mount Wilson).*

It may be proved that the top and bottom of the velocity curve correspond to the points where the star crosses the plane through the center of mass and perpendicular to the line of sight; also, that a horizontal line bisecting the area of the curve represents the velocity of the center of mass. By mathematical analysis all the elements of the true orbit may be computed except the separate values of the major axis and the inclination of the orbit to the plane tangent to the celestial sphere;

instead of these, the quantity $a \sin i$ may be found in kilometers, which sets a minimum value to the major axis. The mass cannot be found without a knowledge of the inclination, but the quantity $m^3 \sin^3 i/M^2$ (called the **mass function**), where $m$ is the mass of the observed component and $M$ the sum of the masses of both, may always be found.

The components of spectroscopic binaries being in general much closer together than those of visual binaries, their periods are much shorter—in some cases, less than a day—and information concerning their orbits accumulates much faster. Although spectroscopic binaries have been studied only since 1888, more than 1200 examples are now (1945) known, and the number of well-determined orbits exceeds 325.

**Astrometric Companions: Stars or Planets?** An unseen companion revealed in the manner in which Bessel discovered the companion of Sirius—by variations in the visible star's proper motion—is called an astrometric companion. Such a discovery is analogous to the discovery of a spectroscopic binary, the difference being that in the latter the observed variation of velocity is in the line of sight, whereas in the former it is in a plane perpendicular to that line.

Since 1937 the astrometric study of stellar companions has been part of the program of the Sproul Observatory under the direction of P. van de Kamp, who has skillfully used the parallax photographs collected over the preceding twenty-five years by his predecessor, J. A. Miller. The work has already revealed astrometric companions to many of the nearer stars, suggesting that singleness among stars in general is the exception rather than the rule. Particularly interesting are the results obtained by Strand at the Sproul Observatory from his study of 61 Cygni, long known as a wide double and one of the nearest stars. Supplementing the Sproul plates with some made by Rutherfurd in 1870 to 1874—among the earliest stellar photographs ever made—Strand found that one of the two visible components is accompanied by an unseen companion revolving about it in an eccentric orbit at a mean distance of two astronomical units in a period of five years, and that the mass of this astrometric companion is only 1/60 that of the Sun or 16 times that of Jupiter. (He finds that each of the two visible components has a mass about 0.6 times the Sun's, that they revolve in a period of 720 years, and that their present distance apart is about 110 astronomical units.) The apparent motion in the large and small orbits is in the same direction, but the true orbit planes have a high mutual inclination.

The question has been raised as to whether this invisible companion is a star or a planet, and the answer must depend on our definition of the words. Some prefer to call it a dark star. If, with Russell and the Sproul astronomers, we make the distinction one of self-luminosity and argue that a body of such small mass cannot shine perceptibly by its own light, we may properly call it a planet. However, the differences between the system of 61 Cygni and our own planetary system (the only one heretofore so-called) are great: the primary in the former system is only about 36 times as massive as the companion, whereas the Sun is more than 1000 times as massive as its greatest planet; the Sun is a single star with a retinue of planets circling nearly in the same plane, whereas 61 Cygni is a pair of stars with a single (known) planet revolving in an eccentric ellipse.

It is noteworthy that Jupiter produces a variation in the Sun's velocity of only 0.03 km./sec. and that, seen (or rather unseen) from $\alpha$ Centauri, Jupiter would be of the 23rd magnitude and would reach a greatest elongation from its zero-magnitude primary of only 4″. With our present instrumental equipment we could not detect a Jupiter belonging to any distant star, to say nothing of lesser planets.

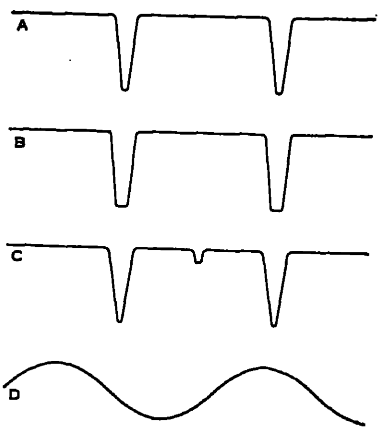


Fig. 241. *Light-Curves of Eclipsing Binaries.*

**The Study of Eclipsing Binaries.** Suppose two equal stars, one luminous and the other opaque, revolving in a plane which passes nearly through the Solar System. At each revolution the dark star must pass in front of the bright one and intercept a part of its light; but during the remainder of the period the bright star would shine with constant luster. Plotting magnitudes as ordinates and time as abscissae, we should then get a light-curve somewhat like *A*, Figure 241. Now suppose the dark companion to be smaller than the bright star and the inclination of the orbit to be such as to project the companion, at eclipse, wholly upon the surface of the primary. The light at minimum would then be constant for

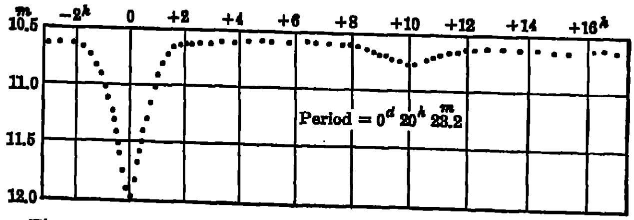


Fig. 242. *Light-Curve of the Eclipsing Binary RT Persei. (From Russell, Dugan, and Stewart's* Astronomy, *Ginn and Company.)*

a short time and the light-curve would be as at *B*. Third, suppose that the companion were not perfectly dark but shone faintly by its own light; the companion would then not only eclipse the primary, but would be eclipsed by it, producing a secondary dip in the light curve as at *C*, or as in Figure 242, which is a light curve derived from actual observations.[4] Finally, suppose the stars to be equal, both in diameter and in luminosity, and that they revolve almost in contact so that the interval between eclipses is short; the light-curve will resemble *D*.[5]

Algol, the eponym of this class of stars, was recognized as a variable star by Goodricke in 1783. Observations with the naked eye result in a light-curve similar to *A* (Figure 241), and Goodricke proposed the explanation which is now known to be correct, namely, that the star, although appearing single in all telescopes, had a close, dark companion revolving in an orbit turned edgewise to the Earth in the period of the observed light changes, $2^d\ 21^h$. This explanation was confirmed by Vogel of Potsdam in 1888 by showing the star to be a spectroscopic binary, the maximum velocity of recession occurring a quarter period before the eclipse and the maximum velocity of approach a quarter period after, while during eclipse and a half-period later the velocity is equal to that of the center of mass (Figure 243). See also page 368. All Algol variables which have been investigated spectroscopically have shown similar variations of radial velocity, and there is no question that all variable stars of this type are binaries. About 200 such stars are now known.

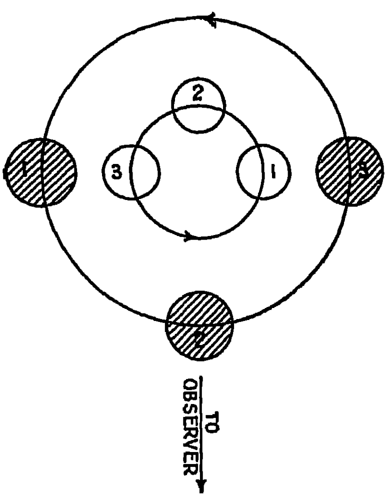



Fig. 243. *An Eclipsing Binary.*

From the light curve of an Algol variable, if it is precisely determined, many details can be learned concerning the two components and their orbits. The theory, begun by Pickering, has been extensively developed by Russell and applied by Shapley and others. The mere existence of an eclipse shows that the inclination of the orbit plane is near 90°. If both primary and secondary minima are observed, the intervals between them

[4] By R. S. Dugan of Princeton University. The abscissae are the number of hours since primary minimum, and each dot represents the mean of many observations made with a polarizing photometer. Although the two stars are evidently of very unequal brightness, they are inferred to be of nearly equal size because the primary and secondary minima are of nearly equal duration. The secondary minima occur halfway between primary minima, showing that the orbit is circular or else that the line of apsides is directed toward the Solar System.

[5] Curve *D* might also be produced by a single star which had the form of a prolate spheroid and rotated around a minor axis with the Solar System nearly in the plane of its equator.

determine the eccentricity and the position of periastron; for example, if secondary minima occur halfway between primary minima (the usual case), the orbit is circular.

The depth, form, and duration of the minima make known the inclination, the diameters of the two stars in terms of their distance apart, and their luminosities in terms of their combined light. If the light-curve slopes upward after primary eclipse and reaches a maximum just before secondary eclipse (as Stebbins's precise observations have shown to be the case with Algol), the companion is brighter on its hemisphere which faces the bright star, doubtless because of reflection or reradiation of the latter's light. If the curve reaches a maximum halfway between eclipses, the stars are ellipsoidal in form with their longest axes directed toward one another (and, in some cases, almost touching)—at syzygy they are placed endwise toward us and at quadrature sidewise, thus giving us maximum light in the latter position. The ellipsoidal form is not uncommon among Algol stars generally, as might be expected in view of the powerful tidal action of two great stars situated close together.

If velocity curves of both components are available from spectroscopic observations, the combination of photometric and spectroscopic data affords a determination of the diameters of the stars and of their orbits in kilometers, of their masses in terms of the Sun's mass, and hence of their densities. In most Algol stars, according to Shapley, there is evidence that the fainter star is self-luminous, and in no case is it proved to be entirely opaque. Often the bright star is smaller than its companion. The densities of all the eclipsing binaries of *B* and *A* spectral type are of the order of a tenth that of the Sun; their diameters and masses exceed those of the Sun. The densities of the few redder Algols known range from 0.000001 to 2.6☉.

**Some Binaries of Special Interest. Mizar, ζ Ursae Majoris,** the bright star in the bend of the handle of the Great Dipper, was the first visual double and also the first spectroscopic binary to be discovered. The visual duplicity was discovered by Riccioli of Bologna in 1650. The two components are 14″ apart, of 2.4 and 4.0 magnitude. No orbital motion has been found, but both components of Mizar, and also Alcor, 11′ away, partake of the common proper motion of the Ursa Major Group (page 353). From the group motion, the distance from the Solar System is well determined at 72 light-years, and so the distance from Mizar to Alcor is no less than a quarter of a light-year, and even the components of Mizar are at least 300 astronomical units apart—ten times the distance of Neptune from the Sun. In 1889 E. C. Pickering found with the objective prism that the lines of the spectrum of the brighter component of Mizar were alternately double and single, showing it to con-

sist of a pair of almost equally luminous revolving stars. The period of this pair is about 20.5 days. In 1925 Pease measured the distance and position angle of this close pair with the interferometer at Mount Wilson, finding a separation of from 0″.011 to 0″.013. By combining the spectroscopic and interferometric data, the elements of the real orbit were determined: the eccentricity is 0.53, the inclination to the plane perpendicular to the line of sight is 50°, and the sum of the semimajor axes of the two absolute orbits is 43,000,000 kilometers—about three-quarters of the mean distance of Mercury. In 1908, both Alcor and the fainter component of Mizar were found, by Frost at the Yerkes Observatory, also to be short-period spectroscopic binaries.

**Castor, α Geminorum,** was found to be a double star by the English astronomers Bradley and Pound in 1719. The magnitudes are 2.8 and 2.0 and the sepa-

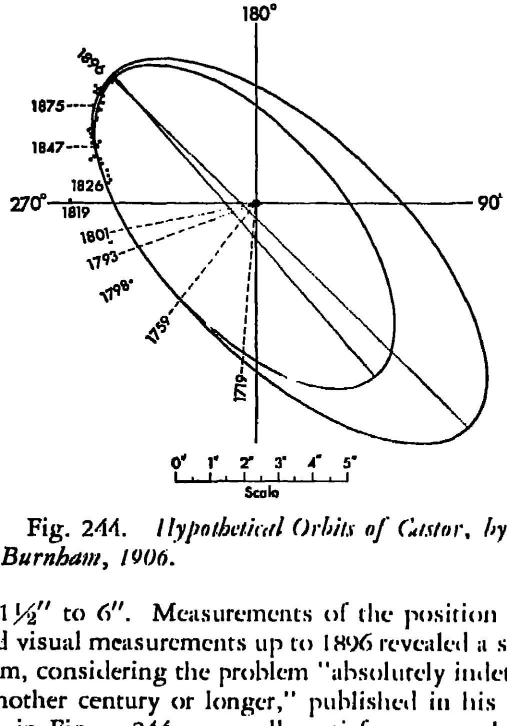



Fig. 244. *Hypothetical Orbits of Castor, by Burnham, 1906.*

ration varies from 1½″ to 6″. Measurements of the position angle beginning in 1719, and continued visual measurements up to 1896 revealed a slow orbital motion; but in 1906 Burnham, considering the problem "absolutely indeterminate and likely to remain so for another century or longer," published in his great catalogue the two ellipses shown in Figure 244 as equally satisfactory to the observations; yet in 1941 Strand, by combining recent photographic observations with the earlier visual ones, derived satisfactory elements and showed that the apparent orbit is the ellipse shown in Figure 245. The period is 380 years, the eccentricity of the real orbit is 0.36, and the real semimajor axis, derived by means of the star's parallax of 0″.07, is 86 astronomical units. Both components are spectroscopic binaries; the orbit of the fainter is nearly circular, with a period of 3 days and a radius which may be as small as 1,300,000 kilometers (about three times the radius of the Moon's orbit), whereas the period of the brighter is 9 days, its eccentricity 0.50, and the minimum possible value of its semimajor axis about the same as that of the fainter. Both components of the visual pair are of A0 type. More remarkable still, at a dis-

tance of 73″ from the bright pair is a ninth-magnitude M-type star which shares the parallax and proper motion of Castor and which, as found by Adams and Joy in

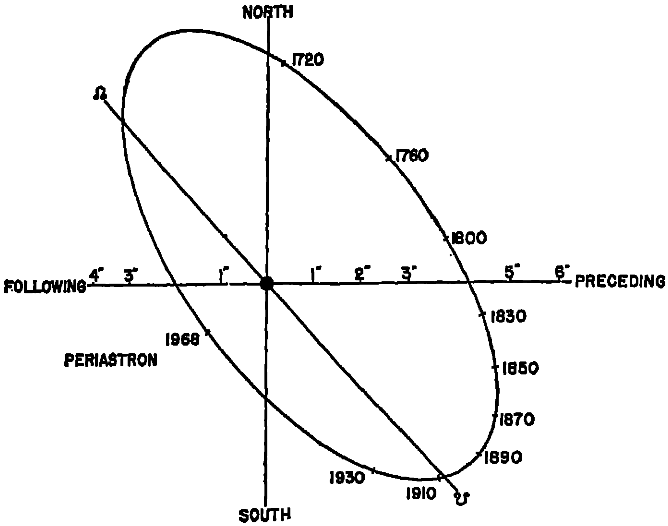



Fig. 245. *Orbit of Castor by Strand, 1941.*

1925, is a spectroscopic eclipsing binary with alternating double and single lines, of which those of hydrogen and calcium are *bright*. The period of this extraordinary pair is about 0.8 day. Its distance from the bright visual binary is about 1000 astronomical units.

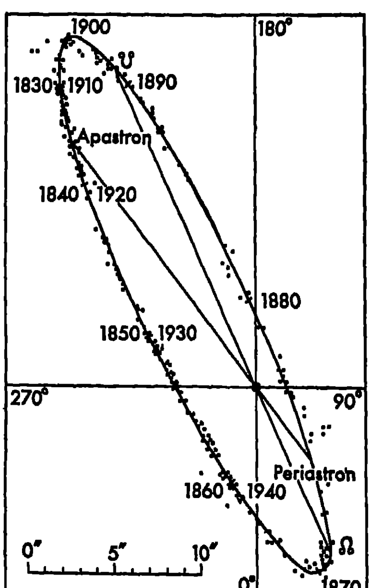



Fig. 246. *Orbit of α Centauri, by Finsen.*

Unusually exact information has been obtained about Capella, which was found to be a spectroscopic binary by Campbell in 1899, with a period of 104 days and a nearly circular orbit. In 1921 Anderson and Merrill succeeded in measuring the position angle and separation with the interferometer (page 375). From their work and that of Campbell are derived the elements $a = 0\farcs0536 = 126{,}630{,}000$ kilometers (between that of the Earth and that of Venus); $i = 41\fdg08$; $m_1 = 4.2 \times$ Sun, $m_2 = 3.3 \times$ Sun; heliocentric parallax $= 0\farcs0632$, distance = 52 light-years. Capella has a tenth-magnitude, M-type, proper-motion companion situated at a distance of 12′.

**α Centauri** has the largest apparent orbit of any known binary. Its orbit is highly eccentric ($e = 0.51$) and also highly inclined ($i = 79°$). Its apparent distance varies from 2″ to 22″. The actual distance varies from 11.4 to more than 35 astronomical units.

The radial velocities of its components have been separately studied (though not through a complete period) and its orbit has been determined spectroscopically as well as visually (Figures 246 and 247).

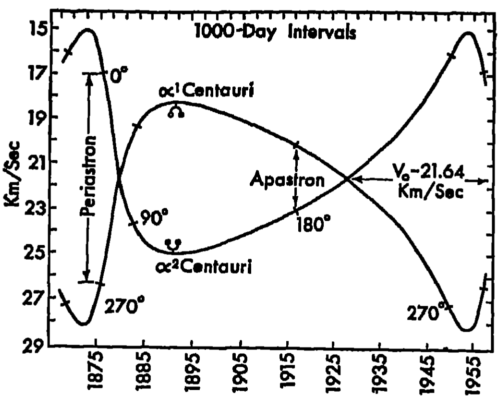



Fig. 247. *Velocity Curve of α Centauri, by Lunt.*

The visual binary **42 Comae Berenices,** period twenty-five years, is distinguished for having an orbit plane which, according to See, passes directly through the Solar System. If this is precisely true, the star must be an Algol-type variable with an eclipse about every thirteen years; but the event has never been observed.

Visual binaries are known with periods less than five years. The star having the shortest known period of velocity variation is **γ Ursae Minoris,** period 0.11 day. Its $a \sin i$ is only 35,000 kilometers, and this is outdone by the brighter visual component of **τ Cygni,** whose velocity curve indicates a value of $a \sin i$ of only 15,000 kilometers—a small fraction of the distance of the Moon from the Earth. It is probable that these two stars are not really binary, and that the observed velocity variation is due to a pulsation or rhythmic expansion and contraction of a single star, as has been inferred in the case of the Cepheid variable stars (page 400).

**ε** and **ζ Aurigae,** two of Capella's three Kids, are eclipsing spectroscopic binaries of special interest. The period of ε is about 27 years, and its eclipse lasts about a year. According to Struve, Kuiper, and Strömgren, the larger star of the pair has a diameter 3000 times that of the Sun but a mass only 30 times the Sun's, so that its density is only about a millionth that of ordinary air; it is invisible because its temperature is only about 1300° and its light is mainly infra-red; and its tenuous gas is made semi-opaque by ionization by the ultra-violet light of the smaller star, itself a supergiant of type F5.

The period of ζ Aurigae is 972 days, and its eclipse lasts 39 days. The primary is a K5 supergiant and the secondary is a B2 star with a diameter about $\frac{1}{80}$ that of the primary. Their brightness is so nearly equal that ordinarily the spectrum appears composite; but during the eclipse of the B star its lines disappear, and for about ten days before and after its eclipse the spectrum shows that the light of the B star is reaching us through a very extensive chromosphere surrounding the giant K star.

The complicated changes in the spectrum and brightness of **β Lyrae,** which are repeated in a period of 13 days, are attributed by Kuiper to a system consisting of two nearly equal egg-shaped stars revolving in that period in a plane passing through

the Solar System, with their sharper vertices almost together, surrounded by a nebula which is expanding at a speed of some 50 km./sec. and rotating at more than 300 km./sec. Kuiper considers that this nebula consists of a stream of gas flowing from the greater star to and around the smaller and that some of this gas is continually flowing out of the system.

The companion of Algol was considered to be perfectly opaque until the work of Stebbins with the selenium photometer in 1910 showed that there was a minute secondary minimum in the light-curve and that the curve sloped slightly upward preceding the minimum and downward following it. From the spectroscopic elements, the form of the light curve, and a study of the rotational velocity (page 375) made in 1924 by McLaughlin, it appears that the brighter star has a radius of 3.12, a mass of 4.72, a density of 0.16, and a luminosity of 160, all in terms of the Sun; and that the companion, of radius 3.68, mass 0.95, and density 0.02, is brighter on the hemisphere which is turned toward the primary, that hemisphere having a luminosity of 17 and the other 10. The distance between the centers of the two stars is constant at 10,500,000 kilometers. The eclipsing pair revolves around an invisible third body in a period of 1.9 years. The distance of Algol from the Solar System is about 100 light-years.

The binary system of Sirius, which is of great interest, is discussed elsewhere (pages 240, 380).

**The Rotation of Stars.** If a star is rotating around an axis which is not pointed directly toward us, one side must be approaching while the other recedes; and so, if the spectra of the two opposite limbs could be compared as in the case of the Sun (page 202), a relative Doppler shift

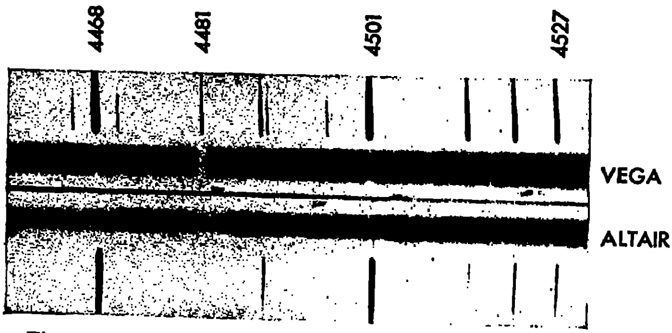



Fig. 248. *High-Dispersion Spectrograms of Vega and Altair (Yerkes Observatory).*

of the spectral lines would be observed. Since the stars are so far away that we can study only the mingled light from an entire hemisphere, what we actually observe is a widening of the lines. For example, the lines in the spectrum of Altair are broad and fuzzy in contrast to those in the spectrum of Vega, which are fairly narrow and sharp. From measurements of the distribution of intensity in the spectral lines, it is inferred that the

radial velocity of the equatorial limb of Altair is about 200 km./sec. (a hundred times that of the Sun's equator), and that Vega either does not rotate rapidly or rotates around an axis which passes nearly through the Solar System.

From extensive studies of this kind, Struve and Elvey of the Yerkes Observatory find that rapid rotation is common among stars of B and A spectral types but rare among single stars of the redder types, and that many close binaries of all types rotate rapidly.

## EXERCISES

1. Note the position of Regulus on the star maps in this book. What is the approximate value of the annual part of the reduction of its radial velocity to the Sun on November 23?

*Ans.* +30 km./sec.

2. At what sidereal time is the diurnal part of the reduction of the radial velocity of Regulus to the Sun a positive maximum?

*Ans.* 4 hours

3. Verify the cross motions and space motions of the first five stars in Table 24.
4. Using Map 1 and data from Table 24, show that $\alpha$ and $\beta$ Centauri will form a striking naked-eye double about 4000 years from now. (The proper motion of $\beta$ is so small as to be negligible.)
5. Refer to Figure 234 and state whether the orbital motion of Krüger 60 causes its position angle to increase or to decrease.
6. Is the orbit of the spectroscopic binary 5 Tauri (Figure 239) circular?
7. Estimate the velocity of the center of mass of the system of 5 Tauri. (This is the ordinate of a horizontal line drawn across the velocity curve in such a position that equal areas lie between the line and the curve, above and below the line. The computed value of the velocity of the center of mass of 5 Tauri is +14.2 km./sec.)
8. What is the period of 5 Tauri?

*Ans.* 960 days

9. What difference is there in the orientation of Figures 244 and 245?

# CHAPTER 17

# PHYSICS OF THE STARS

**Astrophysics.** Until after the middle of the nineteenth century, astronomy consisted almost exclusively of the study of the *positions* of the heavenly bodies—of where they were but not of what they were. With the advent of the spectrograph and other modern appliances, astrophysics was developed and it is now the most rapidly expanding branch of astronomy in both its practical and its theoretical phases. It includes the study of such physical properties of the stars as luminosity, temperature, size, mass, density, and chemical composition.

**Spectral Criterion of Luminosity.** It was discovered by Adams and Kohlschütter at Mount Wilson in 1914 that the relative intensity of certain lines in star spectra of the F, G, K, and M types differed in the spectra of stars of different luminosity. By a study of these slight but unmistakable differences in the spectra of stars whose absolute magnitude could be determined from their measured parallaxes, curves were formed which correlated the relative intensity of the "magnitude lines" (certain lines of hydrogen, calcium, strontium, and iron) with the stars' absolute magnitude. These curves then give the absolute magnitude of any star whose spectrum can be distinctly photographed; and by the formula (page 343)

$$\log p = \tfrac{1}{5}(M - m) - 1$$

it is possible to derive at once the star's parallax. "Spectroscopic parallaxes" thus determined have the great advantage over trigonometric parallaxes of being as reliable for distant stars as for near ones. The method has been extended to other spectral classes, has been applied extensively at Mount Wilson and Harvard, in Canada and England, and has more than doubled the number of stars whose distances are known with a reasonable degree of accuracy.

The reason for this absolute magnitude effect lies in the fact that, in general, intrinsically faint stars are both hotter and denser (they are also smaller) than intrin-

sically bright stars. Since spectral classification is based on properties that depend on ionization in the stellar atmospheres (and therefore on temperature and density), these differences must compensate in the average ionization of two stars that are judged to be of the same spectral type but are of different luminosities. It is because this compensation is not the same in all the chemical elements that there are differences of intensity of some of the individual lines.

**The Range of Stellar Luminosities.** The (intrinsically) faintest known visible star was discovered by van Biesbroeck in 1943. It is an eighteenth-magnitude proper-motion companion separated 74″ from BD + 4°4048 Aquilae, for which Schlesinger has found a parallax of 0″.170. The corresponding distance is 19 light-years, and so the little star's absolute magnitude is 19.2 and its luminosity 1/550000. Seen at the Sun's distance, it would be a little brighter than the full Moon; at a hundred light-years it would be too faint for detection. We may reasonably suppose that there are many stars not yet recorded that are equally feeble.

At the other extreme are a few stars, like Rigel and Deneb, which outshine the Sun thousands of times. Still brighter is the ninth-magnitude star S Doradûs in the large Magellanic cloud, which has, according to Shapley, a luminosity of 600,000, and even this has been exceeded for brief periods by some of the *novae* in spiral nebulae (page 388). The ratio of S Doradûs to van Biesbroeck's star is far greater than that of the most powerful searchlight to a candle, and it is emphatically true that "one star differeth from another star in glory."

The Sun is a modest member of the stellar host, exceeded in real brightness by a great majority of the visible stars. If removed to the distance of Aldebaran it would be invisible to the naked eye. Yet it is probable that a majority of all *existing* stars are fainter than the Sun, many being rendered invisible by their faintness and remoteness.

**Giants and Dwarfs; the Russell Diagram.** Between 1900 and 1910, as knowledge of the distances and luminosities of the stars increased, it became evident that many stars, especially the red ones, fell into two well-defined groups of widely different luminosity, to which Hertzsprung gave the names of **giants** and **dwarfs**. In 1913 the absolute magnitudes and spectral types of about 300 stars were known, and Russell assembled this information in a form now often referred to as a **Russell diagram,** plotting spectral type horizontally and luminosity vertically and representing each star by a dot. Figure 249 is such a diagram, compiled by Ruml and Pěkný of Prague in 1942 and representing 6412 stars.

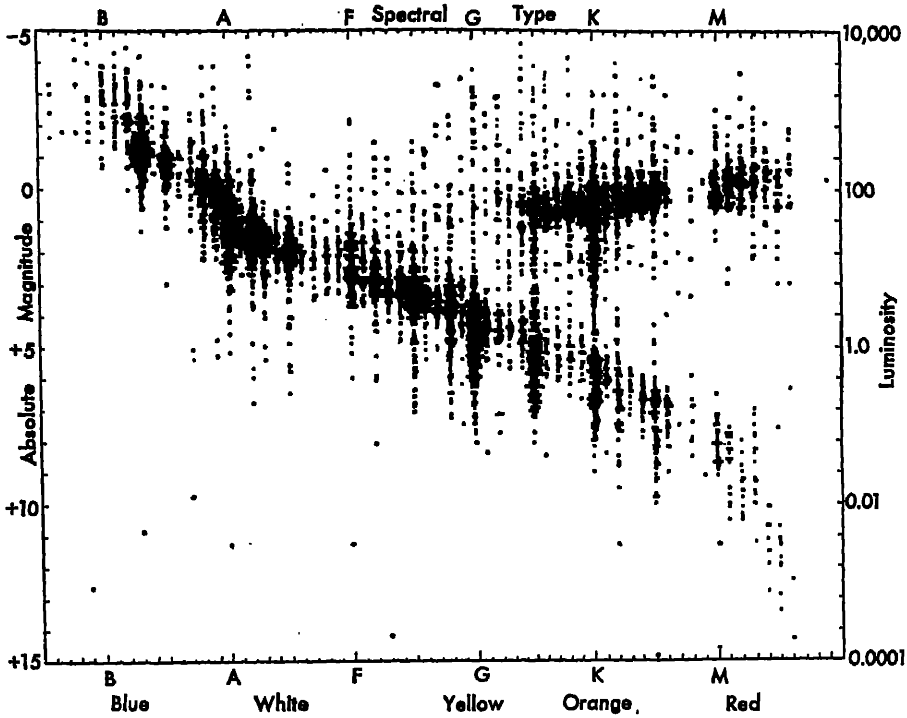



Fig. 249. *Russell Diagram Representing 6412 Stars (from* Astronomische Nachrichten).

Stars brighter than magnitude minus one, represented by scattered dots near the top, are often called supergiants. Note that, the scale of luminosities being logarithmic, they are many thousands of times more luminous than the subdwarfs near the bottom. Most of the dots obviously lie in two main branches of the Russell diagram—the giant branch and the dwarf branch. Although in the diagram about as many giants as dwarfs appear, it is reasonable to suppose that in space the dwarfs are far the more numerous since we can see the distant giants but not the distant dwarfs. The diagonal portion of the diagram which includes the white giants and red dwarfs is called the **main sequence**. The Sun, a rather bright dwarf, is located in the main sequence, about at the center of the diagram.

**White Dwarfs.** A number of stars are known which are both white and of low luminosity. The most famous of these is the companion of Sirius (pages 240, 380); others are van Maanen's star, the companion of $o^2$ Eridani, and probably the companions of Procyon and Mira. They are shown in the lower left-hand corner of Figure 249, at the left of and far

below the main sequence. It is probable that white dwarfs are fairly abundant in space, though they are not so in star lists.

**The Temperatures of the Stars.** The effective temperature of the radiating surface of a star, like that of the sun's surface (page 207), may be determined by means of the laws of radiation; but of course, on account of the faintness of the stars, the observations are more difficult. The principal data of observation are in the form of color indices (page 340) and, for a few bright stars, heat indices (page 341) and spectral energy curves (page 170). The temperature can also be estimated from the degree of ionization shown by the enhanced lines in the spectrum (page 178). The effective temperatures of stars of the principal spectral types are found to be approximately as shown in the table on page 331 and in Figure 227.

**Diameters of Stars Computed from Magnitude and Temperature.** The stars are so distant and their apparent diameters are so small that, even in the largest telescopes, it is impossible to measure them with such an instrument as a micrometer, as is done with the planets. The diameters of stars have, however, been determined by more indirect methods.

A star's rate of radiation in ergs per square centimeter per second may be computed from its effective temperature by Stefan's law. Its total radiation per second may be computed from its absolute magnitude and color index. The total surface in square centimeters equals the total radiation divided by the rate per square centimeter, and the diameter in centimeters is the square root of the total surface divided by $\pi$. To obtain the diameter in terms of that of the Sun, this value must be divided by $1.4 \times 10^{11}$, the number of centimeters in the Sun's diameter. Since the color index depends on the temperature, the latter may be eliminated from the computation and the various steps combined in a simple formula:[1]

$$\log D = 0.82\, I - 0.20\, M + 0.51$$

where $M$ is the star's absolute magnitude, $I$ its color index, and $D$ its diameter in terms of the diameter of the Sun.

Thus, for Sirius, whose color index is 0.0 and absolute magnitude 1.3, we have $\log D = 0.51 - 0.26 = 0.25$, whence $D = 1.8$ diameters of the Sun. At Sirius's distance of 8.8 light-years, this diameter subtends an angle of only 0.″006, and there is no wonder that it is imperceptible to direct observation.

[1] Derived in the compendious treatise on general astronomy by Russell, Dugan, and Stewart, p. 738.

**Direct Measurement of Star Diameters.** Beginning in 1920, the apparent diameters of a few stars have been directly measured at Mount Wilson with the **interferometer.** The method was described by Michelson as early as 1890 and had been suggested by Fizeau many years earlier. Observations have been made by Pease, using, in the earlier work, a special interferometer in connection with the 100-inch Hooker telescope. A beam of structural steel, attached to the upper end of the tube of the telescope, carries four mirrors, the two outer of which can be moved along the beam so that their distance apart may vary up to twenty feet. The light of the star is reflected by these two mirrors to the two others which are near the middle

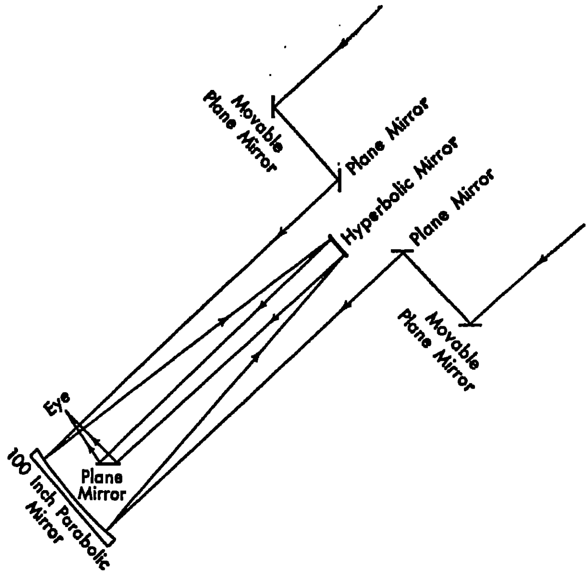



Fig. 250. *The 20-Foot Interferometer of the Mount Wilson Observatory.*

of the beam and by them to the 100-inch paraboloid, after which it follows the usual course of light in the Cassegrain form of the telescope and finally reaches the eye of the observer (Figure 250). The interference of the two beams of light thus coming from a point of the star produces, instead of the usual circular diffraction pattern (page 49), a set of exquisitely fine bright and dark fringes which appear somewhat like the teeth of a comb. If the star's apparent diameter is great enough, then for a certain separation of the mirrors the bright fringes corresponding to one limb of the star fall upon the dark fringes corresponding to the opposite limb, and the appearance becomes that of a continuous band of light. From the distance between the mirrors at which this disappearance of the fringes occurs, the angular diameter of the star may be accurately computed; and the real diameter in miles or kilometers may then be found with a degree of accuracy about proportional to that of the star's parallax. A fifty-foot interferometer, with independent objective and mounting, was later completed at Mount Wilson. Though such instruments can be applied only to the largest, nearest, and brightest stars, enough has been done with them to show that the diameters computed from color indices are not seriously in error.

The interferometer has been used also for the measurement of position angle and separation of close double stars in which the two components are of equal brightness.

**Diameters of Eclipsing Binary Stars.** It has been noted (page 364) that the form of the light-curve of an eclipsing binary makes known the diameters of its components in terms of their mean distance apart. Spectroscopic observations, if both spectra are visible, give the projection of this distance in kilometers as one of the elements of the relative orbit ($a \sin i$); and since $\sin i$ (which is always nearly equal to unity in an eclipsing binary) is also known from the light-curve, the scale of the system and the diameters of the stars in kilometers can then be readily computed.

Even if only the spectrum of the brighter star is observable, its diameter may sometimes be found by taking advantage of the spectroscopic effect of its rotation. It is safe to assume that both stars of an Algol pair rotate in the same direction and in the same period as they revolve, facing each other as the Moon faces the Earth—the tidal friction in so close a pair would inevitably bring about this result. Soon after the beginning of the principal eclipse, when the light-curve is descending to minimum, we receive light mainly from the receding half of the bright star's disk, the other half being hidden by the companion; during the ascent of the light-curve after mid-eclipse, we receive light mainly from the approaching half. Hence, the radial velocity departs from a smooth curve just before and just after the light minimum, and the amount of this departure makes known the rotational velocity of the bright star. From this velocity and the known period of rotation, the circumference of the star and hence its diameter are obtained in kilometers, and as the relative diameters of the two stars are known from the light-curve, the diameter of the companion may also be found.

It will be noticed that the determination of the diameter of an eclipsing binary does not depend on knowledge of the star's parallax or distance—an important advantage not possessed by either of the methods which are applicable to single stars.

**The Range of Stellar Diameters.** The range in the size of stars, like the range in their luminosities, is enormous. The small red dwarfs like Krüger 60 and Proxima are about the size of the larger planets of the Solar System, and the white dwarfs are smaller still, comparable to the Earth. The red giants, such as Betelgeuse and Antares, have diameters hundreds of times that of the Sun and could readily contain the orbit of the Earth; it has been inferred that the invisible component of $\epsilon$ Aurigae could contain the orbit of Saturn. (It should be borne in mind, however, that the terms giant and dwarf, as applied to stars, refer to luminosity and not necessarily to size.) Most stars of the main sequence have diameters

comparable to that of the Sun—ranging from perhaps ten times the Sun's diameter for the average B-type star to one-tenth the Sun's diameter for an M-type dwarf. The diameters of a number of typical stars are illustrated in Figures 251 (giants) and 252 (dwarfs), with Vega included in both diagrams to show the relative scale.

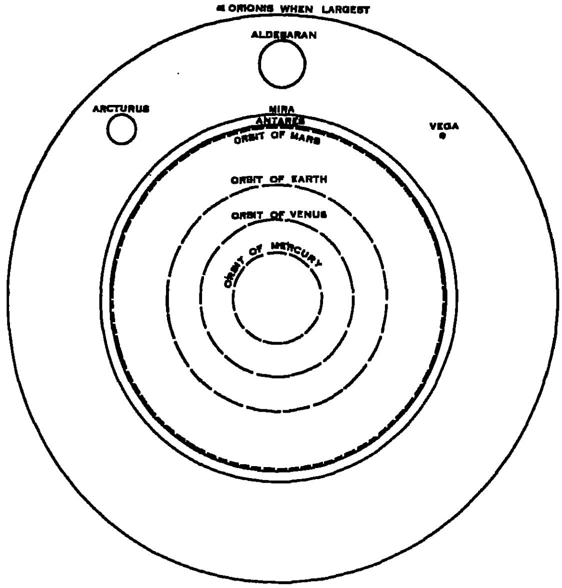



Fig. 251. *Dimensions of Some Giant Stars.*

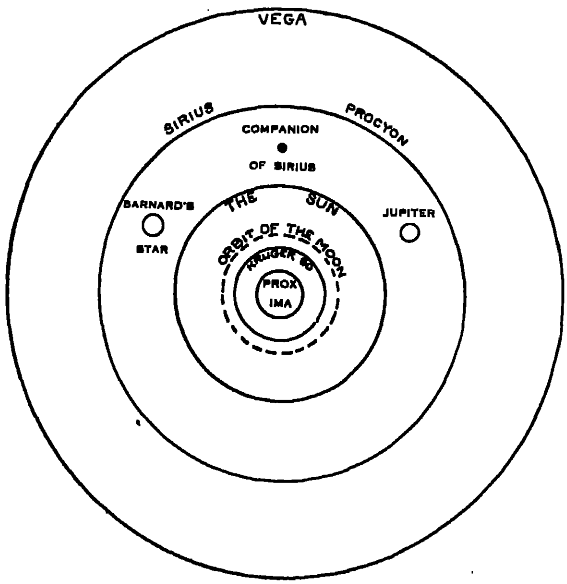



Fig. 252. *Dimensions of Some Dwarf Stars.*

**The Masses of Stars.** The mass of a star can be directly determined only if the star is a member of a binary system of which the parallax and orbit are known. We have (page 244)

$$m_1 + m_2 = \frac{a^3}{P^2} \tag{1}$$

where $m_1$ and $m_2$ are the masses in terms of the Sun's mass, $P$ the period in years, and $a$ the semimajor axis in astronomical units. For visual binaries, the semimajor axis is determined in seconds of arc. Call this value $a''$. If $p$ is the heliocentric parallax in seconds, the value of $a$ in astronomical units is given by

$$a = \frac{a''}{p}, \tag{2}$$

whence the sum of the masses

$$m_1 + m_2 = \frac{a''^3}{P^2 p^3}. \tag{3}$$

The separate masses of the two stars can be found in only a few cases, such as that of Sirius (page 240), in which the motion of one or both stars with respect to the center of mass is known.

From observations of spectroscopic binaries, the mass *function* (page 361) is readily determined, and if the value of $i$ is known also from visual or photometric observations the mass itself can be found. For estimating the average mass of a number of spectroscopic binaries, $\sin^3 i$ is assumed to have the value 0.59, which is the mean of all its values from $i = 0°$ to $i = 90°$.

There is no such wide range in stellar masses as we have noted in stellar luminosities and diameters. Certainly very few stars have masses as great as ten times, or as little as one-tenth, the Sun's mass. Stars even three times as massive as the Sun are rare, and Jeans estimates that not one star in 100,000 is ten times as massive as the Sun, although a very few stars of even greater mass than this are known. The visible star of smallest known mass (the faint component of Krüger 60) is one-fifth as massive as the Sun.

**The Interior of a Star.** A star is a body of intensely heated gas. So long as its form and size do not change, its parts must be in equilibrium under the forces which act upon them. These forces are: (1) the mutual gravitation of its parts, the resultant of which is directed toward the center and tends to shrink the star; (2) "gas pressure," due to the motion and collision of the particles, which increases with the temperature and tends

to make the star expand; and (3) the pressure of radiation (page 172), which also increases with the temperature and assists the gas pressure. At each point within the star, the two forces which depend on the temperature must together balance the weight of the overlying layers, and it is evident that the temperature and pressure must both increase toward the center. The interior of a gaseous star has been discussed from the standpoint of modern theoretical physics with rare skill by Sir Arthur Eddington, who finds that main-sequence stars, including the Sun, have central temperatures of tens of millions of Centigrade degrees. In red giants the central temperature is lower, but still to be expressed in millions of degrees, while in white dwarfs it is estimated by Eddington to be as high as 1,000,000,000°.

Referring to Figure 130 (page 171), which gives the wave length and intensity of radiation at different temperatures according to Planck's formula, the reader will see that at temperatures of millions of degrees the development of energy is stupendous and the radiation is mostly of very small wave-length (X-rays). The effects of this transcendent temperature as inferred by Eddington are:

1. The atoms within the star are ionized to a high degree, being in extreme cases completely stripped of their orbital electrons.

2. As a result the atoms, being much smaller than normal atoms, are packed more closely together and the density is enormous.

3. The pressure of radiation, though a feeble force at the surface of the Sun or of any star, is enormous inside—17 tons per square inch at a temperature of 1,000,000° and 170,000 tons per square inch at 10,000,000°.

4. The stellar material, consisting of partly or wholly ionized atoms and free electrons, is surprisingly opaque to the prevailing short-wave radiation, so that the latter leaks out from the star but slowly, by a process of absorption and reradiation in greater wave-lengths.

**Relation of Mass and Luminosity.** Eddington shows that the rate at which radiation escapes from a star is greater, the greater the ratio of radiation pressure to the total pressure inside the star, and that this ratio increases with the mass of the star. In other words, great mass is associated with high luminosity, and vice versa, notwithstanding the great disparity in the ranges of stellar masses and stellar luminosities. He has calculated the ratio of radiation pressure to total pressure for gaseous spheres of various masses, and finds that it is near zero for all masses up to $10^{32}$ grams, and near unity (i.e., the radiation pressure vastly exceeds the gas pressure)

for all masses over $10^{36}$ grams. The masses of most stars lie between these numbers; that of the Sun is $10^{33}$ grams. Perhaps, therefore, a body of gas having less than a tenth of the Sun's mass would be unable to shine by its own light, whereas one of a thousand times the Sun's mass would be subject to so great an internal pressure of radiation that it would be liable to burst.

Table 25[2] compares masses observed with those computed from Eddington's mass-luminosity curve for stars of different luminosities, the latter being expressed in terms of absolute visual magnitude.

Table 25

| Abs. Mag. | Mass Computed | Mass Observed | Abs. Mag. | Mass Computed | Mass Observed | Abs. Mag. | Mass Computed | Mass Observed |
|---|---|---|---|---|---|---|---|---|
| 17.5 | 0.064 | .... | 7.5 | 0.53 | 0.55 | − 2.5 | 10 | 12 |
| 15.0 | 0.11 | 0.14 | 5.0 | 0.92 | 0.91 | − 5.0 | 35 | .... |
| 12.5 | 0.18 | 0.22 | 2.5 | 1.72 | 1.72 | − 7.5 | 148 | .... |
| 10.0 | 0.31 | 0.34 | 0.0 | 3.70 | 4.00 | − 10.0 | 675 | .... |

**Dynamical Parallaxes.** The parallax of a binary star can be computed simply by equation (3), page 377, by solving for $p$, thus:

$$p = \frac{a''}{\sqrt[3]{P^2(m_1 + m_2)}}.$$

As a first approximation, $m_1 + m_2$ is assumed to be twice the Sun's mass, and the resulting value of $p$ is used for computing the star's absolute magnitude by the relation (page 343)

$$M = m + 5 + 5 \log p.$$

Eddington's mass-luminosity relation then affords a more reliable value of the mass, which is used in the first equation as a second approximation, and the process is repeated until two successive approximations give the same result. Parallaxes determined in this way are called dynamical parallaxes. The method, in the hands of Russell and others, has proved a valuable means of determining the distance of binary stars which are too remote for direct trigonometric measurement.

**The Densities of the Stars.** When the mass and size of a star are known, the average density of its material can be obtained immediately by dividing mass by volume. The densities of stars are found to have an amazingly wide range. A red giant such as Betelgeuse is a prodigious bubble of rarefied gas, as tenuous as the air in a fairly good artificial vacuum; a white dwarf such as the companion of Sirius is composed of

[2] Abridged from Table XXXIII of Russell, Dugan, and Stewart's *Astronomy*.

material thousands of times heavier than lead, of which a pint, if placed at the surface of the Earth, would weigh more than twenty-five tons!

Observations by Adams have confirmed at the same time the incredibly great density of this particular star and Einstein's prediction of a gravitational shift in the spectral lines of a massive source of light (page 252). In the case of most stars, the Einstein effect is too slight to be easily measured and besides is inextricably combined with the Doppler-Fizeau effect of the star's motion. The motion of the companion of Sirius is accurately given by the orbit of the binary as determined from micrometer observations (page 356), and so the Doppler shift can be allowed for; its Einstein displacement, because of the enormous value of its surface gravity, is more than 30 times that of the Sun. After allowing for the known Doppler shift, Adams found a residual redward displacement of 0.32 angstrom units, in almost perfect agreement with the Einstein displacement calculated by Eddington from the star's diameter of 24,000 miles and density of 53,000 × water. The spectrum is of A type.

Despite its amazing density the star must, for Eddington's calculations to be correct, be in the gaseous state—that is, composed of freely moving particles. Eddington's view is that these particles are neither molecules nor atoms in the ordinary sense, but free electrons and **stripped nuclei**, the ionization having proceeded to the extreme point where no revolving electrons are left to the atom. The particles are thus thousands of times smaller than ordinary atoms and so can be packed together thousands of times more closely.

Among stars of the main sequence the range in density is less, though still sufficiently astonishing—from about one-fiftieth that of the Sun (or 25 times that of ordinary air) for the great B-type stars to twice that of platinum for the smallest red dwarfs.

Table 26

REPRESENTATIVE STARS

| Name | Spect. | Eff. Temp. | Diameter in miles | Diam., ⊙ = 1 | Abs. Mag. | Luminosity | Mass, ⊙ = 1 | Density, ⊙ = 1 |
|---|---|---|---|---|---|---|---|---|
| Betelgeuse (at maximum) | M2 | 3100° | 460,000000 | 540 | − 3.4 | 2900 | 15 | 0.0000001 |
| Antares | M1 | 3100 | 290,000000 | 330 | − 3.2 | 1660 | 15 | 0.0000004 |
| β Pegasi (at maximum) | M2 | 3100 | 140,000000 | 162 | − 1.6 | 600 | 9 | 0.0000002 |
| Aldebaran | K5 | 3400 | 37,000000 | 43 | − 0.2 | 105 | 4 | 0.00005 |
| Arcturus | K0 | 4200 | 21,000000 | 24 | − 0.1 | 96 | 4 | 0.0003 |
| α Centauri[a] | G4 | 6000 | 1,200000 | 1.05 | + 4.73 | 1.12 | 1.08 | 1.06 |
| The Sun | G0 | 6000 | 866500 | 1.00 | + 4.85 | 1.00 | 1.00 | 1.00 |
| Sirius[a] | A0 | 11000 | 1,500000 | 1.8 | + 1.27 | 27 | 2.5 | 0.5 |
| Vega | A0 | 11000 | 2,200000 | 2.5 | + 0.60 | 50 | 3 | 0.2 |
| Rigel[a] | B8 | 12000 | 36,000000 | 42 | − 5.77 | 18,000 | 40 | 0.0005 |
| Krüger 60[a] | M3 | 3300 | 300000 | 0.34 | 11.2 | 0.003 | 0.26 | 7 |
| Barnard's star | M5 | 3200 | 140000 | 0.16 | 13.3 | 0.00048 | 0.16 | 40 |
| Sirius's companion | A5 | 7500 | 28000 | 0.034 | 10.0 | 0.10 | 0.96 | 27,000 |

[a] Brighter star of a double.

**Description of Representative Stars.** In Table 26 are statistics concerning a few stars which are representative of giants and dwarfs of various spectral classes. Additional facts concerning some individual stars are given below.

The first five diameters were computed from spectroscopic parallaxes, together with apparent diameters as measured with the interferometer; the others were computed from spectral type and absolute magnitude. The masses of $\alpha$ Centauri, Sirius and its companion, and Krüger 60 are derived from their orbital motion; the others are from Eddington's mass-luminosity relation.

**Betelgeuse.** Though Betelgeuse is the twelfth star in the sky in order of visual brightness, it is first in order of radiometric magnitude. It sends to the Earth $7.7 \times 10^{-10}$ calories per square centimeter per minute. (Compare this number with the solar constant, page 205.) The brightness of Betelgeuse is irregularly variable over a short range. Interferometer measures indicate a pulsation of the star, its apparent diameter ranging from 0″.034 to 0″.054.

**V Puppis.** A spectroscopic and eclipsing binary with well-determined orbital elements. Both components are of spectral type B1, with surface temperature (according to Plaskett) of 22,000°. The masses have the unusually high values of 18 and 19 times that of the Sun.

**H.D. 1337 (Pearce's Star).** A spectroscopic and eclipsing binary, both stars of type B8, temperature 28,000°. These stars have the greatest accurately known masses—34 and 36 times the Sun's mass. They are ellipsoidal, longest diameters 31 and 48 times the Sun's, and they revolve in a period of 3.5 days with their noses only 350,000 miles apart.

**B.D. 6°1309 (Plaskett's Star).** A spectroscopic binary in which the masses are *at least* 63 and 76 times that of the Sun, but as the inclination of the orbit is unknown the actual values of the masses cannot be determined.

**H.D. 698 Cassiopeiae.** A B-type spectroscopic binary in which the masses are *at least* 45 and 113 times the mass of the Sun.

**Van Maanen's Star** (page 349). A white dwarf, effective temperature 7000°, of luminosity only 1/6000 that of the Sun. Its diameter is about that of the Earth. Its mass is unknown, but its density is believed to be even greater than that of the companion of Sirius. The red shift in the spectrum of this star, which in Table 24 is interpreted as a high velocity of recession, may instead be an Einstein effect due to the star's strong gravitational field.

**Profiles of Spectral Lines; Stars with Shells.** The "lines" of stellar spectra are not lines in the mathematical sense; they have perceptible width, over which their intensity is generally not uniform. A curve exhibiting the internal variation of intensity of a spectral line is called a **line profile.** Line profiles are best produced by means of a microphotometer applied to spectrograms of high dispersion. Their interpretation, made with skill and imagination, leads in some cases to interesting pictures of the physical constitution of stars.

The possible effect of rapid rotation of a star upon the lines of its spectrum has already been mentioned (page 368). Figure 253, by Struve, shows line profiles for Vega (broken curve) and the rapidly rotating Altair. The center of the broad Altair-

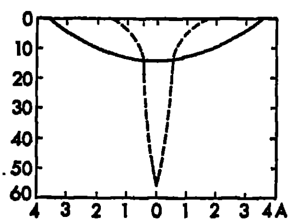



Fig. 253. *Line Profiles of Rotating and Non-rotating Stars (Struve).*

Fig. 254. *Profile of Bifurcated Emission Line.*

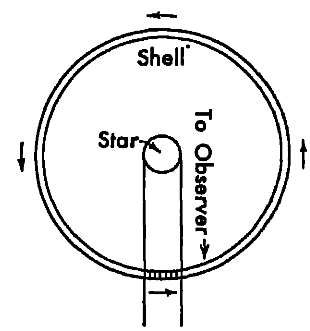



Fig. 255. *Star with Rotating Shell.*

ian line is believed to be due to light coming mainly from the front meridional region of the star where the rotational motion is across the line of sight; and the light of the weaker edge portions, toward violet and red, comes from the approaching and receding limbs respectively.

There are B-type stars having emission lines each split by an absorption line, a representative profile being as shown in Figure 254. This is accounted for by a rotating gaseous shell surrounding the star, as in Figure 255. The emission line is given by the shell and is broadened by

Fig. 256. *Spectrogram of P Cygni (from Lick Observatory).*

the diverse Doppler effects of its different parts; the central dark line, by that part of the shell that moves across the sight line between the star and the observer, absorbing from the star's radiation the waves characteristic of its own spectrum.

Fig. 257. *Profile of Typical P Cygni Line.*

Novae and stars of the type of P Cygni (Figure 256) often have double lines consisting of a wide emission component with the center displaced only slightly or not at all and a sharper absorption component much displaced toward the violet. Such stars are believed to be surrounded by large, rapidly expanding shells, as in

Figure 258. The absorption is attributed to the rapidly approaching segment of the shell that is between us and the star; the emission, to the remainder of the shell in which the sight-line velocity has a wide range of approach and recession.

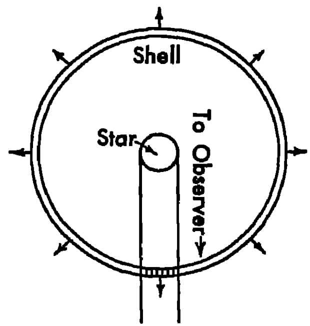



Fig. 258. *Star with Expanding Shell.*

Other possible causes of broadening of spectral lines and of diversity of line profiles include: (1) blending of two or more lines that are too close together for separation by the dispersion employed; (2) the fact that the spectrograph slit is not infinitely narrow; (3) the fact that an atom's emission and absorption are not confined strictly to a precise wave length; (4) Doppler effect of random motions of the atoms of a hot gas; (5) Doppler effect of convection currents in the star; (6) Stark effect of momentary electrostatic fields acting between atoms.

## EXERCISES

1. Prove that a star's apparent diameter in seconds of arc equals the product of its parallax and its real diameter in astronomical units.

2. Find the apparent diameter of the companion of Sirius.

*Ans.* $0''.00011$

3. Assuming the value of radiation pressure given under (3), page 378, for a temperature of 1,000,000°, verify that given for 10,000,000°.

# CHAPTER 18

## STARS OF VARYING BRIGHTNESS

**Discovery and Designation of Variable Stars.** A star whose brightness is known to change is called a variable star. The study of these stars, though an important branch of astronomy today, is of comparatively recent origin. The ancients regarded the "fixed" stars as symbols of immutability, and seem never to have suspected that any of the known stars might vary in brilliancy. There are in ancient chronicles a number of allusions to such phenomena as the "new star" which was observed for a while between $\beta$ and $\rho$ Scorpii in 134 B.C., and which is said to have prompted Hipparchus to make the first star catalogue in order to leave a record of the appearance of the heavens in his time for the use of later astronomers in detecting other changes. These allusions, however, are so vague that it is generally uncertain whether they refer to variable stars of the class now called novae, or to comets, or to bright meteors.

The first recorded discovery of a recurrently variable star occurred in A.D. 1596, when the Dutch astronomer Fabricius noticed in Cetus a third-magnitude star which had not previously been recorded. It faded in a few weeks and was thought to be a nova until 1638, when Holwarda, another Dutchman, again observed it and found that after disappearing again it reappeared eleven months later. The star was then found to have been visible also in 1603, when Bayer had mapped it as *o* (Omicron) Ceti, evidently without suspicion of its identity with the star of Fabricius. The astonishment with which the phenomenon was then regarded is evidenced by the name Mira (wonderful) which was given to *o* Ceti by Hevelius of Dantzig, and by which it is still known.

The Arabs may have noticed the variability of $\beta$ Persei, for the name Algol (*al Ghul*, the demon) is somewhat out of keeping with the more complimentary names which they applied to many stars; but there is no other evidence that they did so. Montanari seems to have been the first,

in 1669, to announce its variability in Europe, but no careful study was made of Algol until that of Goodricke in 1782. Goodricke also discovered the variability of $\beta$ Lyrae and $\delta$ Cephei.

The discovery of other variable stars at first proceeded slowly; in Argelander's list of 1844 there were but 18 entries, and in Chandler's catalogue of 1888 there were only 225; but since the latter date the number of known variables has increased, chiefly through the application of photography to the study of the stars, to many thousands.

The study of variable stars is a branch of astronomical research that is particularly well adapted to the opportunities of amateurs who have small telescopes, and even of those who enjoy observing the sky without instrumental aid. The American Association of Variable Star Observers (AAVSO), which consists largely of amateurs, has made and is making important contributions to knowledge.

Variable stars not already lettered are designated by capital letters of the Roman alphabet beginning with R, followed by the genitive of the constellation name. They are lettered in the order of their discovery, and after nine variables in a single constellation have thus been designated the letters are repeated in pairs, as RR, RS, etc. When ZZ is reached, a new beginning is made with AA. The first variable star discovered after QZ is designated V335. There are many faint variables, mostly in star clusters and nebulae, which have received no designation.

**Classification of Variable Stars.** While stellar variability is of every degree and kind and does not fall readily into separate classes, it is convenient to discuss it under the following heads:

1. Algol-type variables or eclipsing binaries
2. Novae
3. Irregular variables
4. Long-period variables
5. Short-period variables or Cepheids

A star's variation in brightness is best described by the dimensions and form of its **light-curve**, in which are correlated its brightness, expressed by magnitudes as ordinates, and the time from a chosen epoch or phase as abscissae.

**Algol-Type Variables.** It has been pointedly remarked that variable stars of the Algol type are the only ones for which we have a reliable explanation, and that in reality they are not variable at all; they are binary stars, and it is only because of the location of the Solar System in their planes of motion that their light seems to vary. These stars were discussed in Chapter 16 (pages 362 *et seq.*)

**Novae.** A "new" star, temporary star, or nova is a variable star whose light-curve rises very steeply to a sharp, brilliant maximum and then descends more slowly, usually with many secondary maxima and minima at irregular intervals (Figures 259, 260). The customary designation for such a star is the word Nova followed by the genitive of the constellation name and the year of principal maximum; for example, Nova Persei 1901.

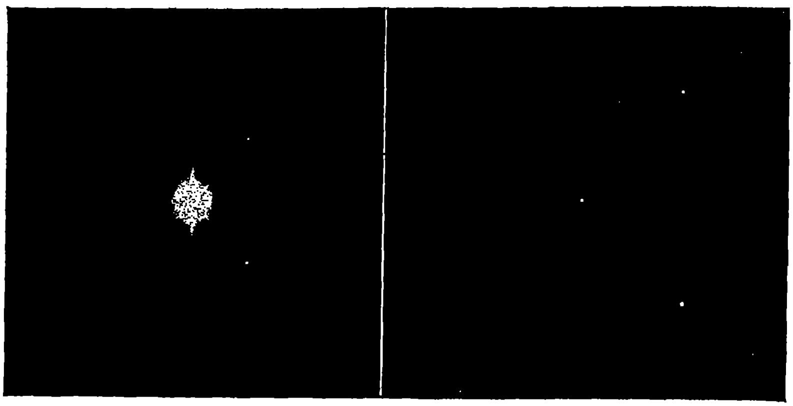


Fig. 259. *Nova Aquilae 1918, Photographed by Duncan at Mount Wilson Near the Time of Maximum Light (Left, with 60-Inch Telescope) and 8 Years Later (with 100-Inch Telescope).*

It was formerly believed that novae were new creations which, after a brief lifetime, went out of existence; but such a view is wholly at variance with what we know of other stars and of the material universe generally; furthermore, a number of recent novae have been identified, on photographs and otherwise, with previously known stars and are still observed as faint stars many years after their outburst.

Up to A.D. 1670, thirteen novae were recorded, all of which were in or near the Milky Way. No more were discovered until 1848, but since that year, and especially since the beginnings of astronomical photography, discoveries of novae have been frequent. Altogether, about sixty have been observed which, from their brightness (most were visible to the naked eye at maximum) and proximity to the Milky Way, are known to be members of the Galactic System.

Undoubtedly many fainter novae have escaped detection. From the number found on Harvard photographs, Bailey has estimated that an average of ten or more reach a maximum of the ninth magnitude or brighter each year. At that rate, in the course of a few hundred million years the total number of old novae in the sky would equal the present number of visible stars, and it is possible that most stars experience such an adventure at some time during their careers.

The most famous novae of the past were Nova Cassiopeiae 1572, known as the star of Tycho Brahe, and Nova Ophiuchi 1604, called Kepler's star. When Tycho first saw the nova of 1572 it was already brighter than Jupiter, and it grew brighter still during the next few days, so that it rivaled Venus and was visible in broad daylight; it then faded slowly and disappeared in about eighteen months. Tycho's deter-

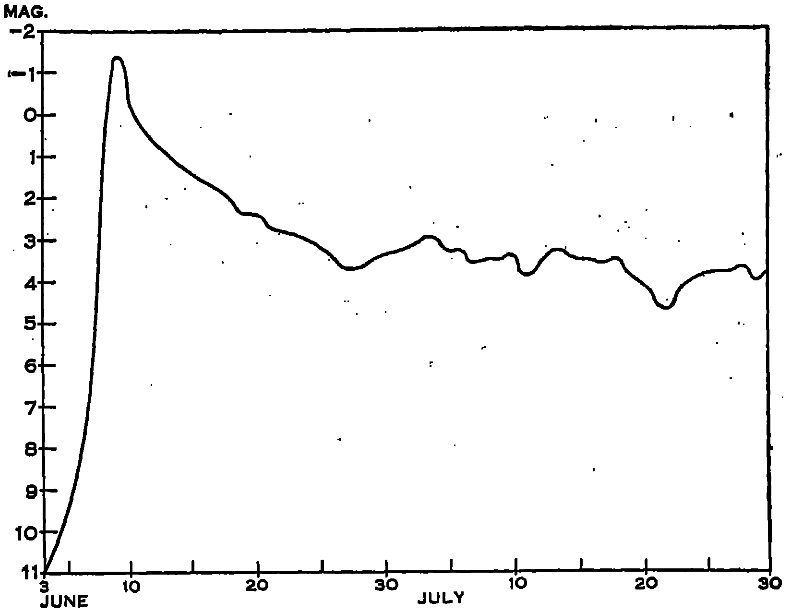



Fig. 260. *Light-Curve of Nova Aquilae 1918.*

mination of its position was not sufficiently accurate to make its identification at the present day certain. Kepler's star became brighter than Jupiter and then faded from naked-eye vision in a little more than a year. Its position was well recorded, and traces of nebulosity have been found there with the 100-inch telescope which are probably the debris of the cataclysm.

Bright novae recorded in the first forty-four years of the twentieth century, with their magnitudes at maximum, are as follows:

| | | | |
|---|---|---|---|
| Nova Persei 1901 | 0.0 | Nova Pictoris 1925 | 1.1 |
| Nova Geminorum 1912 | 3.4 | Nova Herculis 1934 | 1.7 |
| Nova Aquilae 1918 | −1.0 | Nova Lacertae 1936 | 1.9 |
| Nova Cygni 1920 | 1.8 | Nova Puppis 1942 | 0.4 |

In addition, RS Ophiuchi, which had been discovered as a nova in 1898 and had since fluctuated near the eleventh magnitude, repeated the typical nova behavior in 1933 when it rose to magnitude 4.3.

In recent years many faint novae have been observed in the vastly remote extra-galactic nebulae. The great Andromeda spiral (M 31), which has been patrolled rather thoroughly with the Mount Wilson reflectors, is

known to have produced more than a hundred in twenty years, mostly of the fourteenth to eighteenth apparent magnitude.

**Luminosities and Distances of Novae.** As the distance modulus of the Andromeda nebula (page 457) is 22 magnitudes, it is clear that the faint novae found in that nebula have absolute magnitudes, at maximum brightness, of $-4$ to $-8$ and so are comparable in luminosity to the known supergiant stars. By a review of the ordinary novae in the Galactic System, the Magellanic clouds and the nearer external galaxies, McLaughlin finds a mean absolute magnitude at maximum of $-7.0$ (about 50,000 times the Sun's luminosity) with a scatter of a few magnitudes on either side of the mean.

Far more brilliant than ordinary novae and supergiants have been a few stars called **supernovae.** One of these flashed out in 1885 at a point 16″ from the nucleus of the Andromeda nebula, attained an apparent magnitude of 7.5 so that its brightness was a tenth that of the whole nebula in which it originated, and disappeared in even the largest telescopes within a year. With an absolute magnitude at maximum of $-14.5$, it emitted more light in six days than the Sun emits in a million years. Supernovae have been found in several external galaxies, largely by a systematic search that was conducted for some years by Zwicky with the 18-inch Schmidt camera on Palomar Mountain. In the Galactic System, the rank of supernova may have been attained by the star of Tycho Brahe (1572) and by the nova of 1054 A.D., recorded by oriental astronomers, which may have given birth to the Crab nebula (page 390).

The distances of even the nearest known novae are very great—so great that trigonometric measures of their parallax have little significance. Examples of distances inferred by indirect methods: Nova Persei 1901, 1600[1] light-years; Nova Puppis 1942, 4000 light-years; Nova Lacertae 1936, 5400 light-years. Thus the histories of these outbursts have been locked up in trains of light waves a few millions of millions of miles long, which, having left the scenes many centuries ago, have just recently been poured into our eyes and our instruments.

**The Spectra of Novae.** The spectra of novae undergo rapid changes which, no less than their changes of brightness, set novae apart from ordinary stars. Most of the spectral features, and their changes, find a reasonable explanation in the hypothesis of a shell of gas expanding around

[1] This value supersedes that given in earlier editions of this book, which was based on trigonometric parallax.

the star with explosive velocity (page 383). A typical history of a nova spectrum is as follows.

If observed during the star's rapid rise in brilliancy, the spectrum is like that of a star of B or A type, except that the lines are greatly displaced toward the violet. The intensity of the lines increases with the brightness of the star, and at maximum the spectrum is likely to be of a modified A type having very broad absorption lines. As the star starts on the downward portion of its light-curve, its color changes from white to yellow, and *bright* lines, particularly of hydrogen and ionized iron, appear in about their normal positions, on the redward side of the absorption lines. These bright lines soon broaden enormously into wide bands of complicated structure. A few days later a new set of dark lines appears, even more displaced toward the violet than the first set (in the case of Nova Aquilae 1918, the displacement was such as to indicate a velocity of $-1800$ km./sec.), and then the bright lines increase further in intensity, while the continuous background fades. The color of the star then rapidly changes to a deep red, because of the fading of the continuous spectrum and the predominance of the bright $H\alpha$. After a few weeks the bright lines of the nebular spectrum appear and the former spectrum gradually vanishes, leaving the spectrum like that of a gaseous nebula except that the lines are broad. The star then assumes a green color largely because of the intense nebular line at $\lambda 5007$. Later still, as the star subsides to inconspicuousness, the continuous spectrum reappears, and with it emerge the bands characteristic of the Wolf-Rayet stars. In this stage the color of the star is again white.

**Direct Observations of Developing Nebulae.** In several cases the development of a small nebula around a nova not only has been inferred from the changing spectrum but has been observed directly. In 1917, sixteen years after the outburst of Nova Persei, Barnard discovered visually around the star a tiny, unsymmetrical nebula which in subsequent years, as observed both visually and photographically, spread outward at a rate of about $0''.4$ a year. In 1934, Humason photographed its spectrum with a special spectrograph attached to the 100-inch telescope and found bright lines, single at the outer edge of the nebula but double with increasing separation toward the star, indicating an approach of the nearer surface and recession of the farther at a speed of about 1200 km./sec. It is by combining this radial velocity with the cross motion of $0''.4$ per year that the nova's distance given on page 388 was obtained.

Around Nova Aquilae 1918, a few months after its outburst, Barnard and Aitken detected visually a planetary nebula which, while too small to be easily photographed, soon became too faint for visual observation. In subsequent years it was rediscovered photographically with the 100-inch telescope by Hubble and Duncan and was found to grow fainter and to

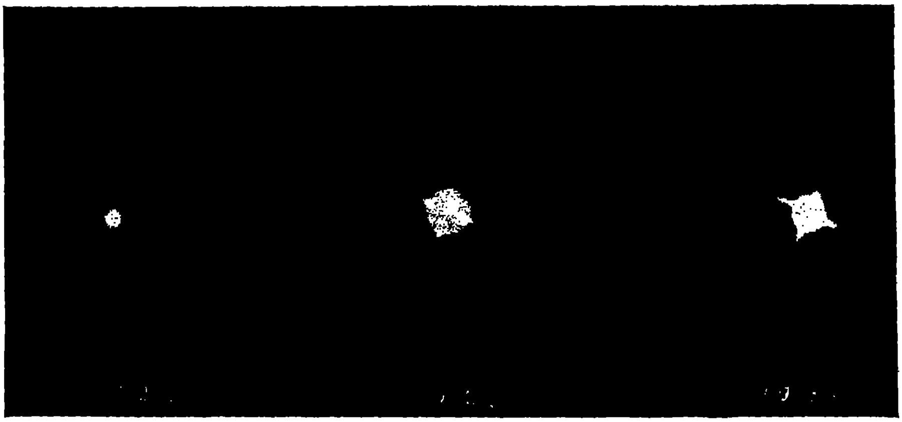


Fig. 261. *Development of a Nebula Around Nova Aquilae 1918. Photographs made with 100-inch telescope in 1922 (Humason), 1926 (Duncan), and 1930 (Hubble).*

increase in radius by about 1.″0 per year (Figure 261). This cross motion, combined with the radial velocity of 1700 km./sec. measured during the first year, gave a distance of about 1200 light-years and a radius of the nebula, at the age of sixteen years, of 7000 astronomical units.

With the 100-inch telescope and light-filters of various colors, Baade has found tiny nebulae attached to other novae several years after their maxima. The forms of these nebulae are not generally simple shells. Nova Herculis 1934, observed in 1940–1942, showed in blue light an elliptical nebula about 3″ in major diameter, while in red light (ionized nitrogen image) there were four distinct condensations, the two brighter being at the ends of the minor axis of the blue nebula and the two fainter at the ends of the major axis.

**The Crab Nebula and the "Guest Star" of 1054 A.D.** About a degree northwest of $\zeta$ Tauri is a nebula about $4' \times 6'$ in apparent size, well known since about the year 1730, which because of its form of spreading tentacles has been called the Crab nebula. Changes in its appearance were suspected by Lampland from his observations with the Lowell 42-inch reflector prior to 1920, and were confirmed and studied by Duncan on plates made with the Mount Wilson 60-inch over an interval of thirty years. There is an unmistakable outward motion of certain details of the nebula (Figure 263) which must have carried them from a point within to their present positions in the short time of eight or nine centuries. Lundmark and others have noted that, in the ancient Chinese and Japanese annals, there is mention

of a "guest star," brighter than Jupiter, appearing in that part of the sky in the year which, in our calendar, was 1054 A.D., and have inferred that it was a nova and that it gave birth to the Crab nebula.

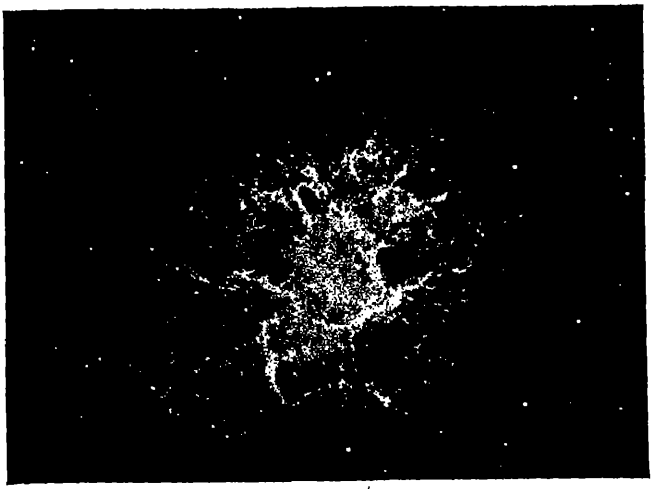


Fig. 262. *The Crab Nebula, Photographed in the Light of Its Red Spectral Lines (by Baade, 100-inch telescope).*

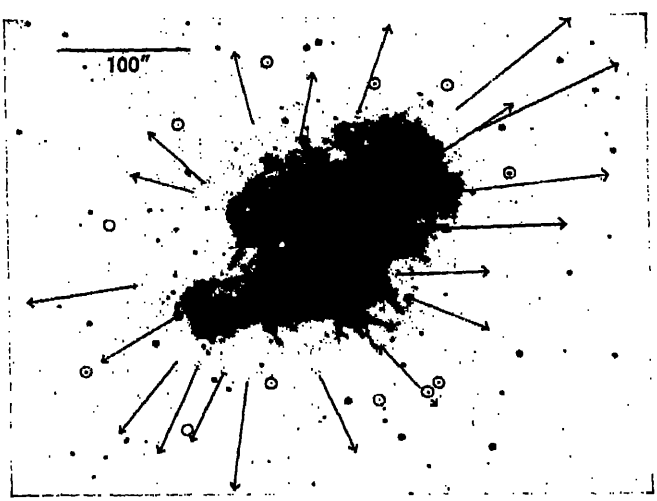



Fig. 263. *Expansion of the Crab Nebula. The vectors indicate the displacement which, if continued at the present rate, will take place in 500 years. Encircled stars were used as reference points in the measurement.*

The spectrum of the nebula, photographed first by Slipher and more clearly by Mayall, is continuous with superposed bright lines which, over the central part of

the nebula, are widely double (Figure 264), indicating a velocity of expansion of 1300 km./sec. With the measured cross motion, this gives a distance of about 4000 light-years and a major axis six light-years long. Baade finds that, photographed with the 100-inch in the light of the continuous part of the spectrum, the nebula is without notable texture, but that in the light of its bright lines it consists of a more extended, elliptical shell of complicated filaments (Figure 262).

Fig. 264. *The Line at λ 3727 in the Spectrum of the Crab Nebula, Photographed by Mayall at the Lick Observatory.*

From the distance of the Crab nebula and the brightness of the nova recorded by the ancient Chinese, Mayall and Oort have inferred that this was a supernova, one of the brightest on record, with absolute magnitude −16.5 and luminosity 350 million times the Sun's.

**The Light-Echo of Nova Persei 1901.** The great nova of 1901, long before its nebulous shell was found, exhibited nebular affiliations of a different kind. Six months after its outburst, when the light of the nova had diminished to less than one per cent of its maximum, Wolf of Heidelberg discovered by photography a faint but sizable shell of nebulosity centered near the star. Successive photographs of this shell, obtained by Ritchey at Yerkes and Perrine at Lick during the following autumn and winter, showed that it was expanding at the prodigious rate of between two and three seconds of arc per *day* (Figure 265). The parallax of the nova had been measured at about 0″.01 (too small to be reliable), corresponding to a distance of about 300

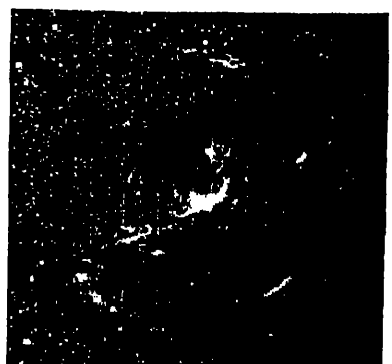


Fig. 265. *The Light-Echo of Nova Persei 1901. Drawings from photographs made with the 24-inch Yerkes reflector. Left, 1901 September 20 (Ritchey); right, 1902 February 8 (Pease). Ruled lines are 2′ apart. Top is north, left is east.*

light-years. At that distance, the observed rate of expansion of the shell indicated an actual velocity about equal to that of light, and Seeliger pointed out that Ritchey and Perrine were witnessing the successive illumination of more and more distant parts of a nebula already existing, as the sudden surge of light from the nova sped outward. The true distance of the nova, as we have seen, was much greater, and the simple assumption of progressive illumination of a homogeneous cloud is not sufficient. Couderc infers that the reflecting nebulosity was in the form of sheets (i.e., with thickness less than a light-year and other dimensions several light-years), irregularly disposed and inclined to the line of sight.

**Conjectures as to the Cause of Novae.** There can be no doubt that the sudden enormous increase in the brightness of a nova is due to some tremendous cataclysm, of a magnitude far exceeding that of any other which man has ever observed. For example, Nova Aquilae 1918 rose in four days from magnitude 10.5 to magnitude $-1.0$, a 40,000-fold increase in luminosity. So far, no really satisfactory theory as to the nature of the cataclysm has been proposed. Among the principal conjectures are the following:

1. A grazing collision of two stars. This theory was propounded as early as 1878 by Bickerton of New Zealand, who believed the nova to be a third body formed of matter torn off the two stars, rapidly expanding because of the enormous evolution of heat. He predicted successfully many of the features since observed in the spectra of novae, but his theory has met with little acceptance because of the vastness of the distances between the stars as compared to their diameters, which should render collisions very rare indeed, whereas novae are rather frequent.

2. A close approach of two stars, resulting in vast tidal upheavals, as in the origin of the Solar System pictured in the Planetesimal Hypothesis. This occurrence, though less improbable than an actual collision, still would happen very infrequently and also would be less effective.

3. The passage of a star through a nebula, the temperature of the star being raised as that of a meteor is raised by passing through the air. While this hypothesis receives support from the dark nebula which evidently had existed around Nova Persei previously to its flaring up and which was illuminated by the star, it is difficult to conceive of so much energy being liberated by the star's passage through a medium of such extreme tenuity as the diffuse nebulae are known to be. Moreover, the hypothesis is ruled out by the brevity of the phenomenon and the enormous size of known nebulae.

4. The sudden release of energy, probably of a subatomic origin, stored up within the star. This release might be brought about, as See and W. H. Pickering have suggested, by a body of planetary or smaller dimensions colliding with the star; the probability of such a collision is much higher than that of a collision of two stars. Russell points out that such a body would strike the star with terrific velocity and, plunging deep beneath the photosphere, be stopped by the denser layers below. Its kinetic energy would thus be transformed into heat, and a pocket formed in which the temperature would be millions of degrees. Russell conjectures that this temperature might act as a trigger to release energy from an unknown

source within the star, the heated gas would become still hotter and, expanding, would blow off the overlying layers of the star with catastrophic force. With the relief of pressure the temperature would drop again so that the brightness of the star would subside, while the expanding gas would form a nebula of increasing size.

**Irregular Variables.** The most famous of irregularly variable stars is probably $\eta$ Argûs (or $\eta$ Carinae), situated in the "keyhole" nebula in the southern circumpolar region (page 434). It was first noticed in 1677 by Halley, and from that year until 1825 it varied irregularly from the second to the fourth magnitude. In 1827 it rose to the first, but fell again to the second by 1830. In 1843 it reached magnitude $-1.0$, and so was the brightest star in the sky except Sirius; but during the following years its brightness greatly diminished, and since 1886 it has been about constant at magnitude 7.5. The spectrum is one of a very rare type, consisting almost wholly of bright lines in which hydrogen and ionized iron predominate. No other bright irregular variable is known which has a range of brightness approaching that of $\eta$ Carinae, though from 1928 $\gamma$ Cassiopeiae displayed changes of an analogous kind on a much smaller scale. $\alpha$ Herculis, $\alpha$ Orionis, and $\beta$ Pegasi are all examples of irregular variables of small range. A number of variables which are sometimes classed with Mira have shorter periods which are much less regular. These stars have spectra which, although of the M type, contain no bright lines.

In the great nebula of Orion, and also in the similar diffuse nebula Messier 8 Sagittarii, there are many faint red stars which vary quite capriciously over a range of several magnitudes. It has been suggested that these are "friction variables," a part of their light being due to friction with portions of the nebula which vary in density; also, that the light of these stars is obscured to varying degrees by passing wisps of dark nebula. Either explanation must be regarded as a mere guess.

**Stars of the Types of R Coronae and SS Cygni.** Somewhat resembling the long-period variables, but much less regular, are a few stars which fall into two classes. Of one of these, R Coronae Borealis is the model. It is ordinarily of the 5.5 magnitude, but is subject to sudden drops of several magnitudes, even down to the twelfth, whose date and duration are unpredictable. Stars of this type lie near the Milky Way.

The other class is represented by SS Cygni, which, usually of the eleventh magnitude, suddenly *rises* to the ninth or eighth. It often shows two alternating types of maximum, one of longer duration than the other, but occurring at irregular intervals.

**The Periods of Variable Stars.** More than two thousand variable stars have known periods. The relative abundance of different periods is

shown in Figure 266, in which the ordinates are numbers of stars and the abscissae (on a logarithmic scale) are periods. The curve makes it evident that periods of half a day, seven days, and especially 300 days are abundant, whereas those between one and two days and between 30 and

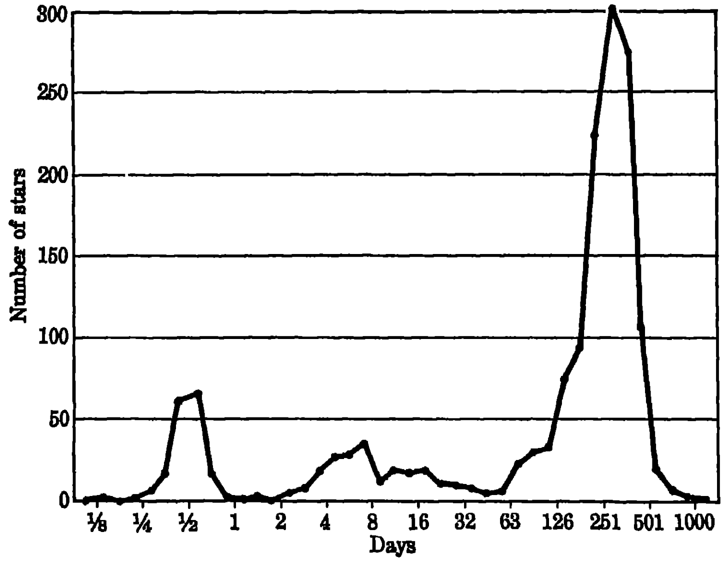



Fig. 266. *Distribution of the Periods of Variable Stars, Compiled by Merrill from Prager's* Catalogue of Variable Stars, 1930.

60 days are rare. Stars whose periods lie in the largest of the three groups are called **long-period variables.** They have characteristics in common that distinguish them from stars of the other two groups which, resembling each other in many ways, are classified together as **short-period variables.**

**Long-Period Variables.** The range of light variation of the long-period variables is from four to nine magnitudes. More than half of these stars have periods between 250 and 400 days. Their light-curves do not repeat themselves exactly, either in the range of brightness or in the interval between maxima. In most cases, the increase of brightness is more rapid than the decrease. All are red stars, mostly of spectral type M, while a few are of types N, R, and S. At and near the time of maximum light, bright lines, particularly those of hydrogen, are superposed upon their spectra. None are known to be connected with nebulae except R Aquarii, in the spectrum of which Merrill found bright lines of "nebulium," and around which Lampland discovered a small nebula of peculiar form.

Variable stars of the long-period class show no marked preference for the Milky Way.

The type star of the class is Mira Ceti, which has probably been studied more fully than any other star. Its period averages 330 days, but varies from this at times by as much as 15 days either way. At maximum light, the star is sometimes as bright as magnitude 1.5 and sometimes as faint as 5.6, but usually is about 3.5; while at minimum it is usually about 9.2, but has been as faint as 10.0 and sometimes has sunk only to 8.0. Figure 267 shows the light curves of Mira and two other long-period variables.

The spectrum of Mira varies from M6 at maximum to M9 at minimum. According to a thorough study by Joy at Mount Wilson, no emission lines are present at minimum, but bright hydrogen lines appear soon after minimum and increase in intensity until about maximum. Low-temperature emission lines of iron, magnesium, and silicon appear some time after maximum. The dark bands of titanium oxide are especially strong at minimum.

The effective temperature of the star varies from about 1500° C. at minimum light to about 2000° C. at maximum. Measures of the total energy made with the thermocouple by Nicholson and Pettit indicate a "radiometric" brightness about eight magnitudes higher than the visual and a variation in total radiation of only about one magnitude. The great difference between the visual and the radiometric range is easily explained by the displacement with changing temperature of the point of maximum energy in the spectrum according to Wien's law; at minimum, the greater part of the energy is contained in the infra-red radiation, but at maximum the peak of the energy curve is shifted into the visible spectrum, and the visual light increases more rapidly than the total radiation. This effect is accentuated by the intensification of the dark titanium oxide bands at minimum, since they lie entirely in the visible part of the spectrum.

The radial velocity indicated by the absorption lines varies through a range of about 12 km./sec. in the period of the light-variation. The emission lines, on the other hand, show a variation of 19 km./sec. At minimum light, their velocity is the same as that of the absorption lines, and at all other times they show a relative motion toward the observer.

Interferometer measures by Pease show that the star is of enormous size, its diameter being about 500,000,000 kilometers. The variation of radial velocity cannot be due to orbital motion, for the semi-axis of the orbit computed on this interpretation is only 26,000,000 kilometers, about 11 per cent of the radius; if the velocity curve is due to the influence of a companion, the companion must be enclosed far within the visible star. The most natural explanation of the oscillation of the spectral lines is that the star *pulsates*, dilating and contracting in the period of its light-changes. The shell of gas which gives rise to the emission lines evidently pulsates through a wider range than the lower-lying reversing layer.

Although not a spectroscopic binary, Mira has a visible companion whose existence was inferred by Joy in 1922 from peculiarities in the bright-line spectrum. At Joy's suggestion, the companion was sought for visually by Aitken with the Lick refractor, and easily found by that skillful observer at a distance of 0.″9, although it had eluded previous scrutiny with large telescopes by several observers, including its discoverer. It is a white star of about the tenth magnitude, but appears to be

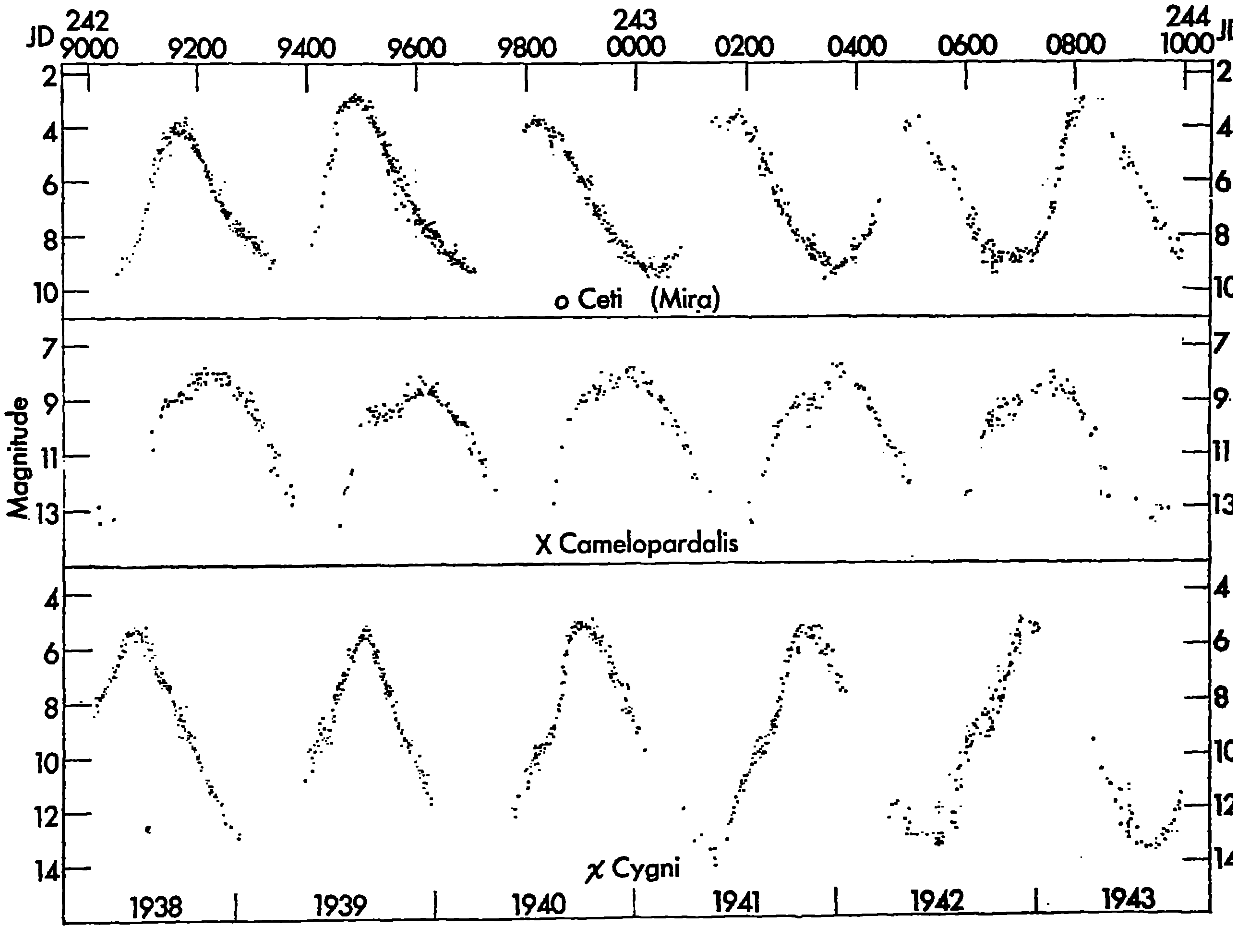



Fig. 267. *Light-Curves of Long-Period Variable Stars, Compiled from AAVSO Observations by L. Campbell.*

slowly growing fainter. Its spectrum exhibits the bright lines of hydrogen, helium, ionized iron, and calcium, but no dark lines except those of hydrogen.

Like the novae, the long-period variables are awaiting an adequate explanation. It is certainly to be expected that a pulsating star would vary in brightness; but it should be hottest and brightest when smallest, and this is not found to be the case with Mira, which is brightest when at about the middle of the contracting phase. An older suggestion postulates a variation of internal activity similar to that which produces the eleven-year cycle in the Sun (whatever the cause of that may be), but the Sun's variation is of so much longer period and so much smaller amplitude than that of the red stars that it would seem to be a different kind of phenomenon.

**Cepheids, or Variable Stars of Short Period.** Though the variable stars of short period are separated naturally into two groups (Figure 266), they all resemble one another in many ways and are often called **Cepheids** from their type-star $\delta$ Cephei. When it is necessary to distinguish between the groups, those of periods less than a day are called **cluster variables** because, though found sparingly in all parts of the sky, they are very numerous in star clusters.

In contrast with variable stars of the long-period or Mira type, the Cepheids are punctual and regular in their variation, the period and form of light-curve changing but little; the range of variation is small, usually less than half a magnitude and seldom exceeding 1.5 magnitudes visually; and the spectral types are A, F, or G.

Most Cepheids have light-curves which rise more rapidly than they fall. Every Cepheid which is bright enough to have been studied spectroscopically shows a variable radial velocity in a period identical with the period of the light curve. The variation, however, cannot be due to eclipses, for the maximum light invariably occurs near the time of maximum velocity of approach (Figure 268), and minimum light at maximum velocity of recession, whereas in the Algol stars the minimum necessarily occurs on the descending part of the velocity curve. The spectra of different Cepheids are related to the periods—stars with periods less than a day are in class A, those of about 4 days in class F, and those over 8 days in class G. The spectral type of a given Cepheid changes slightly with the brightness of the star, being of a cooler and redder type at minimum, so that the range of photographic magnitude is greater than that of the visual magnitude. Cepheids having periods greater than three days are found mostly in

the Milky Way and have small peculiar motions (averaging about 12 km./sec.). The cluster variables which are not in star clusters are scattered in all parts of the sky and are moving much more rapidly than the galactic Cepheids. Stars of both groups are giants; those of longer period are several hundred times, and the cluster variables scores of times, as luminous as the Sun.

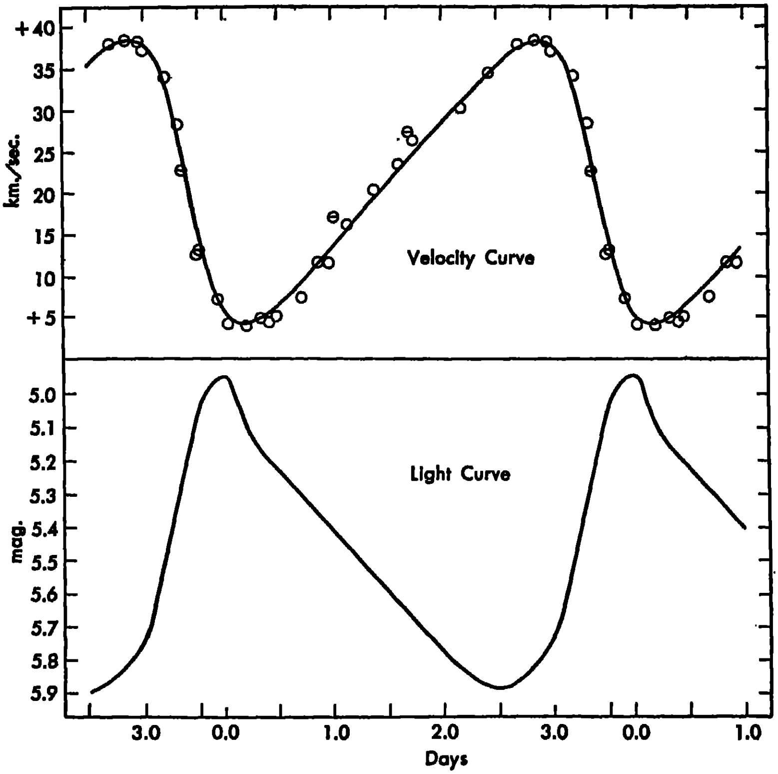



Fig. 268. *Light-Curve and Velocity-Curve of the Cepheid RT Aurigae (Duncan).*

**The Period-Luminosity Relation in Cepheid Variable Stars.** The Cepheids possess a remarkable property which gives an indication of their distance, however great it may be. This property was discovered by Miss Leavitt at the Harvard Observatory in 1908 by a study of the small Magellanic cloud. She found many faint Cepheids in this cloud, and upon determining their periods of variation in brightness, found that *the longer the period the greater was the apparent brightness of the star.* Since all the stars in the cloud are at practically the same distance from the Earth, their apparent magnitude must differ by a constant from their absolute magni-

tude, which therefore is also correlated with their periods. Bailey of Harvard discovered in several of the globular star clusters many Cepheid variables displaying the period-luminosity relationship, and in 1917 Shapley pointed out that, assuming the relationship to hold for all Cepheids, if the distance and absolute magnitude of one or more Cepheids could be determined by independent means, those of all the others and of the star clusters of which they are members could be immediately obtained from their periods. The relationship is exhibited by the curve in Figure 269, drawn according to Shapley. The form of this curve is determined by the behavior of the Cepheids in the small Magellanic cloud; its position on the absolute-magnitude scale (zero point), by the brightness of nearer Cepheids whose distances are known independently. Shapley's original zero point, which depended precariously on eleven galactic Cepheids, has been verified by the much more extensive work of R. E. Wilson. The curve gives at once the absolute magnitude of any Cepheid whose period is known by observation; and from this and the observed apparent magnitude the heliocentric parallax and distance may be derived directly by the methods on page 343. The period-luminosity relation has proved to be of inestimable value in the exploration of the universe.

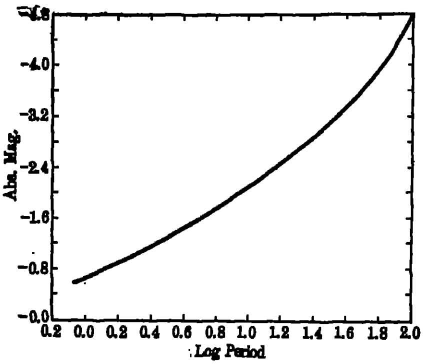



Fig. 269. *Correlation of Period and Luminosity of Cepheids.*

**Theories of Cepheid Variation.** Two main lines of explanation have been offered for the phenomena of Cepheid variation and, as in the case of other variable star theories, neither is wholly satisfactory. They are referred to as the binary and the pulsation hypotheses.

According to the first, a Cepheid is a close binary surrounded by a tenuous envelope through which the two stars move in their orbits. The brighter star, which alone impresses our instruments, either is heated by contact with this medium more on its preceding hemisphere than on the following, or its own slightly opaque atmosphere is brushed back from the preceding side. In either case the star would be, as observed, brightest when at the lowest point of its velocity curve, in the first case because of greater emission and in the second because of lesser absorption of its light. The binary hypothesis accounts for the observed velocity curves, but fails because the orbits derived from them are smaller than these giant stars are themselves

believed to be, so that the companion would have to revolve inside the primary star. Moreover, no Cepheid is known to exhibit periodically double lines, as some might be expected to do if they were binaries.

According to the pulsation hypothesis, each Cepheid is a single giant star, and the variation of its velocity is to be explained by a pulsation, the star expanding and contracting periodically as Mira appears to do. This explains the forms of the velocity curves fairly satisfactorily, fits in with the correlation of period and luminosity, and gives a reasonable explanation of the existence of a variation in light since a body of gas is cooler when in an expanded condition than when compressed; but the theory fails to account for the peculiar relation between light-curve and velocity curve and is somewhat strained to account for the lack of symmetry in the light curve.

Jeans has overcome some of the difficulties of the pulsation hypothesis in a mathematical treatment of a pulsating star which is at the same time rotating, as most stars probably are, and in which the pulsations combined with the rotation are in the act—an act that is likely to be protracted some 100,000 years—of producing a fission of the star into two bodies which will ultimately develop by tidal action into an ordinary binary.

## EXERCISES

1. According to observations made by members of the AAVSO, a maximum of Mira occurred about March 3, 1944. Another took place about January 30, 1945. Using these dates and the light-curve of Mira shown in Figure 267, predict the date of maximum for the present year.

2. Find Mira's place in the sky with the help of Maps 6 and 7. On every convenient night during several weeks before and after maximum, determine the magnitude of Mira by the method of Argelander (page 339). Note the star's color. Convenient comparison stars for naked-eye study of Mira are listed below:

| *Star* | *Mag.* | *Star* | *Mag.* |
|---|---|---|---|
| $\alpha$ Arietis | 2.2 | $\alpha$ Piscium | 3.8 |
| Deneb Kaitos | 2.4 | $\delta$ Ceti | 4.1 |
| $\alpha$ Ceti | 2.7 | $\xi$ Piscium | 4.8 |
| $\beta$ Arietis | 2.8 | $\lambda$ Ceti | 4.9 |
| $\gamma$ Ceti | 3.6 | 75 Ceti | 5.4 |

3. Identify $\beta$ Lyrae by means of Map 2. Observe it by the method of Argelander on every available night for a month, and form a light-curve from your own observations. Convenient comparison stars for $\beta$ Lyrae are:

| *Star* | *Mag.* |
|---|---|
| $\gamma$ Lyrae | 3.3 |
| $\mu$ Herculis | 3.5 |
| $\theta$ Herculis | 4.0 |
| $\kappa$ Lyrae | 4.3 |

4. According to observations made by Stebbins with the photoelectric photometer and kindly communicated for use in this book, a minimum of Algol occurred at J.D. 2430639.6721 (the fraction of the Julian Day is counted from Greenwich mean noon); and the period of Algol was 2.8673218 days. Predict a number of minima for the present year. Compare your predicted dates with those given in the annual *Handbooks* of the British Astronomical Association and the Royal Astronomical Society of Canada, or in the monthly *Sky and Telescope.*

5. Identify Algol among neighboring stars with the help of Maps 6 and 7. Determine its usual magnitude by the method of Argelander. On a night when a minimum occurs at a convenient time, watch the changes of the star's brightness by comparison with stars of the following list:

| *Star* | *Mag.* | *Star* | *Mag.* |
|---|---|---|---|
| $\gamma$ Andromedae | 2.2 | $\epsilon$ Persei | 3.0 |
| $\beta$ Andromedae | 2.4 | $\delta$ Persei | 3.1 |
| $\beta$ Arietis | 2.8 | $\beta$ Trianguli | 3.1 |
| $\zeta$ Persei | 2.9 | $\alpha$ Trianguli | 3.6 |

# CHAPTER 19

## STAR CLUSTERS AND NEBULAE

**The Clustering of the Stars.** The distribution of the stars is neither uniform, as if they had been planted by a gardener, nor wholly random, as if they had been scattered haphazard. Instead, the stars exhibit in many regions a clustering tendency, and form aggregations which range in population from the binary pairs and such multiple stars as Castor and Mizar through the open clusters like the Pleiades, through the great globular clusters of many thousands of stars, and through the vast cumulous star clouds to the mighty concourse of the Milky Way.

**The Milky Way.** No discussion of star clusters or of nebulae can proceed far without a reference to their relations to the Milky Way or Galaxy,[1] the greatest aggregation of them all; so it is well to begin with a description (though it can but be inadequate) of that beautiful and stupendous panorama.

Although to city dwellers the Milky Way is almost imperceptible through the illumination of the sky by artificial light, in country places and at sea it is seen to merit Milton's description as

> A broad and ample road whose dust is gold
> And pavement stars.

To the naked eye it appears as a band of soft, misty light encircling the sky; but even a small telescope, as Galileo discovered, shows it to be composed of myriads of faint stars, and among these a larger instrument reveals many nebulae. Its light is due almost entirely to stars which are far too faint to be seen as individuals without optical aid, so that if all naked-eye stars were blotted out the Milky Way would still shine unchanged upon a dark, blank sky (Figures 270, 271).

[1] The word **Galaxy** has come to be used in reference not only to the Milky Way but to the Galactic System of which the Milky Way is the projection upon the celestial sphere, and even to similar systems beyond the Milky Way—the "external galaxies."

Fig. 270. *The Milky Way in Sagittarius and Scutum, Photographed with Wide-Field Camera by Duncan at Mount Wilson.*

The general course of the Milky Way is shown on the star maps between pages 6 and 7 by the galactic circle, defined (page 27) as the great circle inclined 62° to the celestial equator with its ascending node (galactic longitude 0°) in right ascension $18^h\ 44^m$. The galactic circle almost coincides with the central line of the Milky Way, but not quite; the latter is in fact rather irregular and lies mainly about a degree farther south.

In Cassiopeia, the Galaxy reaches its most northerly point (galactic longitude 90°), which divides it into what may be called the winter and summer halves. On February evenings, for observers in middle northern latitudes, the winter half extends from the northwest point of the horizon, through the zenith, to the southeast point. It is bright in Cassiopeia and Perseus, where it contains the double star-cluster *h* and χ Persei, visible to the naked eye as a small condensation in the misty background; thence, as a broad, faint stream, it flows through Auriga and across the border of Gemini and Orion and the edge of Canis Major into Argo. The summer half, conspicuous on August nights and autumn evenings, extends from Cassiopeia through Cepheus and Cygnus, where, just north of Deneb, it is seemingly perforated by the northern Coalsack. In the midst of

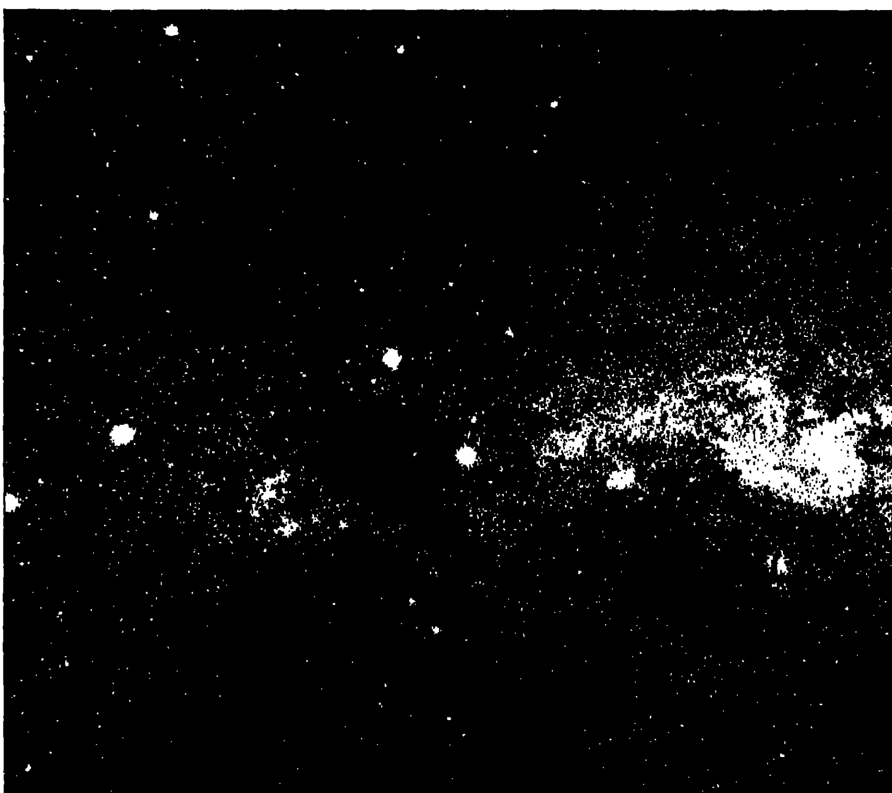


Fig. 271. *The Milky Way in the Region of the Southern Cross, Photographed by Margaret Harwood at Arequipa with 1-Inch Lens, 1923 May 12–13–18. Exposure 11 hours 39 minutes. The Cross is upright near the center of the plate and below it is the Coal Sack. The stars γ and δ Crucis appear relatively faint because of their yellow color. The bright stars near the left edge of the plate are α and β Centauri.*

the Northern Cross are bright star clouds, and here the stream divides into two branches. The western branch, at first the brighter and broader, fades to invisibility in Ophiuchus, but reappears in Scorpius to rejoin the eastern branch, which extends through Sagitta, Aquila, Scutum, and Sagittarius. This is the richest region in the sky, where, as Barnard says, "the stars pile up in great cumulous masses like summer clouds," and where, according to recent researches, is located the vast nucleus of the Galactic System. Joining the Argo and Sagittarius regions, and invisible in our latitude, the southern Milky Way passes through Crux (galactic longitude 270°) with its southern Coalsack, Centaurus, Circinus, Norma, and Ara.

Naked-eye observation shows only the general course of the Galaxy, together with the bright star clouds and the large dark areas such as the Coalsacks. Telescopic explorations such as those made by the Herschels a century ago show much more, but it is only by photography that the real beauty and complexity of the galactic structure are revealed. The photographic studies begun by Barnard about 1890 with an old portrait lens were a revelation. These were followed by Barnard and others with better and larger lenses of the portrait type, and in recent years still better wide-field photographs have been made by Ross with lenses of his own design. The most efficient instrument for the purpose is the Schmidt camera which, with modern sensitive films, photographs in a few minutes what the equipment of the 1890's required hours to reveal.

It will be shown in the next chapter that the appearance of the Milky Way is one of perspective and is due to the fact that we live in the midst of a vast aggregation of stars, star clusters, and nebulae (the Galactic System) which has roughly the form of a disk or of two saucers placed rim to rim and bottom outward. Looking toward the rim, we see vast numbers of stars (the Milky Way), most of which are exceedingly faint because of their great distance. Looking into other parts of the sky, we see fewer stars and especially fewer very faint stars because the stars which we do see in these directions are relatively near.

**The Belt of Bright Stars.** It was pointed out by Sir John Herschel and emphasized by Gould that a belt of bright stars, containing many of the brightest in the sky, intersects the Galaxy in Cassiopeia and Crux at an angle of about 20° and traverses Cassiopeia, Perseus, Taurus, Orion, Canis Major, Argo, Crux, Centaurus, Lupus, Scorpius, Ophiuchus, Hercules, Lyra, Cygnus, and Cepheus. This belt contains practically all of the bright B-type stars. The numerous first-magnitude stars in that portion extending from Perseus to Canis Major, on the west side of the

galactic circle (Map 6), are probably responsible for the often-expressed opinion that the skies of winter are clearer than those of summer; it is not, however, the clearness of the winter sky that gives this impression but the brightness of the winter stars.

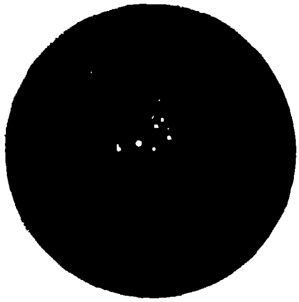


Fig. 272. *The Pleiades. (Short-exposure photograph with small camera by Duncan.)*

**Naked-Eye Star Clusters.** At least three notable star clusters are partly resolvable to the naked eye: the Pleiades, the Hyades, and the little constellation Coma Berenices. Of the Pleiades (Map 6 and Figure 272), seven stars are easily visible to the naked eye, four more may thus be seen on an exceptionally clear, moonless night, and several hundred stars and some great wisps of nebulosity are revealed by photographs (Figure 273). Alcyone, the brightest Pleiad, is of the third magnitude. Many of the brighter Pleiades are B-type giants. The Hyades are apparently more scattered, but, as we have noted, their common motion will carry them so far away that they will eventually appear as a compact cluster. The same is true of the Ursa Major group which, because we are in the midst of it, has not now the appearance

Fig. 273. *The Pleiades. (Photographed by Duncan with 100-inch Hooker telescope and Ross correcting lens, 1933 August 20. Exposure 2h. 10m.)*

of a cluster at all. In Coma, a score or more of stars are visible to the unaided eye, but none is brighter than the fourth magnitude. This cluster is situated almost at the north galactic pole.

Among clusters visible but not resolvable to the naked eye, and so appearing like small, faintly luminous clouds, are Praesepe Cancri, *h* and χ Persei (Figure 274), M 7 Scorpii, and the globular clusters ω Centauri, M 13 Herculis, and M 22 Sagittarii.

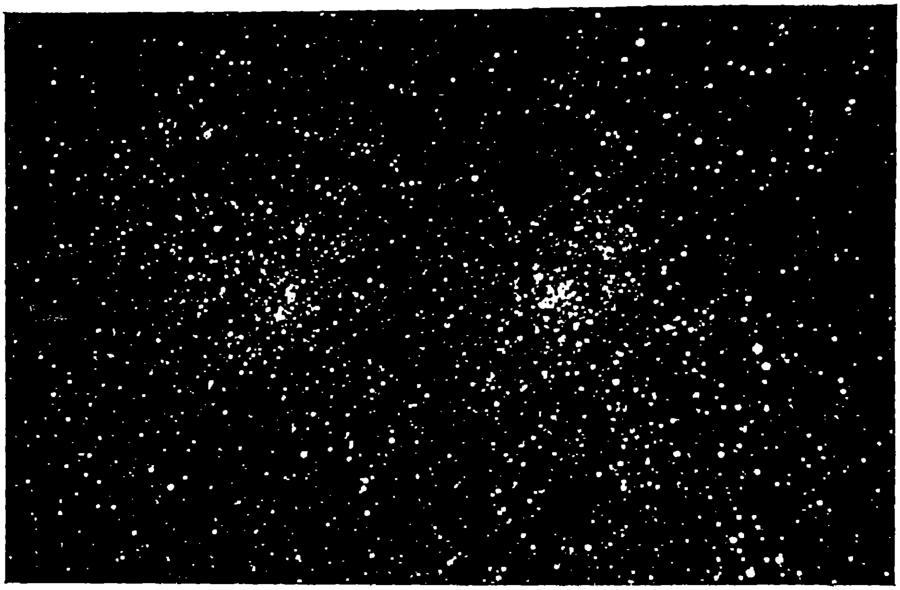


Fig. 274. *The Double Star Cluster* h *and* χ *Persei, Photographed by Isaac Roberts with 20-Inch Reflector, 1890 January 3. Exposure 3 hours.*

**Open or Galactic Star Clusters.** The Pleiades and Hyades are the brightest examples of a large class called, because of the loose distribution of their stars, open clusters. Trumpler, who has studied such clusters extensively, lists 334. All but a very few, and these mostly bright and relatively near, are within 15° of the galactic circle, and so another and perhaps better name for the class is galactic clusters. Those in the direction of the great star clouds of Sagittarius are more closely confined to the galactic plane than those in the opposite direction. Few lie between 0° and 80° of galactic longitude (Aquila to Cepheus).

Most galactic clusters contain a few hundred, some a few thousand, stars. They occupy spaces from a minute or so to more than a degree in diameter. Their stars are irregularly distributed, at such distances apart as to be resolvable in the telescope. Many of their stars are giants, but few, if any, galactic clusters contain variable stars. Nebulosity is evident

in some of the brighter, notably the Pleiades, and may exist in many others which are too remote for the nebulosity to have been noticed.

**Distances and Dimensions of Galactic Clusters.** The stars in a few of the nearer open clusters have perceptible common proper motions, and the distances of these are readily determined by the method discussed on page 352. Estimates of the distances of more remote clusters are made by utilizing the spectral types and apparent magnitudes of their stars: apparent magnitude $m$ is plotted against spectral type for many stars in the same cluster, and the resulting diagram is compared with a standard Russell diagram (Figure 249) in which absolute magnitudes $M$ are used. This comparison gives the value of the distance modulus $m - M$, from which the distance in light-years may be computed at once by the relation

$$\log D_{ly} = \tfrac{1}{5}(7.566 + m - M).$$

By these methods Trumpler finds that the Hyades are about 120 light-years, the stars of Coma Berenices 270 light-years, and the Pleiades 500 light-years away, and that the majority of telescopic galactic clusters range in distance from 1500 to 15,000 light-years. From their distances and apparent sizes he finds that the diameters of galactic clusters range from ten to eighty light-years.

**Globular Star Clusters.** Among the most magnificent objects in the sky, as seen with a large telescope, are the crowded swarms of stars which are known as globular star clusters. Each contains many thousands of faint stars which at the center are crowded so closely together (Figure 275) that they are resolved with difficulty even by the largest instruments. Photographs made with great reflectors add more and fainter stars with each increase in length of exposure, and it is probable that in none of the globular clusters have all the stars yet been perceived. Away from the center, the congestion grows gradually less and the precise boundary of the cluster is difficult to determine; but in general the boundary is approximately circular or elliptical, and the impression made upon the beholder is that of looking into a vast globe filled with stars. All such clusters are probably in slow rotation, and therefore slightly oblate, the axes of those which appear circular being directed nearly toward the observer.

Only about a hundred of these great clusters are known, and these lie mostly in the constellations Argo, Centaurus, Sagittarius, and Ophiuchus, between galactic longitudes 235° and 5°. Most of them are near the Milky Way, but none are in its middle; they are almost completely absent in

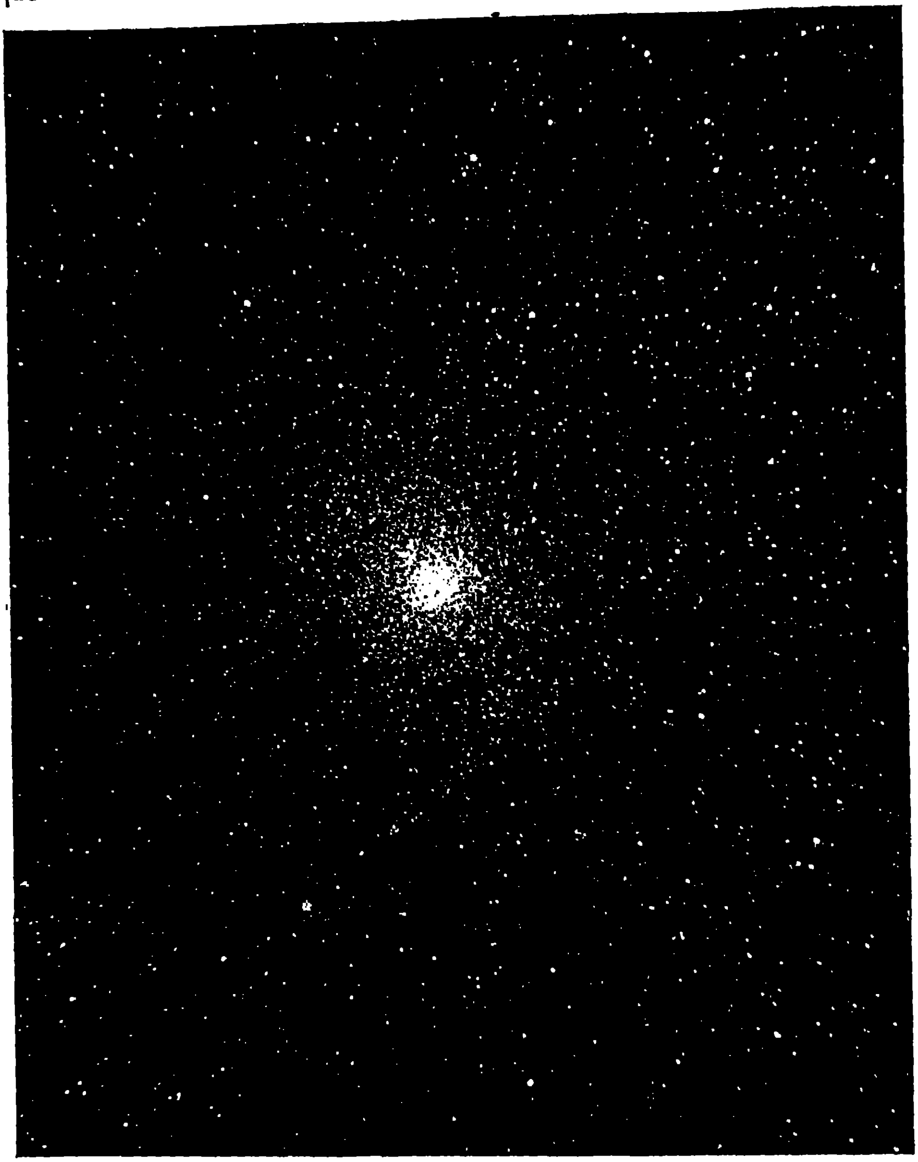


Fig. 275. *The Globular Star Cluster M 22 Sagittarii, Photographed by Duncan with 60-Inch Mount Wilson Reflector, 1918 August 6. Exposure 3½ hours.*

galactic latitudes $-10°$ to $+10°$. Their apparent distribution is thus in sharp contrast to that of the galactic clusters. Their radial velocities are large ($-410$ to $+225$ km./sec.), but their proper motions are too small to be detected with certainty, showing that globular clusters are vastly remote.

The globular cluster having the greatest apparent brightness and size is ω Centauri, which appears to the naked eye as a hazy, fourth-magnitude star, while in a large telescope it is seen as a mass of stars of the twelfth magnitude and fainter, occupying a circular area in the sky of about two-thirds the diameter of the full moon. It is too far south for observation in the northern half of the United States. The second-largest cluster, 47 Tucanae, is still farther south, only about 17° from the south pole, and near the lesser Magellanic cloud. The finest clusters visible in our latitudes are M 13 Herculis and M 22 Sagittarii, each barely visible to the unaided eye but containing more than 60,000 stars of the thirteenth magnitude and fainter. The most distant known globular clusters, such as NGC 2419 Lyncis and 7006 Delphini, have apparent diameters less than 2′ and contain no stars brighter than the seventeenth magnitude.

**Distances and Dimensions of Globular Star Clusters.** The distances of the globular clusters are too great to be determined by the methods which succeed with galactic clusters. Fortunately, however, a better method is offered by many of the globular clusters, as they are rich in variable stars of the Cepheid type, from a study of which Shapley has determined their distances by the method outlined on page 400. From the distances he computed the linear diameters of the clusters and the absolute magnitudes of their stars, and found that the globular clusters thus studied were all of about the same size and that the brightest stars in them (which are all red giants) were of a remarkable uniformity of brightness, of about −1.5 absolute photographic magnitude. By assuming these uniformities to extend to all globular clusters, he then found the distances of those which do not contain known variable stars from their apparent diameters and the apparent brightness of their brightest stars. His results, the first of which were obtained in 1917, were a revelation. The nearest globular clusters, such as ω Centauri and M 22 Sagittarii, were shown to be about 20,000 light-years away, and the farthest yet studied, NGC 2419, is about 250,000. At such distances, stars fainter than the Sun would be invisible, and the many thousands of stars that appear on long-exposure photographs of these clusters must be only the giants, while still greater numbers of lesser stars doubtless escape our detection.

Some dim conception of the meaning of 250,000 light-years may be attained if we reflect that the light by which we study these remote stars is older than the oldest evidence of the existence of humanity on the Earth. As Sir James Jeans says, "Through the childhood, youth and age of countless generations of men, through the long prehistoric ages, through the slow dawn of civilization and through the whole span of time which history records, through the rise and fall of dynasties and empires, this light has traveled steadily on its course, covering 186,000 miles every

second, and is only just reaching us now. And yet this enormous stretch of space does not carry us to the confines of the universe. . . ."

The diameter of the dense central part of each individual cluster is about 15 light-years, and that of the outlying regions fully 100 light-years.

**Internal Conditions of Star Clusters.** In the central portion of a globular star cluster there are about 1500 times as many giant stars as there are stars of all kinds in an equal volume of space near the Sun; but even so, the average distance between giants is some 50,000 astronomical units, so that there is still plenty of room for these and for numerous dwarfs to move without collision. (The only suggestion on record of a collision in a star cluster is that afforded by a 7th-magnitude nova that appeared in M 80 Scorpii in the year 1860.) In galactic clusters the star density is much less; at the center of M 11 Scuti, one of the densest galactic clusters, the average distance between stars is one light-year.

Trumpler remarks that "an observer at the center of M 11 would find about forty stars which would appear three to fifty times as brilliant as Sirius shines in our sky"; Shapley comments that "this display would be very dull compared with the show at the center of the Hercules cluster."

**Star Clouds.** In parts of the Milky Way, especially from Cygnus to Centaurus, the stars appear to be gathered into great clouds of hundreds of thousands each. Some of these, doubtless, are apparent only, being sections of a distant background outlined between intervening dark nebulae; but it is difficult to believe that this is true of all. A fine example is the main feature of the little constellation Scutum Sobieskii, between Aquila and Sagittarius, which is about 7° in diameter and conspicuous to the naked eye. South of this, near the Milk Dipper of Sagittarius, the Milky Way is a tumbled mass of star clouds so complex as to defy description (Figure 270).

About 45° from the galactic circle and 20° from the south pole, and hence always invisible in the latitudes of the United States, are the **Clouds of Magellan** (Figures 276, 277), named for the great Portuguese navigator. To the naked eye they appear like detached portions of the Milky Way, about 7° and 4° in diameter, respectively. Their brightness, as described by Sir John Herschel,[2] "may be judged of from the effect of strong moonlight, which totally obliterates the lesser, but not quite the greater."

[2] *Outlines of Astronomy*, 1857, p. 515.

Fig. 276. *The Greater Magellanic Cloud, Photographed at the Arequipa Station of the Harvard College Observatory.*

From studies of the many Cepheid variables contained in the Magellanic clouds, Shapley finds a distance of 86,000 light-years for the greater and 95,000 for the smaller. With the angular extent given above, these distances would imply diameters of 11,000 and 6000 light-years; but recent work at Harvard shows that faint outlying portions almost double these dimensions. The larger cloud contains many star clusters and diffuse nebulae. One of the nebulae (30 Doradus, the Looped Nebula) is itself more than 100 light-years wide. According to Shapley, if this nebula were

placed among the bright stars of Orion, it would fill that entire constellation and be so bright as to cast perceptible shadows of objects on the Earth.

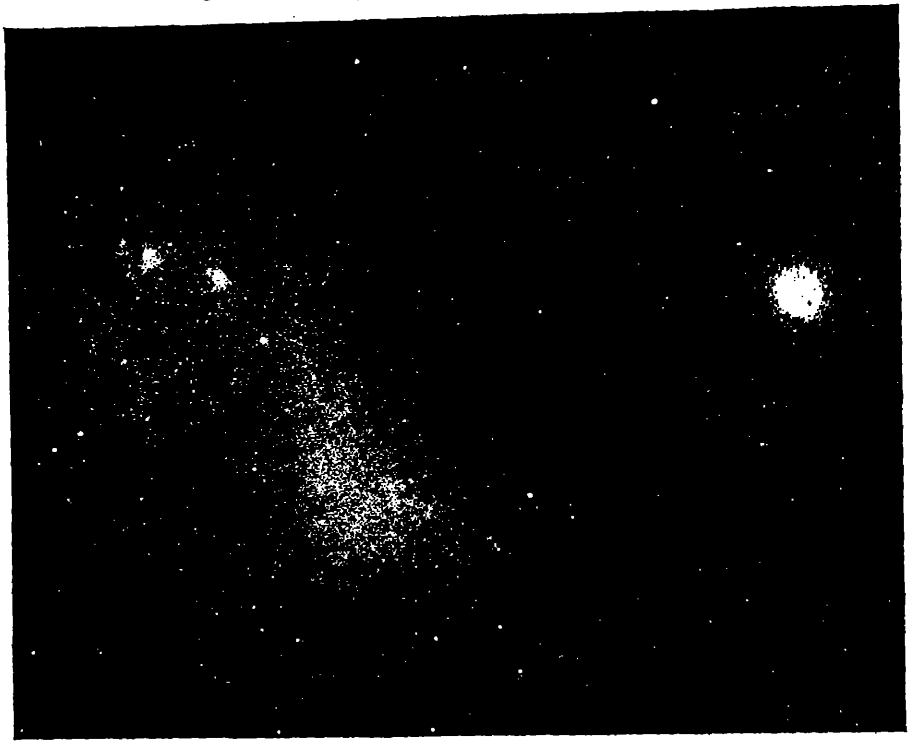


Fig. 277. *The Lesser Magellanic Cloud and the Globular Star Cluster 47 Tucanae, Photographed by Bailey with 24-Inch Telescope at Arequipa, 1898 November 10. Exposure 5 hours.*

## THE NEBULAE

**Observations of Nebulae Prior to the Use of Photography.** The word nebula is used in astronomy to denote any object outside the Solar System which occupies a perceptible area in the sky and which cannot be resolved by large telescopes into stars.

The term *stella nebulosa* was applied by the early astronomers to any hazy, luminous spot which seemed fixed among the stars. In his Almagest, Ptolemy listed six, all of which have proved to be star clusters or loose groups of stars. They include Praesepe, the double cluster in Perseus, a bright cluster (Messier 7) in the tail of the Scorpion, and three loose groups such as the one composed of $\varphi_1$, $\varphi_2$, and $\lambda$ Orionis in the head of Orion, which, with attention, are easily resolvable by the naked eye. Some of the *stellae nebulosae*, such as *h* and $\chi$ Persei, $\omega$ Centauri, and 47 Tucanae, were given stellar designations on the maps of Bayer and Flamsteed.

Very few of the nebulae which are still recognized as such are visible to the unaided eye; among these few the only one easily seen is the great nebula in Andromeda (Messier 31), which was recorded by Al Sufi in the tenth century. Remarkably enough, Galileo makes no mention of true nebulae, although he made a careful examination of the region of Orion which contains one of the brightest. The honor of the first telescopic discovery of a nebula seems to belong to an admirer of Galileo's, Peiresc of Provence, who found the great nebula of Orion (Messier 42) with a telescope in December, 1610. Two years later, the Andromeda nebula was first observed telescopically by Simon Marius in Germany, who compared it to a candle shining through a plate of horn.

The first catalogue of nebulae and star clusters was made by the French astronomer Messier in 1781. Messier's chief interest lay in the discovery of comets and, as many of the nebulae resemble faint comets when seen in a small telescope, he found it expedient to list the brighter ones with their right ascensions and declinations and a brief description in order that they might be readily recognized. His list contains 103 objects, more than half of which are star clusters, although Messier described many of these as "*nébuleuse sans étoiles*." For many of these nebulae and clusters, their numbers in Messier's list are the most common designation.

No great interest seems to have been taken in the nebulae until 1783, when Sir William Herschel began his systematic study of the heavens with an eighteen-inch reflector (page 440), during the course of which, with the aid of his sister Caroline, he found more than 2500 new nebulae and clusters which he classified and described. This work was extended to the southern hemisphere by his son Sir John Herschel, who took the eighteen-inch telescope on an expedition to the Cape of Good Hope in 1834. In 1864 Sir John published a *General Catalogue* describing over 5000 nebulae and clusters, only 450 of which were discovered by other observers than the Herschels. This catalogue is now superseded by the *New General Catalogue* published in England by J. L. E. Dreyer in 1888. With two supplementary lists, the *Index Catalogues*, Dreyer's compilation contains more than 13,000 objects. Most nebulae and star clusters are commonly known by their numbers in Dreyer's catalogue, the name of which is abbreviated to NGC. Thus, the great nebula of Andromeda is referred to as M 31 or as NGC 224.

Details of the structure of many nebulae were revealed by the six-foot reflector built in Ireland in the 1840's by the Earl of Rosse, the most important discovery being that of a spiral or whirlpool-like form. Other valuable observations of nebular details were made by the English astronomer Lassell, who transported a 24-inch reflector for this purpose to the island of Malta. The early observers with large telescopes gave some of the nebulae names which are still used, although the details revealed by modern instruments largely obliterate any resemblance between these nebulae and the objects for which they were named. Examples are the Crab nebula

(M 1), the Swan or Omega nebula (M 17), the Dumbbell nebula (M 27), and the Owl nebula (M 97).

It was commonly believed in Herschel's time that all nebulae were "resolvable" if only a sufficiently large telescope were used, and that many were "island universes" or sidereal systems comparable in size with the Galactic System; but Sir William Herschel later expressed the view that some of the nebulae were composed of a shining fluid which was not of a starry nature. This view was strongly confirmed by Sir William Huggins, when in 1864 he examined the spectrum of the little bright nebula (NGC 6543, Figure 278) which is very near the north pole of the ecliptic, and saw but a single bright line in the green (many fainter lines have since been observed), showing that the nebula was "not an aggregation of stars, but a luminous gas."

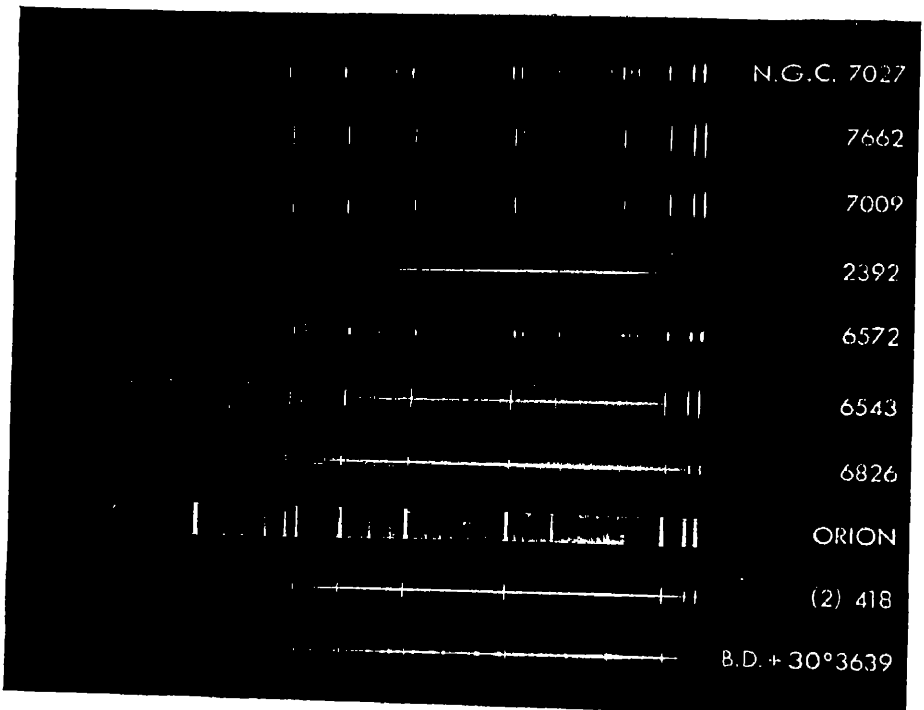



Fig. 278. *Spectra of Gaseous Nebulae, Photographed at the Lick Observatory.*

**Photographic Studies of Nebulae.** In the study of nebulae, photography possesses two great advantages over visual observation: first, a well-made photograph is a permanent and accurate record of the appearance of the object observed, a record which is free from personal bias or misjudgment on the part of the observer. Second, by prolonging the exposure of the plate, often to a duration of many hours, details are shown which are much too faint to be perceived by the eye. For this reason, more can be learned about the forms of nebulae by a study of even the

reproductions of photographs such as appear in this book (though they fall far short of the original negatives) than by visual study with the most powerful telescope.

The first successful nebular photograph was made by Henry Draper of New York in 1880, when he photographed the great nebula of Orion with an eleven-inch refractor, exposing the plate fifty-one minutes. This was soon surpassed by plates taken with a three-foot reflector by Common in England. In 1886 Isaac Roberts began in England an extensive program with a twenty-inch reflector, and by the end of the century he had published two fine volumes of photographs of nebulae and star clusters. His three-hour exposure on the Andromeda nebula in 1888 was a great revelation, for it showed that certain vague dark lanes previously noted by visual observers were really a part of a convoluted structure and that this great nebula is one of the spirals. Roberts's photographs were improved upon by Keeler at the Lick Observatory, who used a three-foot reflector from 1899 to 1901; and during the last decade of the nineteenth century and the first two of the twentieth, wonderful photographs of extended nebulosities and of Milky Way structure were made with large lenses of the portrait type by Barnard at the Lick and Yerkes Observatories, Wolf at Heidelberg, and Bailey at the Harvard Observatories in Massachusetts and Peru.

Perhaps the most perfect nebular photographs yet obtained are those made by Ritchey with superior reflectors of his own construction, first a twenty-four-inch which he used at the Yerkes Observatory from 1900 to 1904, and later the sixty-inch Mount Wilson reflector, which he used principally from 1908 to 1910. The 100-inch mirror of the Hooker telescope at Mount Wilson was also constructed under Ritchey's supervision, and has been used in nebular photography with excellent results since its completion in 1918.

For photographing objects whose diameters are less than a degree, or for the detailed study of larger objects, no instrument is better than a reflecting telescope; its mirror is perfectly achromatic, and if constructed with a large ratio of aperture to focal length (in most modern reflectors this ratio is 1:5), it is very rapid. For the general study of more extended bodies, such as the galactic clouds and the largest nebulae, the field of good definition in the reflector is too small, and better results are obtained with wide-field lenses and Schmidt-type telescopes.

**Method of Reproduction of Photographs of Nebulae in This Book.** Most nebulae are extremely faint, even as they appear on the photographic plate. In many of the illustrations reproduced in this book, the contrast of the nebula with the sky background was heightened by successive copying on slow, fine-grained plates. A glass positive was made from the original negative, a second negative from this, and a paper print from this second negative, and the copper plate used in printing the half-tone illustration was made by a photographic process from the paper print. This procedure is often necessary in order that the faint details of the original negative be not lost in reproduction. The scale of the photograph was in many cases

enlarged or reduced to suit the available space. The long rays which appear attached to the bright stars in the photographs made with reflecting telescopes are caused by diffraction of the star's light by the thin steel webs which support the secondary mirror.

**The Classification of Nebulae.** Twentieth-century research has made clear that the objects called nebulae fall into two main classes which have nothing in common except their non-stellar appearance under limited telescopic power. The first of these classes is the **galactic nebulae,** so called because they are all situated in or near the Milky Way and are believed with good reason to belong to the Galactic System and to have distances of only a few hundred or a few thousand light-years. This class, which will be discussed immediately, is subdivided into **planetary** nebulae and **diffuse** nebulae.

The other of the two principal classes consists of the **extra-galactic nebulae,** which are distributed profusely in all parts of the sky *except* the Milky Way. These are now known to be mostly at distances of millions of light-years, and each is believed to be a vast system of the same nature as the Galactic System itself—what Herschel called an island universe and is now called an external galaxy. The extra-galactic nebulae will be discussed in the last chapter of this book.

**The Spectra of Gaseous Nebulae.** Nebulae which have spectra characterized by bright lines (Figures 278, 289) are known as **gaseous nebulae.** They include all the planetaries and many of the diffuse nebulae. The brightest *visible* line is usually one at $\lambda 5007$, caused by doubly ionized oxygen. Near it are another strong line ($\lambda 4950$), also caused by doubly ionized oxygen, and the $H\beta$ of hydrogen; the dominance of these lines gives to many such nebulae a vivid green color. The emission which is *photographically* strongest is usually a pair of lines at $\lambda 3727$, caused by singly ionized oxygen. The Balmer hydrogen series is always conspicuous but, unlike the case of the solar chromosphere, the red $H\alpha$ is relatively weak. Neutral and ionized helium are represented, the latter in many nebulae by a strong line or band at $\lambda 4686$. Nitrogen, carbon, and many other elements of low atomic number are represented in various states of ionization by faint lines.

The lines at 5007, 4959, and 3727 were long attributed to a hypothetical element which Huggins named "nebulium." It was expected that this element would eventually be discovered in the chemical laboratory—an expectation which was strengthened by the chemical discovery of helium in 1895 but which was gradually

abandoned as the gaps in the periodic table (page 175) were filled one after another by newly discovered elements, none of which showed the nebular lines. In 1927 Bowen of Pasadena cleared up the mystery by showing, on the basis of modern atomic theory, that emissions having the wave lengths of the "nebulium" lines would be expected from ionized oxygen under nebular conditions, although "forbidden" (page 178) under conditions known on the Earth. Although, in the spectra of most gaseous nebulae, oxygen lines are the strongest, Bowen shows that hydrogen is by far their most abundant constituent, with helium second, while oxygen is much rarer than either. In the spectra of NGC 7027 and some other planetary nebulae, Bowen and Wyse have found faint lines of many highly ionized elements, including metals.

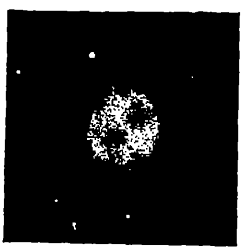
**Planetary Nebulae.** The term **planetary nebulae** was applied by Herschel to objects having small, well-defined, circular or elliptic disks which, as seen in moderate-sized telescopes, often resemble those of planets (Figure 279). They range in apparent size from mere points, which can be distinguished from stars only by their spectra, to the helical nebula NGC 7293 Aquarii, which is about 15′ in diameter. Their distances are believed to be thousands of light-years, and their diameters thousands of astronomical units. Their densities must be exceedingly low, even in comparison with those of giant stars.

The large planetaries, of which the Owl and the Dumbbell are additional examples, are very few in number and rather faint; the small ones, mostly a few seconds in diameter, number about 150, and many of them are so bright that they can be photographed with present-day reflectors with only a few seconds' exposure. Most of the known planetary nebulae contain centrally located stars which, in the few cases that are bright enough for spectrographic observation, are of the B type or hotter. Sometimes the central star is much brighter than the nebula, producing what Herschel called a **nebulous star**; more frequently it is fainter than the nebula and its light is often so concentrated in the violet end of the spectrum that the star is imperceptible visually in all but the largest telescopes, although showing conspicuously on photographs. Fluorescence in a flood of ultra-violet starlight is the most probable explanation of the luminosity of the nebula.

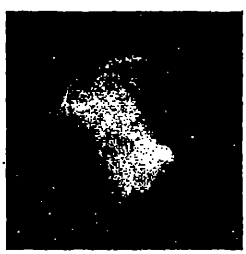
As photographed with large telescopes, the disks of the planetary nebulae show numerous details. Many are brighter at the circumference, giving the appearance of a ring, as in the famous nebula in Lyra (NGC 6720); some, like NGC 7009 and 7662, have two or more concentric rings; and a few appear to be spirals, not of the flat or watch-spring variety, but helical like a corkscrew. All are fairly symmetrical with respect to a central

Fig. 279. *Planetary Nebulae. Photographed at the Mount Wilson Observatory. Scale for bottom row, 1 inch = 50″; for others, 1 inch = 6.′5.*

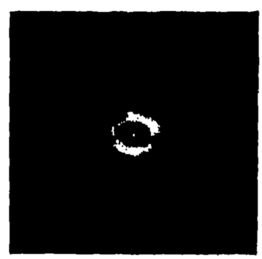
NGC 7293 (*Hubble, 100-inch telescope*)



M 97 (*Ritchey, 60-inch*)



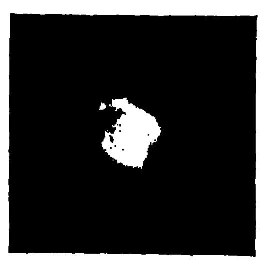
M 27 (*Duncan, 100-inch*)



M 57 (*Duncan, 100-inch*)

NGC 7009 (*Pease, 60-inch*)

NGC 7662 (*Pease, 60-inch*)



NGC 6543 (*Pease, 60-inch*)

point or star. It is practically certain that few of these nebulae are really flat rings, for only one or two present an appearance that can be due to a thin ring turned edgewise; they must be translucent shells of gas, the appearance of a ring being due to the greater thickness of gas in the line of sight at the apparent edge of the shell.

The spectra of planetary nebulae consist of bright lines only or of bright lines on a continuous background. Photographed with a slitless spectrograph, the bright lines become monochromatic images of the nebula, and it is thus often found that the form of the nebula appears different in light of different wave-lengths, probably because of a variety of distribution of the constituent gases.

The spectral lines of most planetary nebulae are considerably displaced, indicating velocities which average about 30 km./sec.—noticeably higher than the average velocities of the stars. The lines, when the slit is placed centrally, are often inclined in such a way as to suggest a rotation around the shortest axis of the nebula, and in some cases they are doubled and in some distorted in a manner difficult to interpret. A simple doubling of the spectral lines where the slit of the spectrograph crosses the center of the nebula is attributed to an expansion of the nebulous shell; for these nebulae seem to be nearly transparent so that the light comes to us from both the front and the rear; and an outward motion would shift the lines toward the violet for the particles on the nearer side and toward the red for those on the farther.

The central stars of planetary nebulae are mostly of the O type of spectrum, a state to which some novae are eventually reduced. This fact and the further fact that such nebulae have actually been observed to form around novae suggest such a catastrophic origin of all planetary nebulae. However, the observed nova shells have expanded with speeds hundreds of times greater than those that are indicated by the spectra of planetary nebulae and have faded, some to invisibility, in a few years. If novae gave birth to the planetary nebulae they must have been explosions of a different order from, say, those of Nova Aquilae 1918 and Nova Puppis 1942.

**The Colors of Planetary Nebulae.** As observed visually with the 100-inch telescope, many of the planetary nebulae present beautiful colors. The brightest ones are mostly of a greenish blue, while others are predominantly red. The two bright planetaries NGC 7662 and NGC 3242 have very bright greenish-blue rings enclosed in fainter and larger shells of a rosy hue. The central stars, which are prob-

ably white, appear yellow in contrast with the blue ring. The "Saturn" nebula (NGC 7009) consists of a bright greenish-blue ring or "ball" with reddish ansae resembling in form the ring of Saturn, and with a yellow-appearing central star. The celebrated O-type star BD $+30°3639$, which was found spectroscopically by Campbell to be surrounded by a small gaseous nebula, appears in the 100-inch telescope in the center of a beautiful rosy ring about 4″ in diameter.

**Diffuse Nebulae.** Galactic nebulae of the second class display a great variety of form, size, and brightness. Few show any of the sharpness of outline that characterizes the planetary nebulae, and most of them fade imperceptibly into the background of the sky; hence the term **diffuse.** Some, like the great faint wreath in the constellation Orion (Figure 280), extend over many degrees on the celestial sphere, while some are only tiny wisps. The brightest of the diffuse nebulae is the central portion of the great nebula in Orion (Figures 281, 287), which is very dimly visible to the unaided eye as a slight mist around the middle star of Orion's sword. Some are so faint as to require many hours' exposure to record them on photographic plates at the focus of a modern reflector; and some, the dark nebulae, are manifest only as they are silhouetted against a brighter background of stars or luminous nebulosity.

Many luminous diffuse nebulae have continuous or dark-line spectra, but some conspicuous examples, including the great nebula in Orion, have emission spectra like those of planetaries. The radial velocities shown are mostly small.

The parallaxes of such ill-defined objects could not be directly measured even if they were large. From the *motus parallacticus* of the stars involved in the Orion nebula, Kapteyn has estimated its distance at 600 light-years.

The density of these objects must be exceedingly small. It may be shown that if the mass of the Orion nebula were as great as 100,000 times the Sun's mass, its gravitational attraction would perceptibly affect the proper motions of the neighboring stars, which it does not; and yet even the bright portion near the center is about a third of a degree in diameter, corresponding to a real diameter of about 3 light-years, or 20,000,000 times the diameter of the Sun. The mean density is therefore at most of the order of $10^{-17}$, or about a millionth that of the best vacuum yet produced artificially on the Earth.

**The Source of the Light of Galactic Nebulae.** That such extraordinarily tenuous objects as the galactic nebulae could shine by virtue of their own high temperature is difficult to believe. It has often been suggested that nearby stars are responsible for the illumination, and Hubble,

Fig. 280. *Orion and Its Nebulosities, Photographed by F. E. Ross, 1927 January 6, with 3-Inch Ross Lens. Exposure 5 hours.*

from a comprehensive study of galactic nebulae, supports this view with the following evidence:

1. With few exceptions, the diffuse nebulae surround or lie near bright stars of the hot spectral types, and planetary nebulae possess central stars which, in all cases where there is spectral evidence, are of the O or B type.

Fig. 281. *The Great Nebula in Orion (M 42), Photographed by H. W. Babcock with 36-Inch Reflector of the Lick Observatory, 1939 January 17.*

2. For a given surface brightness, the extent of the illumination is greater, the greater the brightness of the star, the relation being that which would be expected if the intensity varies inversely as the square of the distance.

3. A definite relation exists between the spectrum of the nebulosity and that of the associated stars: if the stellar spectrum is of the B0 or hotter type, the nebular spectrum consists of bright lines only or of bright lines on a faint continuous background; if the star is of type B1, the continuous background is conspicuous and the nebular spectrum is intermediate or mixed; and if the star is cooler than B1, the nebula has an absorption spectrum similar to that of the star.

The light of the *gaseous* nebulae, which surround high-temperature stars, is believed to be emitted by the atoms of gas as their electrons return to them after ionization by extreme ultra-violet starlight (fluorescence). That of nebulae near cooler stars, such as the nebulous brush around Merope in the Pleiades (Figure 273), is explained simply as reflected starlight.

It is probable that the apparent form of a luminous diffuse nebula seldom corresponds to the actual distribution of matter in space, since we perceive only the portion of it which is brought to luminescence by the influence of the associated stars. The visible portion may usually be surrounded by dark nebulosity which is of the same nature as the bright.

**Obscure Nebulae.** In many parts of the Milky Way, even in the midst of the densest star clouds, there are areas almost totally devoid of stars. Sir William Herschel is recorded to have exclaimed, when he encountered one of these in a star cloud in Scorpius during his systematic sweep of the sky, "*Hier ist wahrhaftig ein Loch im Himmel!*" As holes in the sky, or at least as vacancies among the stars, they were long regarded, notwithstanding the fact that, to appear starless, they must be tunnels extending through vast depths of starry space and pointed directly toward the Solar System—a highly improbable circumstance even if the stars were motionless, and practically impossible after they had moved about at random for a few millions of millions of years.

During the present century the conviction has grown in the minds of astronomers that these dark areas are not vacancies but obscuring masses—**dark nebulae** of vast dimensions which obscure the stars beyond them. Barnard, who discovered many of them, was at first doubtful of this interpretation, but later became its chief advocate and convinced its opponents. Some of these objects, like the dark "horse-head" south of $\zeta$ Orionis (Figure 285) and the dark markings in Ophiuchus (Figure 282), when photographed with large telescopes, leave no doubt as to their character of opaque clouds. Many of them are feebly luminous, as Barnard first

pointed out, and Bailey has shown from photographs made at Arequipa that faint luminosity pervades them almost universally. It is evident that in many regions luminous and obscure nebulae are continuous with

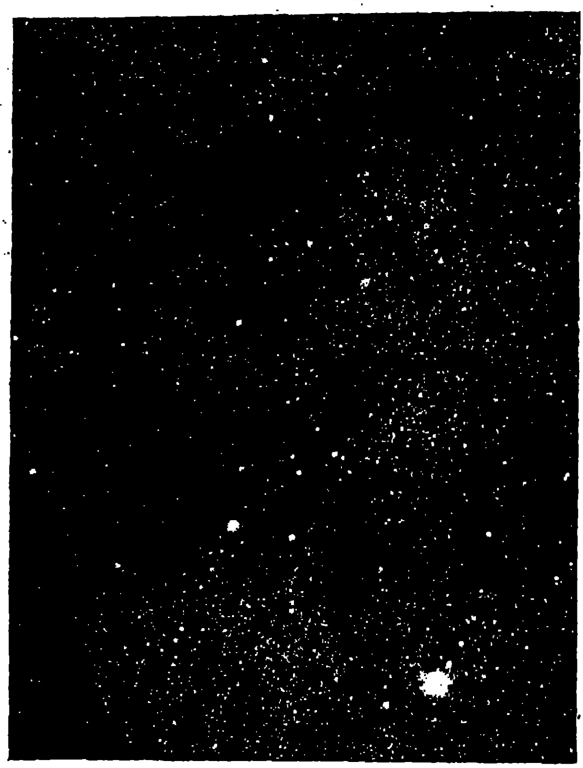


Fig. 282. *Dark Nebulae near θ Ophiuchi, Photographed by Barnard with 10-Inch Camera at the Yerkes Observatory.*

one another and form parts of the same object, as in the neighborhood of ρ Ophiuchi, where a luminous nebula with a spectrum like that of the star which it surrounds is itself surrounded by a border of comparatively starless sky.

Probably the most conspicuous (visually) of all the obscure nebulae is Number 86 of Barnard's list (Figure 286), which was discovered by Barnard with a five-inch telescope in 1883. It is about 2′ in diameter, "like a jet black nebula" (in Barnard's words), and has a seventh-magnitude orange-colored star on its northwest border. Just east of it is a fine open star-cluster.

Bailey held the view that obscuring nebulosities are the principal cause of all the irregularities in the Milky Way, including the great galactic star clouds; and that if the dark nebulae were absent so as to reveal the Galaxy as it really is, "we should

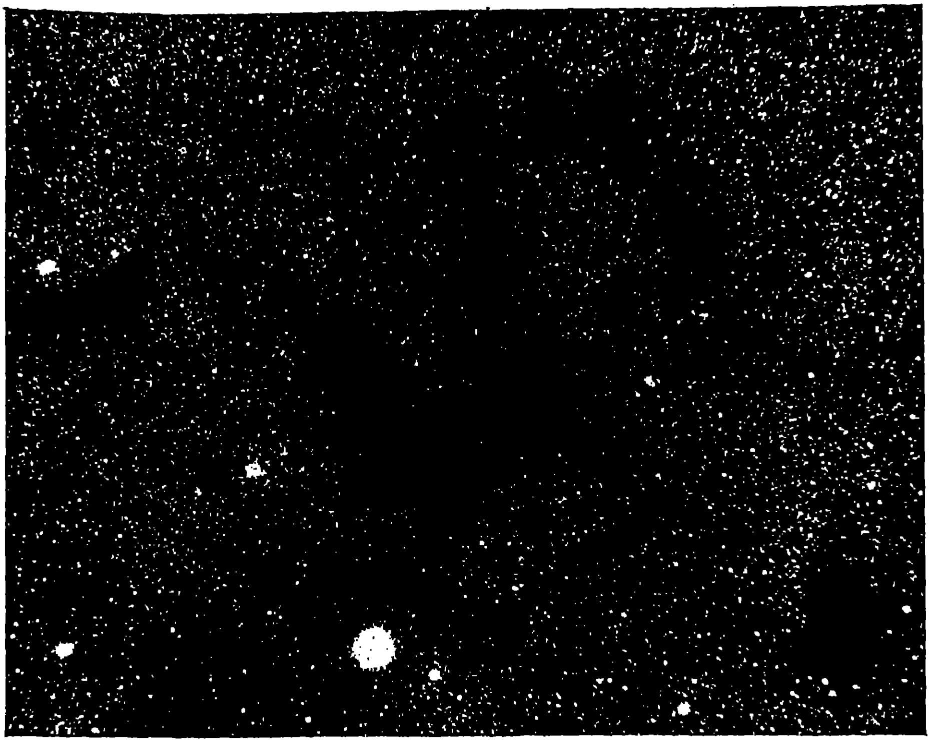


Fig. 283. *Detail of Figure 282: S-Shaped Dark Nebula, Photographed by Duncan with 100-Inch Reflector, 1921.*

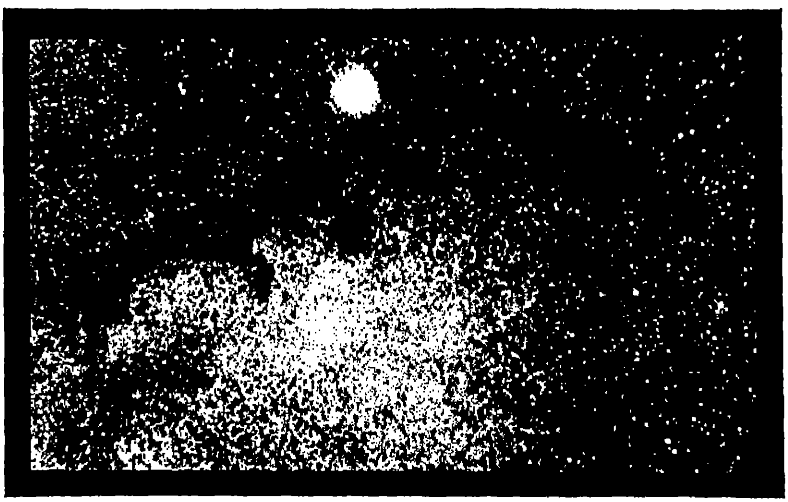


Fig. 284. *Detail of Figure 282: Foliated Dark Nebula, Photographed by Duncan with 100-Inch Reflector, 1941.*

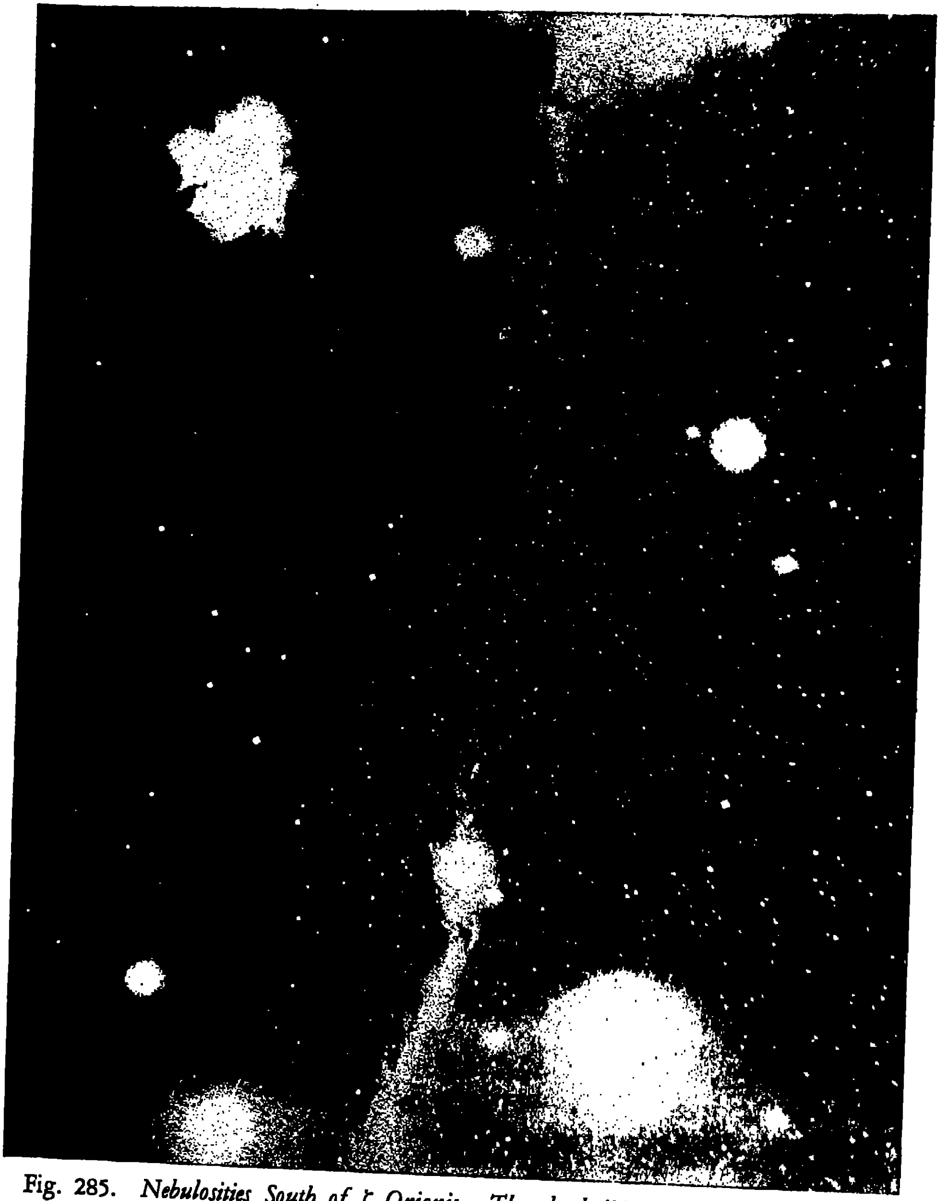


Fig. 285. *Nebulosities South of ζ Orionis. The dark "Horse-head" nebula, Barnard 33, is near the center. Photographed by Duncan, 1920 November 13, with 100-inch reflector. Exposure 3 hours.*

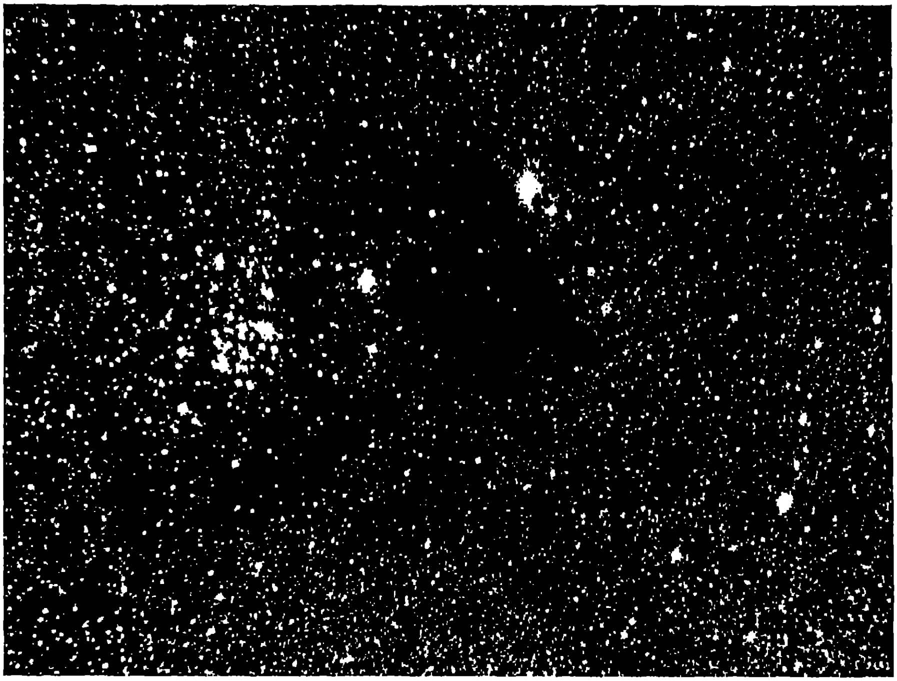


Fig. 286. *Dark Nebula Barnard 86 Sagittarii, Photographed by Duncan with 100-Inch Reflector, 1925 July 19.*

behold a zone of faint stars, little if at all broken by rifts or holes, without definite limits in width."

A given mass is most effective in obstructing light if it is in the form of microscopic, widely separated, solid particles as in a cloud of dust or smoke. A puff of smoke, for example, may be quite opaque even if only a few inches thick and practically imperceptible in weight. The nebulae being certainly of extremely low density, their opacity, wherever it occurs, is best explained by assuming the presence of a small quantity (per cubic mile, say) of fine dust.

**Spectral Absorption Lines of Interstellar Origin.** It has been known since 1904 that in the spectra of many spectroscopic binaries the H and K lines of ionized calcium do not partake of the periodic shift of the other lines caused by the star's velocity, and therefore cannot originate in the star's atmosphere. In some cases, probably in all, the D lines of sodium behave like H and K; and in the 1930's many other such detached lines, mostly faint, were found at Mount Wilson and Victoria. Some have been identified with the atoms of titanium and potassium and the molecules of carbon compounds; some have not been identified at all. The detached lines of calcium and sodium are weak and narrow, and so can be detected only in the spectra of high-temperature stars; in other spectra they are masked by the broad

corresponding stellar lines. The velocity indicated by their position, when freed from the motion of the Solar System, is always small, and they have been recognized in the spectra of many non-binary stars in which the velocity shift is sufficient to uncover them. In 1943, Adams showed that on high-dispersion spectrograms the calcium lines are in many cases multiple, indicating an origin in layers of cloud having slightly different radial velocities. The studies of Struve and of Plaskett and Pearce show that the detached lines are stronger in the spectra of distant stars than in those of nearer stars, so that the strength of such lines affords a criterion of distance.

The generally accepted explanation of detached spectral lines is that the atoms which produce them are not in the stars' atmospheres but are distributed thinly in the interstellar space through which the stars' light travels to us. Eddington pointed out that, as the diffuse nebulae have no definite boundaries, the outer portions of one nebula may merge with those of another, "so that there is always some residual density in interstellar space." He held the view that the ionized state of the calcium is due to starlight which, even in the depths of space, is strong enough to ionize an atom occasionally, whereas the atoms are so far apart that their chance of picking up an electron and so restoring the normal state is small. He calculated that the density is so low that there is but one atom to each cubic inch. A tube having a cross section of one square inch, containing atoms so distributed, and extending a hundred times the distance from the Sun to $\alpha$ Centauri, would contain about the same number of atoms as a cubic inch of ordinary air. Eddington remarked, "It depends upon our point of view whether we regard this as an amazing fullness or an amazing emptiness of space."

**Diffuse Nebulae of Special Interest. The Orion Region.** Preëminent among all nebulae as observed visually is the great nebula of Orion, Messier 42, NGC 1976, which by many is considered the most beautiful object in all the sky. It surrounds the star $\theta$ Orionis, described by Tennyson as

> "A single misty star
> Which is the second in a line of stars
> That seem a sword beneath a belt of three."

A small telescope shows this star to consist of several, four of which, of magnitudes five to eight, form a Trapezium (Figure 42) about 20″ in length, while about 2′ southeast of this group extends a row of three which beginners, forgetting the magnification of the telescope, often mistake for Orion's belt. All are B-type stars and glitter with intense blue-white light. Surrounding the Trapezium is a right triangle of nebulosity whose hypotenuse, about 2′ long, extends approximately parallel to the celestial equator on the north of the Trapezium. This nebulosity, which is the brightest part of the whole formation, presents a curdled appearance. Toward the southeast, beyond the row of three stars, extends a curved arm of nebulosity sometimes called the Waterspout, and to the northwest extends a brighter, broader curved portion like a wing. East of the Trapezium is a dark rift often called the Fishmouth or Sinus Magnus, which is easily seen on photographs to be slightly luminous. North of the rift is a circular wisp of nebulosity (M 43) surrounding a star of the eighth magnitude.

The color of the great nebula is greenish and is easily noticed in a small telescope. In larger telescopes, especially reflectors, this color is intense. Along the eastern

edge of the bright triangle, however, and also on the concave edge of the Waterspout, a rosy tinge is perceptible, as Barnard noted years ago. In the 100-inch reflector this is very evident, and so is the red color of many of the fainter stars (irregular variables) which contrasts strongly with the green of the main nebula and the blue-white of the bright Trapezium. The stars down to about the twelfth magnitude are so numerous that, were the nebulosity removed, the region would still be remarkable as a star cluster (Figure 287).

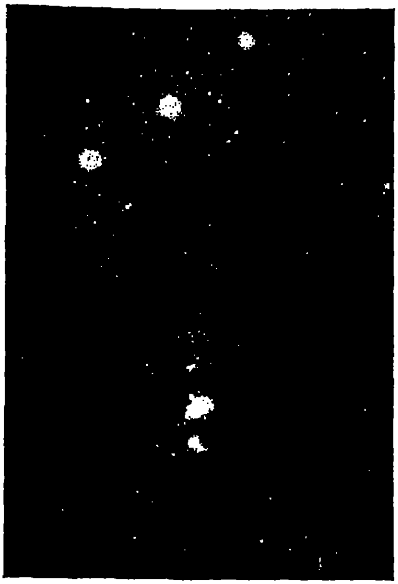


Fig. 287. *Blue light.*

Fig. 288. *Red light.*

*The Belt and Sword of Orion, Photographed by Zwicky with 8-Inch, f/1 Schmidt Camera on Palomar Mountain, 1941.*

On photographs (Figures 281, 288), much of the visual appearance is lost; the central stars are overwhelmed in bright nebulosity, and the regions which appeared dark or only faintly luminous show a wealth of nebulous detail. It requires but a moderate exposure to show that the main nebula extends more than half a degree toward the south, while on the north it joins the nebulosity around c Orionis, the most northerly star of the Sword. Longer exposures, especially if made on red-sensitive emulsions (Figure 288), practically fill the whole constellation of Orion with faint nebulosity.

The spectrum of the great nebula consists of bright lines on a very faint continuous background (Figure 289). The radial velocity ranges in different parts of the nebula from about +10 to about +25 km./sec.

South of $\zeta$, the eastern star of Orion's belt (Figure 286), extends an irregular line of luminous nebula, very faintly visible in the telescope and distinct on long-exposure photographs. East of this line the sky is largely devoid of faint stars, whereas west of it they are numerous. Evidently the eastern side is the location of a dark nebula, for a great dark cloud (Barnard 33), about 5′ wide and shaped somewhat like a

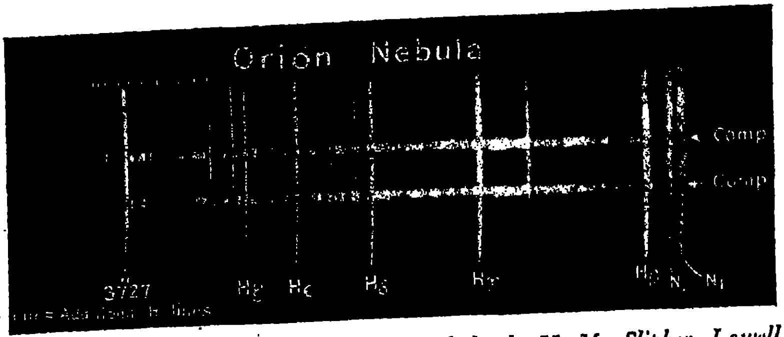


Fig. 289. *Spectrogram of the Orion Nebula, by V. M. Slipher, Lowell Observatory.*


Fig. 290. *The Great Diffuse Nebula M 8 Sagittarii, Photographed by Duncan with 60-Inch Mount Wilson Reflector, 1919.*

horse's head, protrudes from it over the bright nebulosity. This cloud has a "silver lining," as if a bright star or stars were concealed behind it. At the upper left corner of Figure 286 is the bright nebula NGC 2023 which surrounds (and, on the photograph, completely conceals) an eighth-magnitude star. North of the region shown in the figure is a remarkable combination of bright and dark nebulae. None of these nebulae are easily discerned visually, even with the largest telescopes.

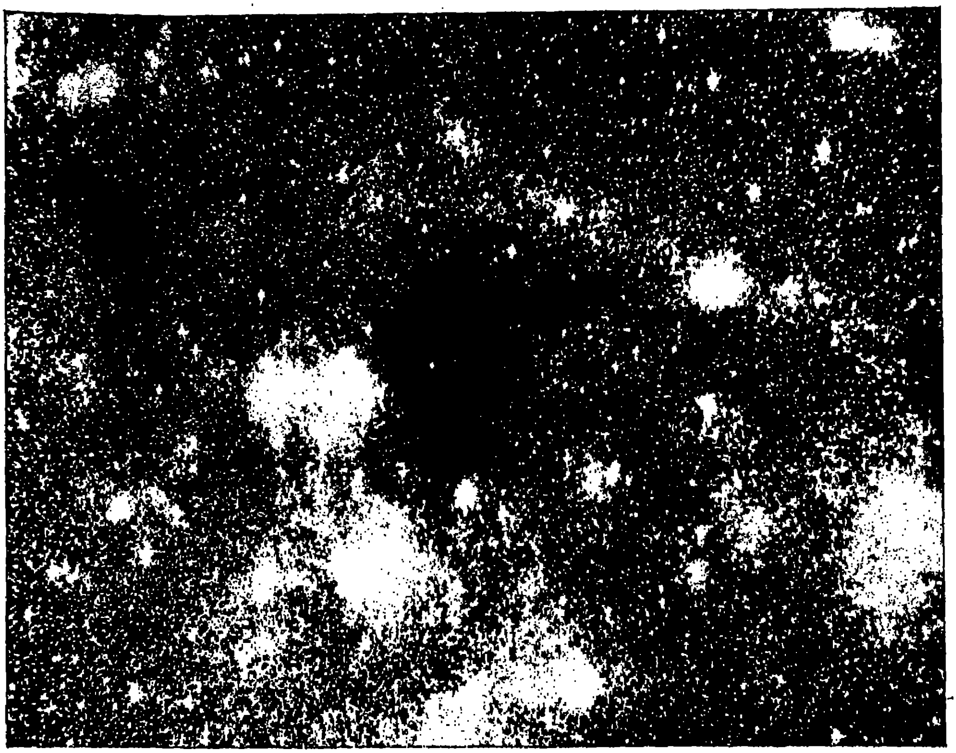


Fig. 291. *The Dark Nebulae Barnard 92 and 93 Sagittarii, Photographed by Duncan with 100-Inch Reflector, 1922.*

**The Region of Sagittarius and Scutum.** The Milky Way in Sagittarius and Scutum abounds in wonderful nebulous forms, both bright and dark. The brightest is Messier 8 (Figure 290), which is faintly visible to the naked eye. It surrounds a cluster of stars of the eighth to tenth magnitude and, as shown on photographs, is an extraordinary combination of bright and dark nebulosities. In color and spectrum it resembles the great nebula in Orion. About a degree north of Messier 8, and faintly connected with it, is the beautiful Trifid nebula, M 20 (frontispiece). A little farther north are M 16 and M 17, both remarkable combinations of bright and dark nebulosities.

In a dense star cloud situated between M 17 and the Trifid nebula are the two dark nebulae Barnard 92 and 93 (Figure 291), the former of which is noticeable in a small telescope.

**Region of $\eta$ Carinae.** Surrounding the irregular variable star $\eta$ Carinae 394) in the south circumpolar regions and covering about a square degree of th is the great mass of luminous and obscure nebulosities known as the Ke nebula. It derives its name from the dark nebula which is situated in the brig mass. The famous variable star is near the eastern edge of this "keyhole," wh not very conspicuous on long-exposure photographs.

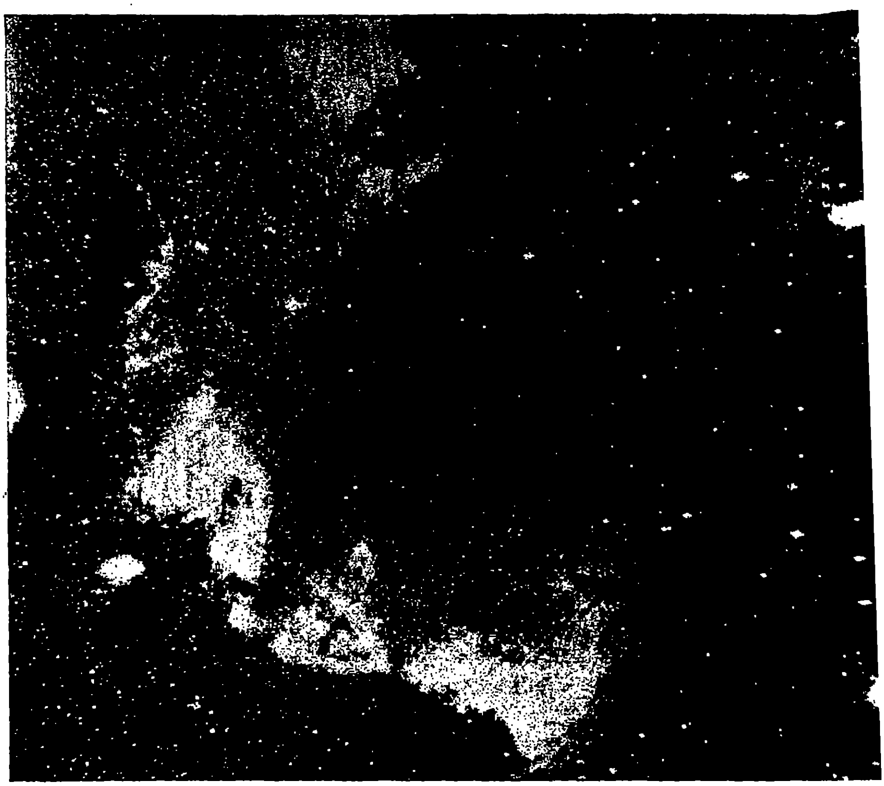


Fig. 292. *Southern Part of the America Nebula, NGC 7000 Cygni, Photograp by Duncan with 100-Inch Reflector, 1922.*

**The Cygnus Region.** The constellation Cygnus contains the vast but fa America nebula, NGC 7000, which is about three degrees long and lies about th degrees east of the giant star Deneb from which, on Hubble's theory, it derives light. It is bordered on either side by dark lanes, presumably nebulous, and t "Gulf of Mexico" (Figure 292) and "Hudson Bay" are marked by dark nebulositi West of the America nebula is a formation which, from its appearance on the Hook telescope photograph (Figure 293), has been called the Pelican nebula.

Surrounding $\gamma$ Cygni, the giant star at the intersection of the arms of the Northe Cross, are a number of faint nebulosities, and south of $\epsilon$ are the two beautiful Ve nebulae, NGC 6960 and 6992 (Figures 294–297), which are faintly connected, forr ing a wreath. Hubble, by comparing a photograph made by him in 1925 with o

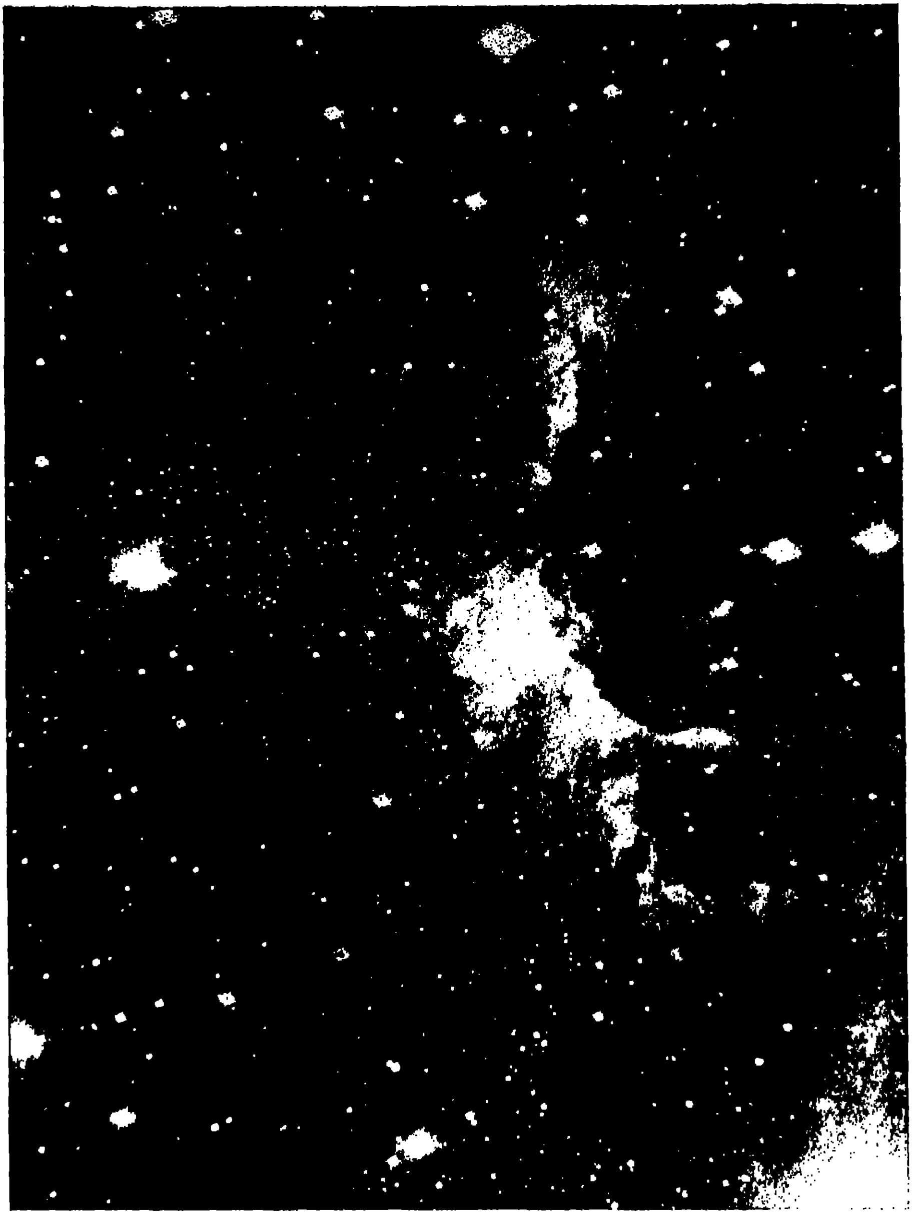


Fig. 293. *The Pelican Nebula in Cygnus, Photographed by Duncan with 100-Inch Reflector, 1925.*

made by Ritchey in 1911, both with the 60-inch reflector, detected in NGC 6960 a proper motion in a direction away from the opposite side of the wreath and at a rate (0".05 per year), which suggests that the wreath originated in an explosion about 150,000 years ago. Figure 297, an enlargement of a pair of plates made by Duncan with the 100-inch with an interval of twenty-two years, shows the middle part of NGC 6960 on a large scale (1 mm. = 8"); yet even here the displacement is so small

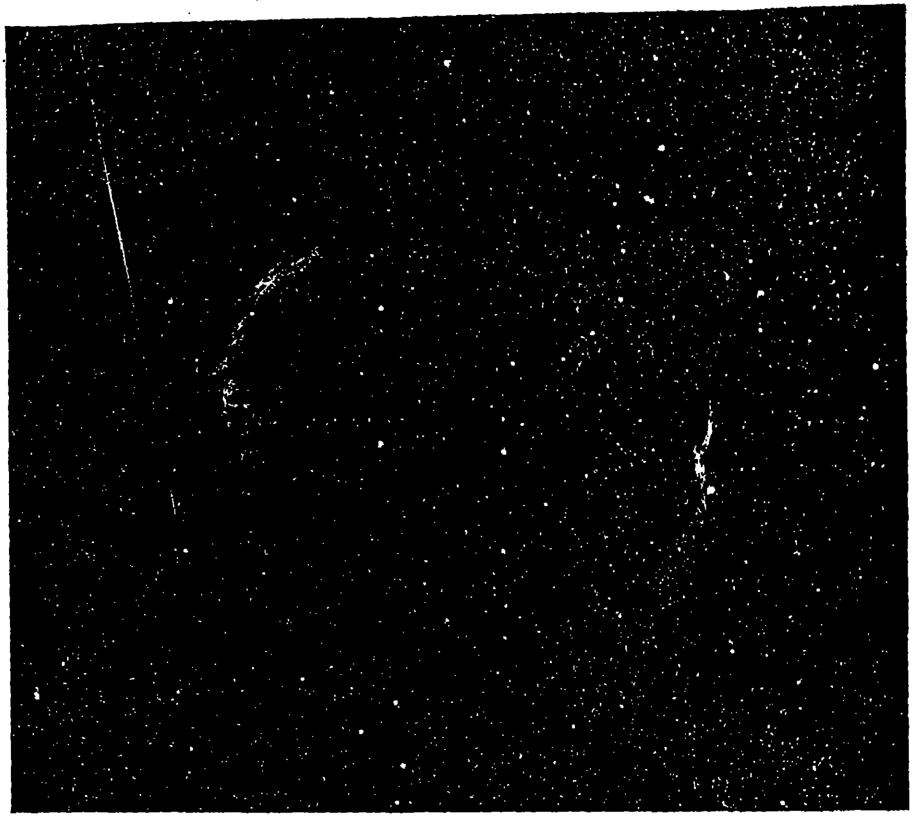


Fig. 294. *The Nebulous Wreath in Cygnus, Photographed by Barnard with 10-Inch Camera. (Meteor trail at left.)*

as scarcely to be noticed without optical aid. If the reader examines the pair of pictures through a stereoscope, the displacement will cause the nebulous filaments to seem to stand out in space between the observer and the stars.

**Variable Nebulae.** The few nebulae that are known to vary in brightness are associated with variable stars. The most notable ones are connected with T Tauri, R Coronae Australis, and R Monocerotis. There is little doubt that the variation of the nebula is in each case due to the variation of the exciting influence of the star. The nebula surrounding

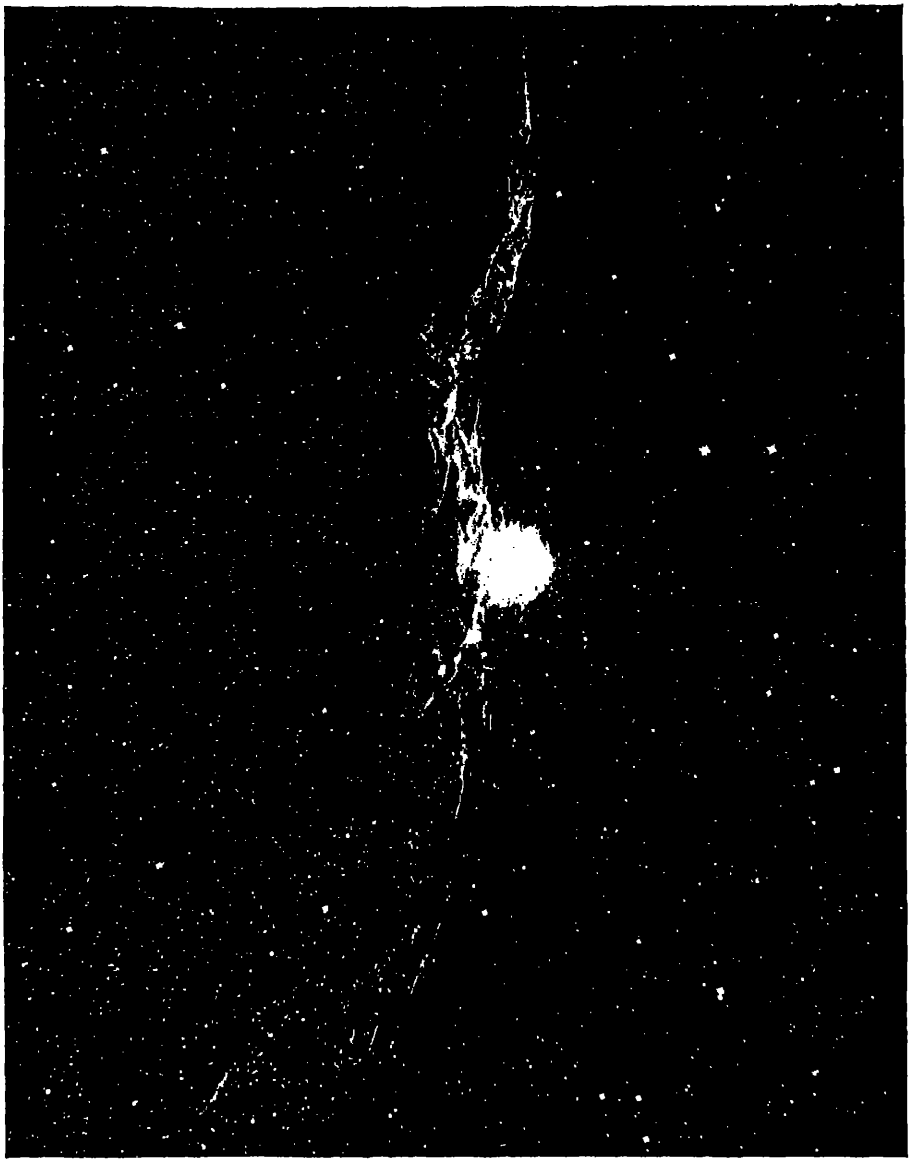


Fig. 295. *Detail of Figure 294. The Veil Nebula NGC 6960 Cygni, Photographed by Duncan with 100-inch Reflector, 1921 August 3. Exposure 7 hours. (Meteor trail at right.)*

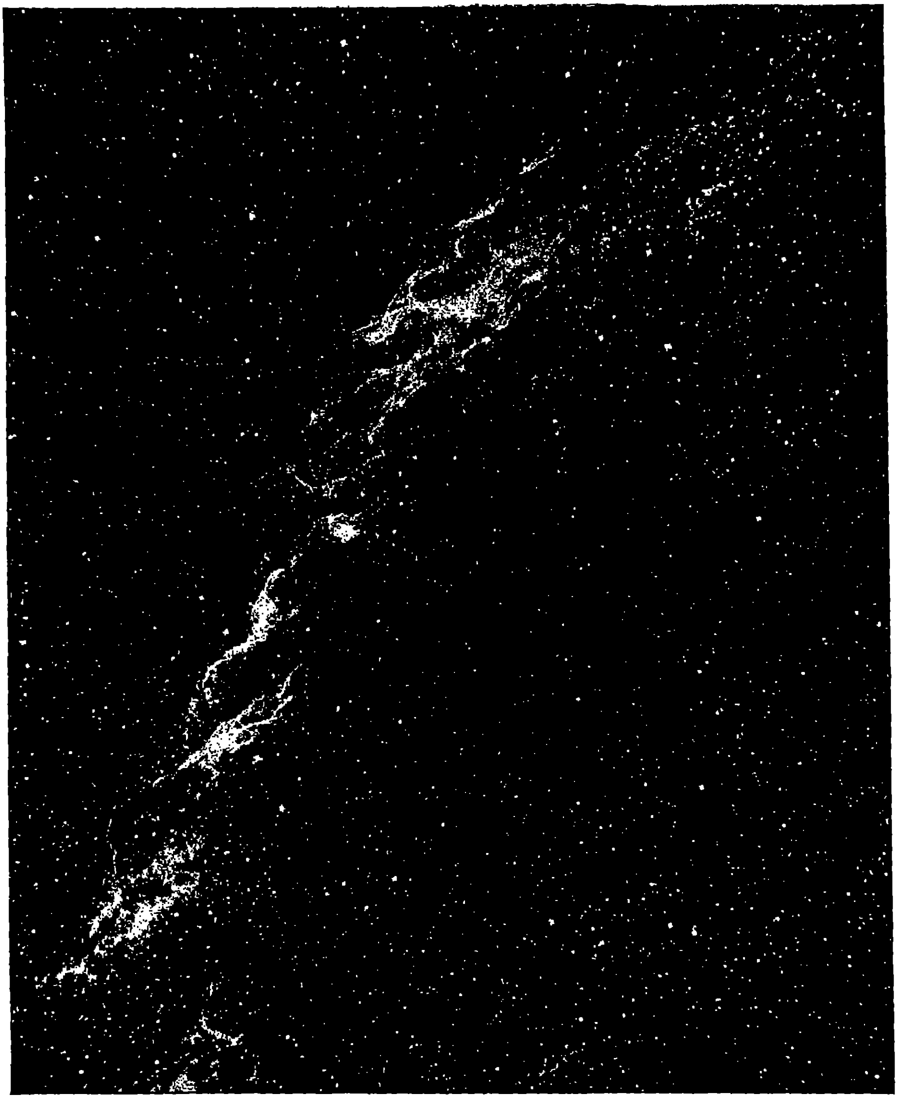
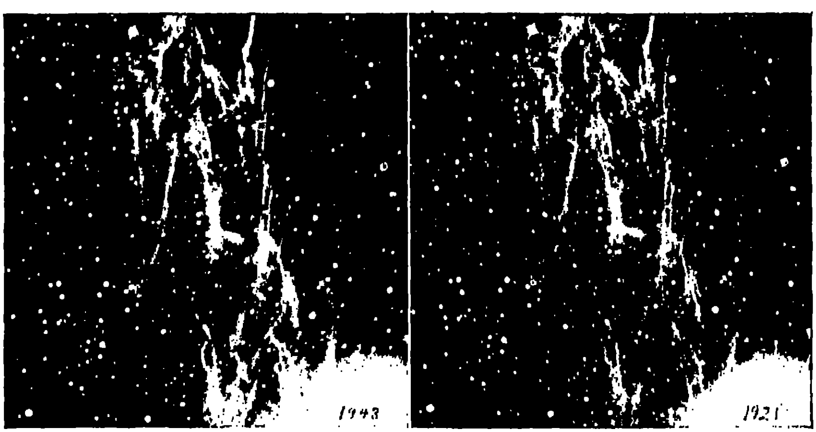


Fig. 296. *Detail of Figure 294. The Veil Nebula NGC 6992 Cygni, Photographed by Duncan with 100-inch Reflector, 1928 August 15. Exposure 6½ hours.*

R Coronae Australis is enclosed in a comparatively starless border probably due to dark nebulosity. Observations at Flagstaff, Mount Wilson, and Helwan, Egypt, have shown that, soon after the star sinks to a minimum, the nebula fades almost to imperceptibility; and that after the star brightens, a wave of luminosity sweeps out over the nebula. The natural interpretation is that the illumination is due to a light-echo similar to the one which swept out from Nova Persei 1901 (page 392), and this view is supported by the fact that the spectrum of the nebula is very similar to that of the star, both including bright hydrogen lines. The form of the nebula is not the same at successive brightenings, perhaps because of changes in the distribution of the nebulous material and to relative motion of the star and the nebula. Similar phenomena have been observed by Lampland, Slipher, and Hubble in the comet-shaped nebula attached to R Monocerotis.

Fig. 297. *Detail of Figure 295. Part of NGC 6960, Photographed by Duncan with 100-inch Reflector, 1921 August 3 and 1943 August 3. The filaments have moved, but almost imperceptibly.*



# CHAPTER 20

## THE GALACTIC SYSTEM

**Studies of the Arrangement of the Stars.** All the stars, star clusters and nebulae such as we have discussed so far are members of the vast aggregation called the Galactic System. We ourselves, along with the rest of the Solar System and its neighbors, are imbedded far within this aggregation, and this situation is a serious obstacle to our perception of its structure—we are so surrounded by the forest that we cannot see the woods for the trees.

The oldest method of attacking the problem of the arrangement of the stars is a statistical one based on observations of their number in different parts of the sky. Systematic study of this kind was begun late in the eighteenth century by Sir William Herschel. His method, which he called "star gauging," consisted in counting the stars visible in an eighteen-inch reflector having a field 15′ in diameter. Sir William made 3400 "gauges" in England, and his son Sir John made 2300 more with the same telescope at the Cape of Good Hope. During the present century, more extensive statistics have been gathered from photographs, and attention has been largely concentrated on "selected areas" distributed about 15° apart over the entire sky. The selection of these areas was made by the Dutch astronomer Kapteyn in 1906, when he proposed a comprehensive plan of coöperative work upon them which has been followed by many of the great observatories of the world.

A second method of determining the extent and structure of the Galactic System makes use of the spatial distribution of some of its members such as the star clusters, the distances of which, as explained in the preceding chapter, are made known by the apparent brightness of their stars, especially the Cepheid variables.

A third method is based on dynamical considerations of the rotation of the whole system, which is disclosed by a statistical study of the apparent motions, especially the radial velocities, of distant stars.

Finally, the general structure of our Galaxy is inferred by analogy with the other galaxies, the so-called extra-galactic nebulae.

**The Number of Stars of Different Magnitudes.** To the casual observer of a clear, moonless sky it often seems that the stars visible to the unaided eye are innumerable; but if attention is concentrated on a limited area, such as the square of Pegasus or the bowl of the Dipper, the stars in that area are found to be easily counted. In fact, only a few thousand stars can be seen in the whole sky without optical aid even on the best nights, and a little moonlight or haze conceals the majority of these, proving that the faint stars outnumber the bright. This increase in number with faintness is continued among the telescopic stars.

Table 27

NUMBER OF STARS BRIGHTER THAN GIVEN MAGNITUDES, WITH RATIO OF EACH NUMBER TO ITS PREDECESSOR

| Visual Magnitude | Number | Ratio | Visual Magnitude | Number | Ratio |
|---|---|---|---|---|---|
| 2 | 40 | ... | 6 | 4,800 | 3.2 |
| 3 | 135 | 3.4 | 7 | 15,000 | 3.1 |
| 4 | 450 | 3.3 | 8 | 46,000 | 3.1 |
| 5 | 1,500 | 3.3 | 9 | 134,000 | 2.9 |

The stars brighter than the ninth magnitude have been catalogued and counted. Table 27 gives the total number of stars brighter than each visual magnitude as determined at Harvard (the numbers cannot be given with perfect precision because of the variability of certain stars and the fact that many lie near the border line between two magnitude-classes).

By a laborious investigation of the number of stars in selected regions of the sky, photographed principally with the sixty-inch Mount Wilson reflector, Seares and van Rhijn have formed curves and tables giving the numbers of stars down to the twenty-first photographic magnitude in different galactic latitudes and to the twentieth visual magnitude for the whole sky. From their curves are derived the numbers in Table 28, which extends Table 27. The numbers for the last three or four magnitudes are of course extrapolations.

From the first of these tables we find that the number of stars visible to the naked eye is about five or six thousand; but since only half the celestial sphere is visible at one time and the faint stars are extinguished by atmospheric absorption when several degrees above the horizon, two

thousand is a liberal estimate of the number that are visible at one time even under the best conditions. The total number down to the twentieth magnitude, which is about a magnitude fainter than can be perceived by the eye at the 100-inch telescope, appears to be about 1,000,000,000.

Table 28

NUMBER OF STARS BRIGHTER THAN GIVEN MAGNITUDES (*Continued*)

| Vis. Mag. | Number | Ratio | Vis. Mag. | Number | Ratio |
|---|---|---|---|---|---|
| 7......... | 14,300 | ... | 14........ | 13,800,000 | 2.4 |
| 8......... | 41,300 | 2.9 | 15........ | 32,000,000 | 2.3 |
| 9......... | 117,000 | 2.8 | 16........ | 70,800,000 | 2.2 |
| 10......... | 324,000 | 2.8 | 17........ | 148,700,000 | 2.1 |
| 11......... | 868,000 | 2.7 | 18........ | 296,000,000 | 2.0 |
| 12......... | 2,260,000 | 2.6 | 19........ | 560,000,000 | 1.9 |
| 13......... | 5,700,000 | 2.5 | 20........ | 1,000,000,000 | 1.7 |

**Significance of the Ratio of Increase.** In the third column of Tables 27 and 28 is printed the ratio of each number to its predecessor. It is evident that, though the number of stars increases with increasing faintness throughout the table, the ratio of these numbers diminishes and apparently would become unity at about the 27th magnitude. This would mean simply that the total number of stars brighter than the 27th magnitude is the same as the total number brighter than the 26th—that is, there are no stars of the 27th magnitude.

If the stars were all of the same actual brightness, their differences of magnitude being due to difference of distance only, their apparent brightness would be inversely as the square of their distance; that is, each star of magnitude $n$ would be $\sqrt{\rho}$ times as far away as one of magnitude $n - 1$ where $\rho$ is the light ratio (page 336). All stars brighter than the $n$th magnitude would be contained in a sphere of radius $\sqrt{\rho}$ times the radius of the sphere containing stars brighter than the $(n - 1)$st magnitude, and the volume of the larger sphere would be $\sqrt{\rho^3}$ times the volume of the smaller. If, further, these stars of uniform intrinsic brightness were distributed uniformly through space, the number of stars brighter than the $n$th magnitude would be to the number of those brighter than the $(n - 1)$st as the volume of the larger sphere is to that of the smaller. Now, $\rho$ being equal to 2.512 . . ., $\sqrt{\rho^3}$ is very nearly equal to 4. Although there is known to be a vast range in the intrinsic brightness of stars, yet when great multitudes are dealt with, the numbers in different magnitude classes must still give an indication of their distribution; and the fact that the ratio of successive numbers in Tables 27 and 28 is less than four shows that the stars are not distributed uniformly throughout infinite transparent space.

Seares finds that an extrapolation indicates 30,000,000,000 as the total number of stars in the Galaxy; but the uncertainty of this estimate is evident when we consider that, if correct, it is based on about thirty invisible stars for every star observable with our greatest telescopes. Moreover, there is no doubt that vast numbers

of stars are hidden from us by dark nebulae and so could not be accounted for by any reasoning based on these tables alone. The above estimate of the number of stars in the Galaxy must therefore be *far too low.*

**General Distribution of Stars with Reference to the Milky Way.** Counts of stars, both bright and faint, in different parts of the sky show that the **star density**, or average number of stars per square degree, is greatest near the galactic circle and, in general, diminishes with increasing galactic latitude. This fact was clearly established by the work of the Herschels, and is strikingly evident in the star counts of Seares and van Rhijn, which are summed up in Table 29. This table gives, for certain galactic latitudes, the average number, per square degree, of stars brighter than photographic magnitude *m*. In the last column is given the **galactic concentration**, by which is meant the ratio of the star density on the galactic circle to that at the galactic poles. Two facts shown by the table are especially noteworthy: first, that there is a marked crowding of stars of each magnitude-class toward the galactic circle (the star density decreases with increasing galactic latitude); and second, that this galactic concentration is greater for faint stars than for bright stars.

Table 29

AVERAGE NUMBER OF STARS PER SQUARE DEGREE, BRIGHTER THAN PHOTOGRAPHIC MAGNITUDE *m*, IN DIFFERENT GALACTIC LATITUDES

| *m* | Galactic Latitude | | | | Galactic Concen-tration |
|---|---|---|---|---|---|
| | 0° | 30° | 60° | 90° | |
| 4.0 | 0.0156 | 0.0074 | 0.0051 | 0.0045 | 3.5 |
| 5.0 | 0.0449 | 0.0214 | 0.0148 | 0.0130 | 3.4 |
| 6.0 | 0.128 | 0.0614 | 0.0421 | 0.0372 | 3.4 |
| 7.0 | 0.361 | 0.173 | 0.118 | 0.103 | 3.6 |
| 8.0 | 1.01 | 0.482 | 0.325 | 0.278 | 3.6 |
| 9.0 | 2.81 | 1.31 | 0.871 | 0.723 | 3.9 |
| 10.0 | 7.71 | 3.49 | 2.23 | 1.81 | 4.3 |
| 11.0 | 20.8 | 9.06 | 5.47 | 4.33 | 4.8 |
| 12.0 | 55.6 | 22.7 | 12.8 | 9.89 | 5.6 |
| 13.0 | 146 | 54.4 | 28.6 | 21.4 | 6.8 |
| 14.0 | 371 | 125 | 61.0 | 44.3 | 8.4 |
| 15.0 | 910 | 272 | 123 | 87.1 | 10.4 |
| 16.0 | 2140 | 561 | 236 | 163 | 13.2 |
| 17.0 | 4780 | 1090 | 428 | 288 | 16.6 |
| 18.0 | 10200 | 1990 | 733 | 482 | 21 |
| 19.0 | 20800 | 3440 | 1190 | 769 | 27 |
| 20.0 | 40100 | 5620 | 1820 | 1160 | 34 |
| 21.0 | 73600 | 8690 | 2650 | 1670 | 44 |

**Galactic Concentration and Spectral Type.** Studies in the distribution of stars of different spectral type, especially those made at Harvard from the data of the *Henry Draper Catalogue*, show marked differences in galactic concentration. It is found that, for stars brighter than magnitude 8.25, there is little or no galactic concentration of F and G stars, that the K stars are concentrated only moderately toward the Galaxy, and that the concentration of A and especially of B stars is very marked indeed. Different degrees of concentration are shown by the M stars of different magnitudes and different subtypes. While the brighter M stars show little or no galactic concentration, that of the faint Ma and Mb stars is strongly marked.

**The "Grindstone Theory."** The first noteworthy speculations on the structure of the sidereal universe were published in 1750 by Thomas Wright, English amateur astronomer, whose theory was advocated and developed by Sir William Herschel. According to this theory, the majority

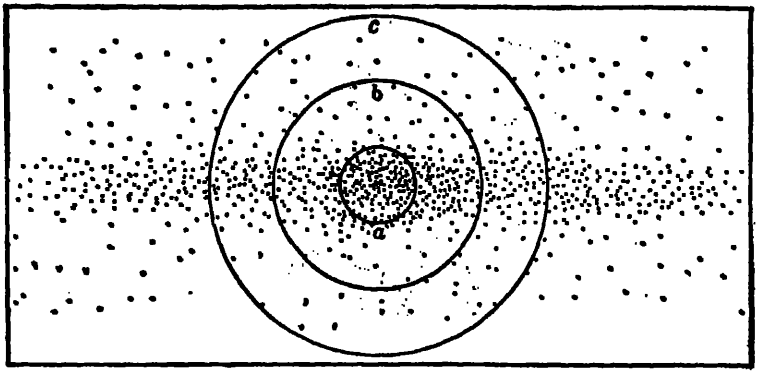



Fig. 298. *Galactic Concentration. (From Russell, Dugan and Stewart's* Astronomy, *by permission of the publishers, Ginn and Company.)*

of stars which are visible as such in our telescopes are included in a space shaped like a disk or grindstone with a diameter several times its thickness, or like two saucers placed rim to rim and bottom outwards; the Solar System is far inside the disk, and the star density thins out on both sides. Thus are explained the bisection of the celestial sphere by the Milky Way, the galactic concentration of stars, and the increase of galactic concentration with diminishing brightness. Figure 298 represents a hypothetical edge-on view of such an aggregation. The brightest stars are in general the nearest, and for an observer at the center these would be the stars included in the circle *a*, which would have a fairly uniform distribution in the sky. Fainter and more distant stars, such as those in circle *b*, would be pronouncedly concentrated toward the plane of the disk, and

the galactic concentration would increase with increasing faintness as it is actually observed to do.

In Herschel's time the existence of dark nebulosities was unsuspected, and, attributing the bifurcation of the Milky Way and the irregularities of its structure to actual irregularities in the form of the disk or the distribution of its stars, he suggested the cross section of the Galactic System shown in Figure 299.

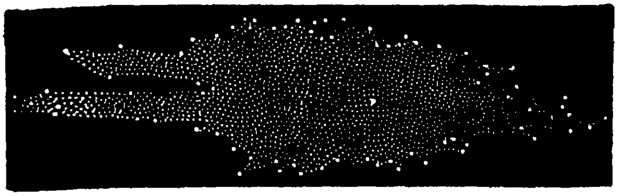


Fig. 299. *Cross-Section of the Galactic System According to Herschel.*

Very elaborate investigations based on the proper motions of stars and on counts of stars of different magnitudes, together with the known distances and luminosities of the nearer stars, were made by Kapteyn, but were unfinished at the time of his death in 1922. The resulting picture, which, though incomplete, was intended to represent in a general way the form and size of the Galactic System (without taking star clouds, star clusters, and opaque matter into account), is sometimes called the "Kapteyn universe." Its form is that of an oblate spheroid, with equatorial diameter about five times the polar, and with star density diminishing outward from the center so that there is no definite boundary. The layer in which the star density is 1/100 of that in the neighborhood of the Solar System has a thickness of about 11,000 light-years and an equatorial diameter of about 55,000.

Trumpler finds (page 409) that the galactic star clusters are distributed sparsely in a disk-like space about 3000 light-years thick and 30,000 light-years in diameter. The center of this disk is about 1000 light-years distant from the Solar System in the direction of Carina (galactic longitude 247°), and the plane of symmetry is inclined about 2° to the galactic plane.

**The Spiral Theory.** The flat spiral form is so common among external galaxies that there is a strong temptation to attribute this structure also to our own Galaxy, although there is little direct evidence either for or against such a theory. The suggestion was first published in 1900 by Easton of Rotterdam, who constructed the chart shown in Figure 300 to indicate in a general way how the stellar accumulations might be arranged in a flat spiral (inner part of the chart), the Solar System being at *S* and the Milky Way appearing as in the outer portion of the chart.

**Obscuring Material in the Galaxy.** During the present century it has become increasingly evident that light from distant bodies is obstructed by dark nebulae, particularly in and near the galactic plane. The presence of obscuring material is manifested to the naked eye by the Coalsacks and the Great Rift in the Milky Way, and more clearly on photographs of star fields such as some of those reproduced in this book.

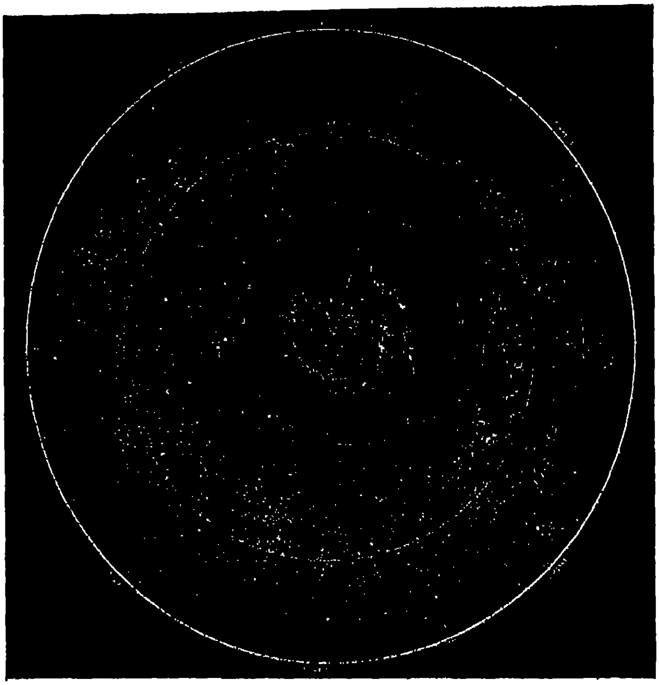


Fig. 300. *Easton's Representation of the Galactic System.*

The dark material is not all completely opaque. Studies such as Trumpler's show that when the distance and linear diameter of galactic clusters are computed from the apparent brightness of their stars, the most distant clusters seem to be the largest. Trumpler attributes this effect to a dimming of the remote stars by intervening matter which makes the computed distance and hence the computed diameter too great, and he infers an absorption of photographic light of about one magnitude per 5000 light-years in the galactic plane—far less than the absorption that would be produced in such a depth by a medium such as our air.

Stebbins, by observations with his photoelectric photometer which he reported in 1934, finds that remote B-type stars which are situated in regions of obscuration are redder than those observed elsewhere, so much so in maximum cases that a star of the type of Rigel appears of the color of Arcturus. The stars thus appear to be reddened by the dark nebulae as the setting Sun is reddened by the Earth's atmosphere.

Obscuration by dark matter in the Galactic System is betrayed by the apparent distribution of the extra-galactic nebulae, which are observed in great numbers in all parts of the sky with the exception of an irregular zone of avoidance along the Milky Way. This zone is delineated clearly by counts, published by Hubble in 1934, of about 44,000 such objects on more than 1200 photographs made with the reflectors at Mount Wilson

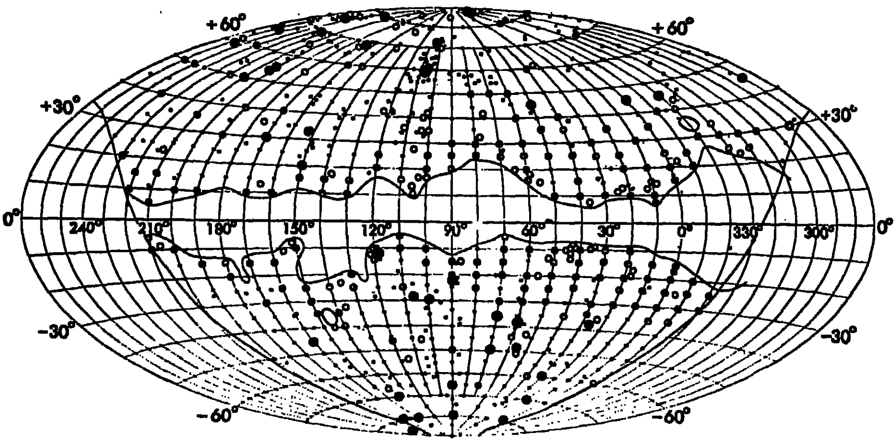



Fig. 301. *Hubble's Counts of Extragalactic Nebulae. The map represents the entire sky on an equal-area projection. The central horizontal line is the galactic circle, and the figures show galactic longitude and latitude. Small dots indicate a normal number of nebulae per plate, large dots a larger number, and open circles a smaller number. The irregular vacant strip along the galactic circle is the* Zone of Avoidance. *The vacant areas along the right and left edges of the map represent the portion of the sky which is not available for observation at Mount Wilson.*

and distributed over the three-quarters of the sky there observable. Figure 301 gives the results of these counts and exhibits the zone of avoidance, which follows the general pattern of the dark areas of the Milky Way and is convincing evidence of a layer of dark nebulosity lying along the galactic plane and concealing the external galaxies beyond. Stebbins traces this zone also by the reddening of B-type stars and globular clusters.

**The System Outlined by Globular Star Clusters.** Shapley finds that most of the globular star clusters are distributed throughout a roughly spherical region more than 100,000 light-years in diameter, which is bisected by the plane of the Galaxy and has its center in galactic longitude 327°, at a distance of about 35,000 light-years from the Solar System. No globular cluster is visible within 4000 light-years of the galactic plane, a circumstance readily explained by obscuration.

The distribution of these clusters, symmetrical with respect to the Milky Way, connects them obviously with the Galactic System, and astronomers generally agree with Shapley in regarding them as outposts which mark its boundary. In that case it is much larger than the Kapteyn "universe" or the Trumpler discoid system of open clusters (a great part of the Galaxy being concealed), and larger than any known external galaxy. Shapley has found cluster-type variable stars at great distances on either side of the galactic plane, and suggests that the main discoid aggregation is enclosed in a roughly globular "haze" of outlying stars.

The location of the center of the greater system is significant: it is among the great star clouds of Sagittarius, a region of much obscuration, and it is possible that, in addition to the visible clouds, there are around this center other vast, dense clouds of stars and nebulae which are hidden from our view and which form a nucleus proportionate in size to the greater Galaxy.

**The Local System.** The studies of Shapley, Charlier, Seares, and others lead them to suspect the existence of what Shapley calls the local system or local star cloud, occupying a flat region within the main Galactic System about 3000 light-years in diameter and 800 light-years thick, and containing nearly all the naked-eye stars, nearly all the bright B-type stars, a considerable share of all A-type stars, and many open clusters and galactic nebulae. The principal plane of this local system makes an angle of about 14° with the plane of the Milky Way and is nearly identical with that of the Herschel-Gould belt of bright stars. Its center is about 300 light-years distant from the Sun in galactic longitude 240°. The interpenetration of the local system with the main Galactic System is blamed for much of the uncertainty attaching to statistical investigations of the stars such as Kapteyn's.

**Rotation of the Galaxy.** A rotation of the Galactic System around an axis passing through the galactic poles was suggested long ago by its flattened form; and this suggestion was strengthened, when the extra-galactic nebulae were recognized as similar systems, by the fact that some of these objects were shown by the inclination of the lines of their spectra to be in similar rotation. It is known from gravitational principles that if the material of the system were distributed uniformly in a spheroid, the resultant attraction on any star would vary directly as its distance from the center, and all parts of the system would have the same period of revolution—the rotation of the whole would be like that of a wheel. Such rotation would be impossible of detection by observation of stars within the system, because it would involve no *relative* motion of the star and the observer. If, on the other hand, practically all the mass is collected

in a nucleus surrounding the center, as may be inferred from the great size of the Sagittarius array of star clouds and dark nebulae, the attraction on stars outside the nucleus must vary inversely as the square of their distance from the center and the revolution of different parts must be in accordance with Kepler's third law, the inner stars revolving more rapidly than the outer. Rotation of this kind is exemplified by Saturn's rings and also by the whole Solar System. Such motion should become evident in the radial velocities of distant members of the system and also, eventually, in their proper motions.

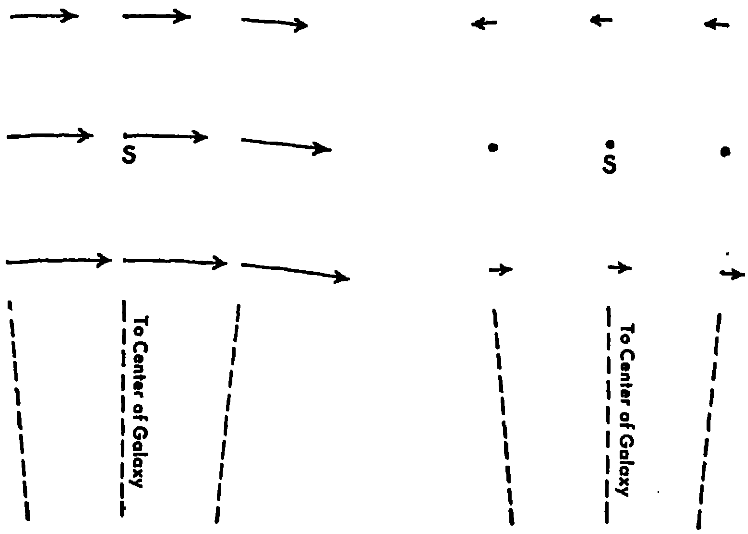



Fig. 302. Fig. 303.

*Effect of Galactic Rotation on Apparent Stellar Motions.*

The nature of the evidence may be understood from a study of Figures 302 and 303, which represent a large group of stars revolving around a distant center in accordance with Kepler's third law, the Sun (and Earth) being in their midst, at *S*. The arrows in the left figure represent the velocities of revolution (the longer the arrow, the greater the speed). The arrows in the right figure represent the velocities relative to the Solar System, obtained by applying to each star a velocity equal and opposite to that of the Sun. The figures show that (1) the stars in directions 90° from that of the center of the Galaxy show us no motion at all; (2) all other stars possess proper motions parallel to the galactic plane and of various small amounts; (3) stars in directions toward and opposite the center have no radial velocity since their motion is transverse to the line of sight; and (4) maximum positive radial velocity is possessed by stars situated along one diagonal of the group, and maximum negative radial velocity by stars along the other diagonal. It is clear also that the rotational effect in proper motions is slight since the cross

motion is small for nearby stars, and that the effect in radial velocities is greater the more distant the star from the Solar System. The most advantageous test for galactic rotation is therefore in spectrographic observations of remote stars situated in the galactic plane in directions 45° from the galactic center. Statistics of the apparent motions of remote stars may disclose not only the fact that the system rotates, but also the speed and direction of the rotation, the location of the center, the mass and approximate size of the system, and the ratio of the gravitational force of the nucleus to that of the system as a whole.

The work of a number of investigators, beginning with that of Lindblad of Stockholm and Oort of Leyden in 1926, has left little doubt that the Galaxy rotates in a manner approximating that of Saturn's rings and in the direction opposite that of increasing galactic longitudes—that is, clockwise as seen from the pole in Coma Berenices. The following results, published by Plaskett and Pearce of Victoria in 1934, were obtained mainly from the radial velocities of 849 stars of spectral types O5 to B7—stars situated near the galactic plane and known to be at great distances from the Sun:

| | |
|---|---|
| Rotational velocity in neighborhood of Sun.... | 275 km./sec. |
| Period of revolution of Sun.................. | 224,000,000 years |
| Total mass of Galaxy....................... | $16.5 \cdot 10^{10}$ times mass of Sun |
| Ratio of gravitation of nucleus to total........ | 0.75 |
| Galactic longitude of center.................. | 324°.4 |
| Distance from Sun to center.................. | 33,000 light-years |
| Diameter of Galaxy......................... | 98,000 light-years |

Observations by Joy at Mount Wilson on the radial velocities of 134 distant galactic Cepheids confirm the above results, giving the same position from the center of the Galaxy, a rotational speed of the Sun only 9 per cent less, and a period of the Sun only 7 per cent greater. Observations by Gerasimovich and Struve on the detached K line in stellar spectra show clearly that the interstellar cloud (page 430) shares in the galactic rotation.

It is noteworthy that the position of the center of the Galactic System found by studies of galactic rotation agrees very nearly with that of the center of the system of globular star clusters as determined by Shapley.

**Summary.** The sidereal system in which we are immersed may, according to the foregoing discussion, be described as follows:

1. Although vast, it is not infinite in extent.
2. It contains many thousands of millions of stars.
3. It contains many diffuse nebulae, planetary nebulae, and open star clusters.

4. Most of its contents lie in a region whose thickness is small compared to its diameter, and whose principal plane is the plane of the Milky Way.

5. Much of the contents of the Galactic System is concealed from us by obscuring matter, the greater part of which lies near the galactic plane.

6. The center of the Galactic System is among the great star clouds of Sagittarius.

7. It is probable that the galactic center is within a vast nucleus, of which the Sagittarius clouds are a part but of which more is concealed than revealed by intervening dark nebulae, and which possesses a large share of the mass of the system.

8. The Solar System is about 35,000 light-years distant from the galactic center.

9. It is possible that the Solar System is near the center of a local star cloud which extends a few thousands of light-years.

10. The Galactic System rotates, in the clockwise direction as viewed from its north pole, around an axis normal to the Milky Way plane; and the rotation is like that of Saturn's rings, not like that of a wheel.

11. The total mass of the Galactic System is at least $10^{11}$ times the mass of the Sun.

12. In the neighborhood of the Solar System, the speed of revolution is between 200 and 300 km./sec., and the period is more than 200,000,000 years.

13. The Galactic System is permeated by an attenuated cloud of atoms which produces detached lines in stellar spectra and which shares in the galactic rotation.

14. The globular star clusters, though belonging to the greater Galaxy, lie outside the main body.

15. The diameter of the system, as outlined by the globular star clusters, is about 100,000 light-years.

16. A thin "haze" of stars extends far on either side of the main discoid system, perhaps giving to the total Galaxy a roughly spherical form.

**The Galactic System Seen from the Outside.** It is evident that, seen from a distance of a million light-years or so, our Galactic System would have the appearance of a nebula, or perhaps of a group of nebulae. If seen edgewise, it would show a spindle form, and if it is a spiral such as Easton and others have imagined, it might resemble NGC 4565, Figure 304, with a longitudinal dark lane due to a peripheral band of dark nebulosity

and a central bulge due to the great nuclear star clouds of Sagittarius. Its spectrum would be of stellar type and would have inclined lines because of rotation. If seen in plan, it might exhibit a form like that of M 33, Figure 305, with a great central nucleus and numerous condensations. The larger of the condensations would represent star clouds, and one of

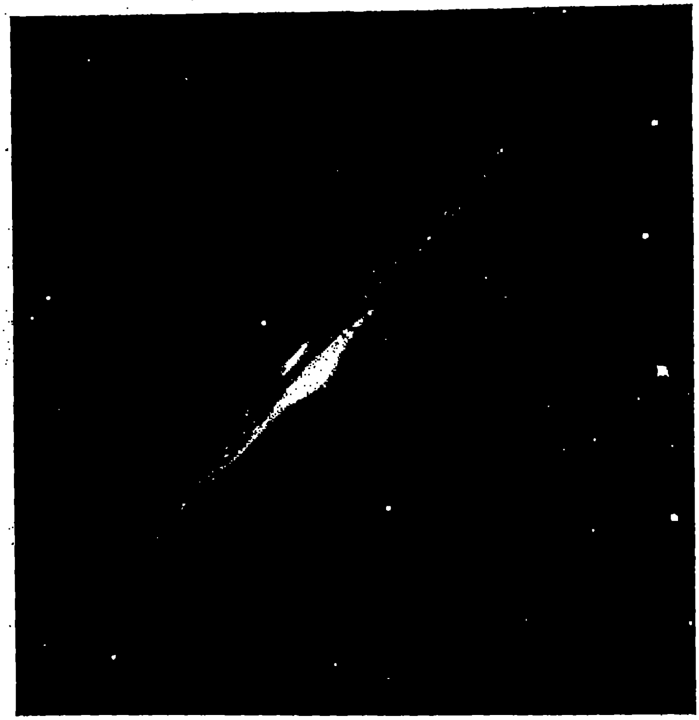


Fig. 304. *The Extragalactic Nebula NGC 4565 Comae, Photographed by Ritchey with 60-Inch Mount Wilson Reflector.*

these might perhaps contain the Solar System; but of the planets and even of the Sun as an individual star there would at that distance appear no trace. Possibly some of the brightest giant stars would be detected as faintly luminous points. Surrounding the main system would be the globular star clusters, distinguishable from single stars by a slight fuzziness in large telescopes only. Not far from the main aggregation would be seen two small companion nebulae—the Magellanic clouds. In the background would appear the myriads of kindred objects already known to us as extra-galactic nebulae, against which our system would be conspicuous for its size.

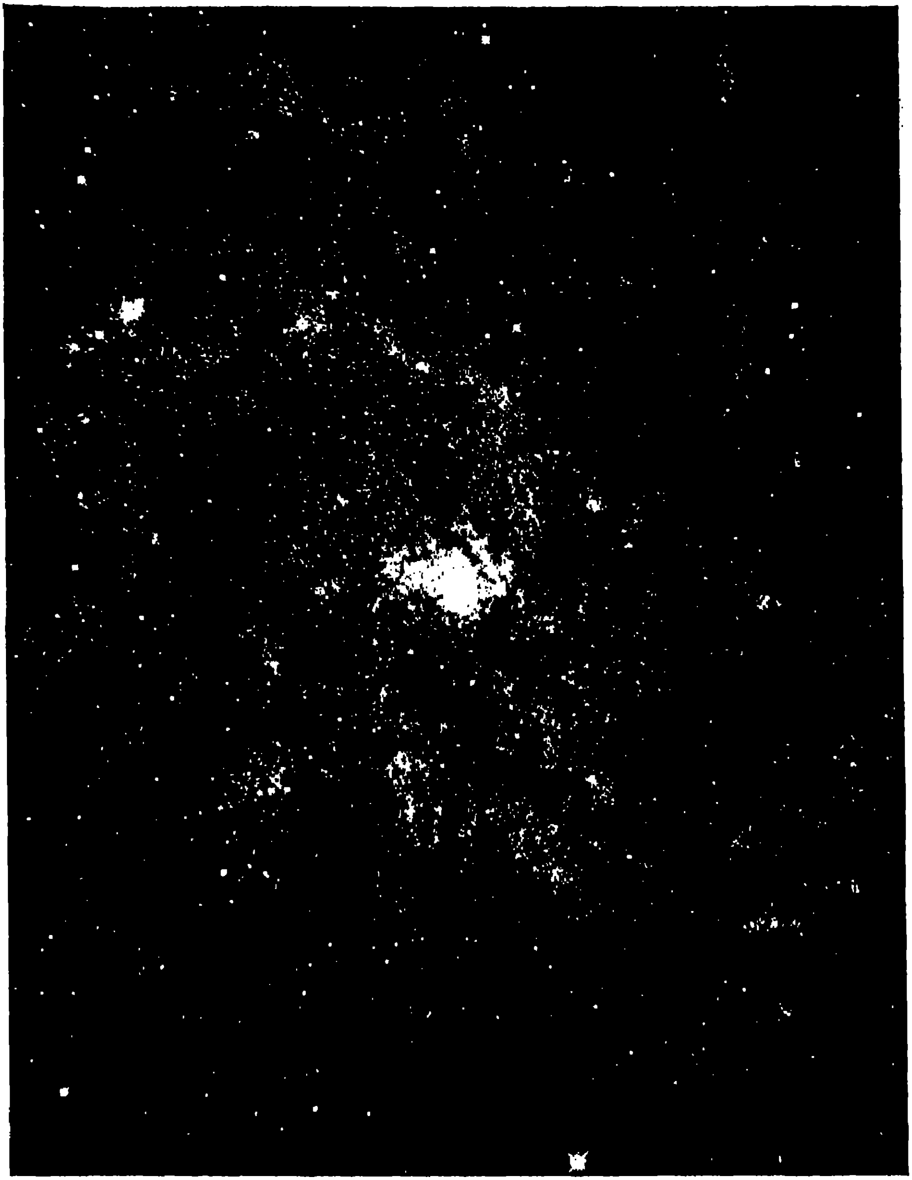


Fig. 305. *The Great Spiral Nebula M 33 Trianguli, Photographed by Ritchey with 60-Inch Reflector, 1910 August 5 7. Exposure 8½ hours.*

# CHAPTER 21

## BEYOND THE MILKY WAY

**Other Galaxies.** That the nebulous-appearing objects which are found outside the Milky Way zone are aggregations comparable with the Galactic System and separated from it and from each other by vast distances—i.e., that they are other galaxies—is indicated by the following facts:

1. Their apparent avoidance of the Milky Way finds a reasonable explanation in the assumption that external bodies in that plane are hidden from us by the obscuring material which is known to exist in the Galactic System.
2. The appearance of most extra-galactic nebulae is in harmony with a greatly flattened form, and there is spectral evidence of rotation in those which are seen nearly edgewise.
3. Parts of the two brightest spirals and some other extra-galactic nebulae have been resolved by the 100-inch telescope into stars.
4. The 100-inch telescope has revealed faint, fuzzy objects like very remote globular star clusters, on and around the Andromeda and some other extra-galactic nebulae, which seem analogous to the globular clusters of the Galactic System.
5. Many novae have been found in extra-galactic nebulae, but they are rare elsewhere in the sky except in the Milky Way. The nebular novae are very faint, and if comparable with galactic novae in actual luminosity they are necessarily very distant.
6. Cepheid variable stars, found in the Andromeda nebula and others, have periods and apparent magnitudes indicating distances which are in no case less than 600,000 light-years.
7. The spectra of extra-galactic nebulae are about what would probably be given by the mingled light of such an aggregation as the Galaxy.
8. The high velocities of extra-galactic nebulae, detected spectrographically, place them in a class apart from galactic objects.

9. Not only are measurable parallaxes lacking, but, notwithstanding the high spectrographic velocities, no motion at right angles to the line of sight has been certainly detected.

10. The spectral lines of distant extra-galactic nebulae are displaced toward the red by a large amount which seems to be proportional to the distance of the nebulae, in harmony with recent cosmological theories which describe the universe as expanding.

**Changing Views on the Plurality of Galaxies.** The conception of "island universes" which was prevalent in the eighteenth century was based insecurely on the fact that many so-called nebulae, when studied with large telescopes, turned out to be star clusters; and in 1864 this theory was easily unseated by Huggins's discovery that some nebulae had bright-line spectra. The opinion then became general for many years that all nebulae were merely members of the Galaxy in which we dwell and that this Galactic System included all the visible universe, or even the entire material universe. The faintness of the spectra of the extra-galactic nebulae prevented their adequate study during the nineteenth century; and although dark lines in the spectrum of the Andromeda nebula were suspected by Huggins and Scheiner, these spectra were generally thought to be continuous and the objects emitting them were often called "white nebulae." As the bright lines of many gaseous or "green" nebulae appeared on a background of continuous spectrum, it was thought probable that all nebulae could be arranged, like the stars, in an unbroken spectral sequence. Moreover, spiral forms were detected or fancied in many of the diffuse and planetary nebulae, "so that" (as one writer [1] of high standing remarked in 1905), "to admit some to membership of the sidereal system while excluding others would be a palpable absurdity." The founders of the Planetesimal Hypothesis betrayed a similar conception of the relative nearness and moderate size of the spirals when they mentioned these objects as examples of the possible result of an encounter of two ordinary stars.

In 1909 Fath showed, from spectrograms which he had made at the Lick Observatory, that a number of spiral nebulae had dark-line spectra similar to the integrated spectrum of the stars in the globular clusters, and argued that these spirals must be composed of stars which, since they could not be seen individually in the largest telescopes, must be either vastly remote or abnormally small.

In 1917 and the following years a number of novae were discovered in spiral nebulae, and they were so much fainter than the usual galactic nova that Curtis and others strongly advocated the opinion that these nebulae were many times more remote than the farthest stars of our system and that, to appear as surfaces at such distances, they must themselves be galaxies. Since 1924, when Hubble discovered a number of Cepheid variable stars in the Andromeda and two other extra-galactic nebulae indicating, by their periods and magnitudes, distances several times greater than the accepted diameter of the Galaxy, this opinion has scarcely been opposed.

**Brightness and Number of the Galaxies.** The extra-galactic nebulae are mostly very faint. Only one (the great nebula of Andromeda) is

[1] A. M. Clerke, in *The System of the Stars*, page 350.

distinctly visible to the naked eye, and only five are brighter than the eighth photographic magnitude. In a thorough survey of those brighter than the thirteenth photographic magnitude, Shapley and Miss Ames have catalogued 1023. With increasing faintness, their number increases amazingly; and Hubble finds that, in high galactic latitudes where there is no obscuration by dark nebulosity, and at magnitude 21, photographs made with the 100-inch telescope reveal about as many nebulae as stars. He estimates that if there were no obscuration in our own Galaxy, about 75,000,000 could be photographed.

**Apparent Distribution.** The most extensive and reliable statistics on extra-galactic nebulae are those being gathered by Hubble at Mount Wilson and by Shapley at Harvard. The apparent distribution of the brighter nebulae is well established: of the thousand brighter than the thirteenth magnitude, twice as many are north of the Milky Way as south of it. The inequality in the two hemispheres diminishes with diminishing brightness, and disappears about the eighteenth magnitude.

There are clusters of nebulae ranging in population from mere pairs to groups of several hundred. The largest and brightest one extends over several degrees, mostly in the constellations of Coma Berenices and Virgo. More concentrated is Wolf's "*Nebelnest*" (Figure 306), also in Coma, where more than 300 nebulae of about the seventeenth magnitude have been photographed in an area of sky no larger than that occupied by the Moon. Still richer clusters of fainter nebulae are in Ursa Major, Leo, and elsewhere. Shapley regards the clustering tendency as an important and characteristic property of the galaxies, but Hubble infers from his counts that, on the grand scale and allowing for local obscuration, there is a random distribution of faint nebulae over the whole sky.

**Distances.** The distances of these nebulae cannot be determined as accurately as can those of stars in the Galactic System, yet most nebular distances are so great that the percentage error is not grotesque, a few million light-years being small compared with the whole distance. In this field also Hubble and Shapley are most active. Hubble was the pioneer and has extended his investigations, by means of photographs and spectrograms made with the 100-inch reflector, to the most remote objects yet detected by man.

The successive steps in this work are as follows:

1. In a half-dozen of the nearest galaxies, it is possible to recognize individual stars of various classes such as novae, irregular variables, Cepheids of different

periods, and helium stars. The absolute magnitudes of typical stars of these classes being known (those of Cepheids most definitely) from studies in the Galactic System and Magellanic clouds, the distance modulus is easily found. That of the Andromeda spiral, for example, is 22 magnitudes. Distances so determined by Hubble and Humason range from 625,000 light-years for the irregular nebula NGC 6822 to more than 2,000,000 light-years for the spiral M 81.

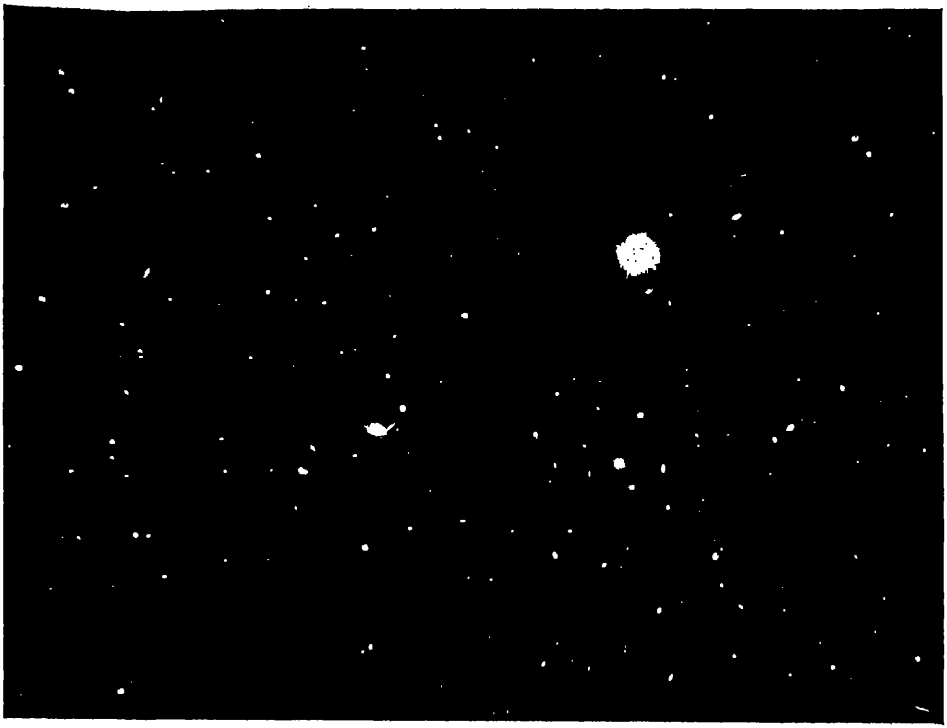


Fig. 306. *Cluster of Several Hundred Extragalactic Nebulae in Coma Berenices, Photographed by Duncan, 1921 May 8, with 100-Inch Telescope. Exposure 4 hours. Most of the nebular images are so small that, in the engraving, they cannot be distinguished from stars.*

2. In about forty more nebulae, it is possible to detect a few stars but not to recognize their types. These are evidently the brightest stars in the nebula. As the brightest stars in the Galactic System, the Magellanic clouds, and the nebulae studied by the first method have an absolute magnitude close to −6, this figure is assumed for the outstanding stars in the more remote nebulae and their distance moduli are determined accordingly.

3. The third step makes use of the total luminosity of the nebula, and is applied to objects in which no stars can be discerned. It is linked with the second step by means of the Coma-Virgo cluster, which contains several hundred nebulae, some of which have perceptible stars and some do not. By the second method Hubble and Humason find about six million light-years for the distance of the cluster, and

using this distance and the observed apparent magnitudes they find that the absolute photographic magnitudes of the nebulae in the cluster average about −14, with no great scattering of values on either side of the average. As this is also the average value for the forty isolated nebulae considered in the second method, it is taken as the mean absolute magnitude of extra-galactic nebulae in general. With a value of the absolute magnitude, the distances of other nebulae may be obtained from their apparent magnitudes. Though this method is quite untrustworthy in the case of a single nebula, it may be applied with confidence to clusters and to the average distance of the hosts of remote nebulae which make up the fainter magnitude classes. Hubble finds a distance of 45,000,000 light-years for the Coma nebelnest, 104,000,000 for the Leo cluster, and 130,000,000 for a faint cluster in Corona Borealis; and estimates that the most remote nebulae observable with the 100-inch telescope are 500,000,000 light-years away. Shapley derives greater distances for the clusters: ten million light-years for the Virgo group and fifty-five million for the cluster in Coma.

Hubble estimates that, in the portion of the universe so far observed, the average distance between nebulae is about 1,500,000 light-years and that, on the average, 2500 nebulae are contained in each volume of space represented by a sphere ten million light-years in radius. For the smaller region occupied by the thousand brightest nebulae, Shapley estimates a density of nebular population about half that found by Hubble.

The Galactic System, the Andromeda nebula, the spiral M 33, the two Magellanic clouds, and eight smaller galaxies are included in the so-called "local" group, all being less than a million light-years distant.

**Classification.** Hubble classifies extra-galactic nebulae as (1) *spiral*, with subclasses of *normal* spirals and *barred* spirals; (2) *elliptic* (or spheroidal); and (3) *irregular*. Of nebulae brighter than the fifteenth magnitude outside of clusters, he finds that 77 per cent belong to the first class, 20 per cent to the second, and 3 per cent to the third. The images of fainter nebulae are mostly too small to show distinct details of structure, and in fact the faintest are difficult to distinguish from stars. In clusters of nebulae, the elliptic form appears to predominate.

**Spiral Nebulae.** The normal spiral nebula, of which Messier 51 (Figure 307) is a good example, consists of a central nucleus and two whorls which in many cases are branched and which, joining the nucleus on opposite sides, wind around it in the same plane and in the same direction. Most such nebulae have approximately the form of an equiangular or logarithmic spiral—that is, a flat spiral in which the tangent to the curve makes a constant angle with the line joining the center to the point of tangency. Some are presented to us edgewise and some in plan, and

others at every angle between. In many of those which we see edge-on (Figure 304), dark lanes extend lengthwise through the center, betraying the existence of a dark extension which would otherwise be invisible.

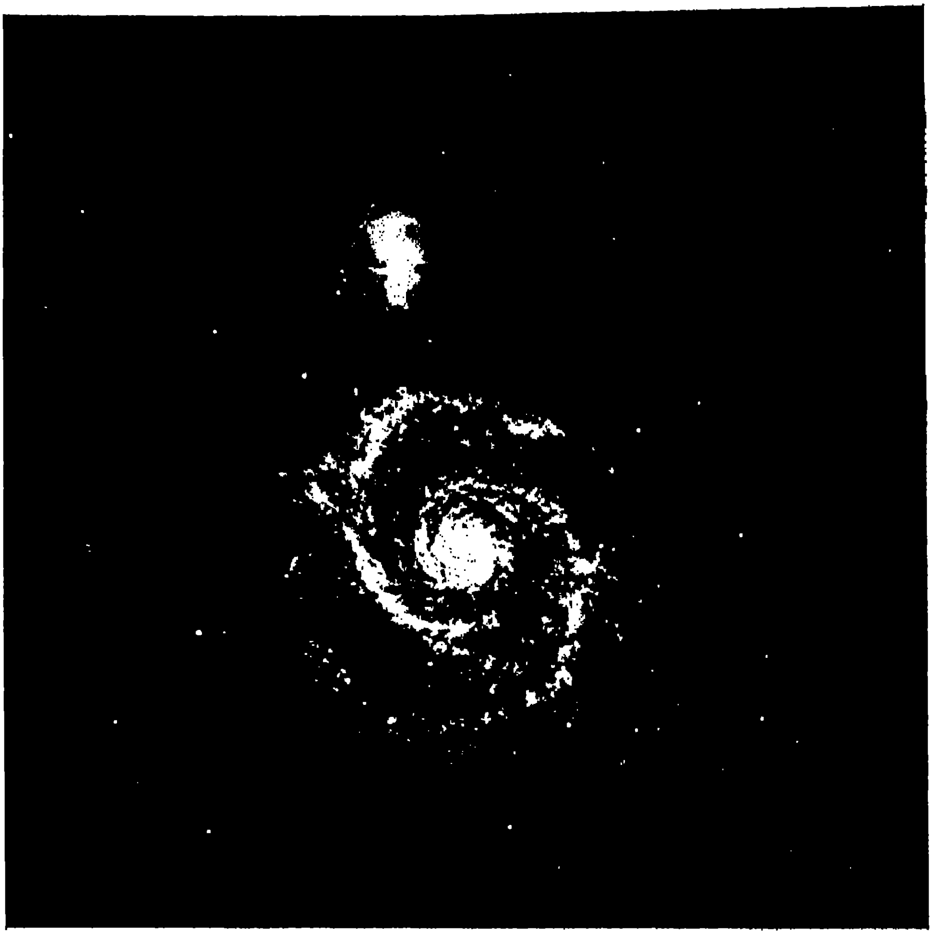


Fig. 307. *The Spiral Nebula M 51 Canum Venaticorum, Photographed with the 100-Inch Reflector.*

The barred spirals are characterized by a straight bar of nebulosity passing through the nucleus; whorls similar to those of the normal spirals spring from the ends of the bar. Barred spirals are only about one-third as numerous as normal spirals.

Photographs of spiral nebulae of either type may be arranged in a sequence characterized by a building up of the whorls at the expense of the nucleus. At one end of the sequence the picture is dominated by the

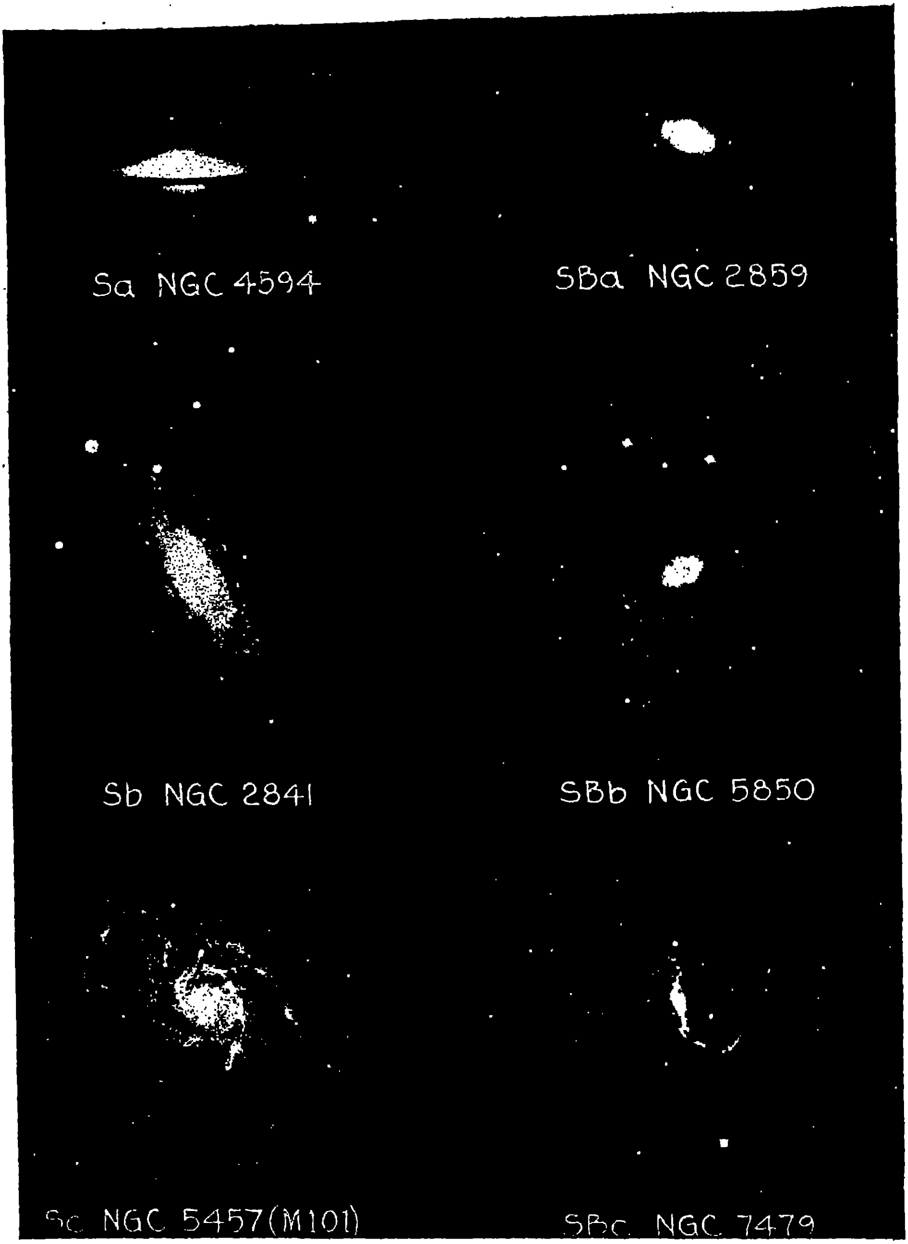



Fig. 308. *Typical Spiral Nebulae (Hubble).*

nucleus, and the nebula glows with soft, misty light nearly free from knots and condensations; at the other end the nucleus is relatively small and the whorls are composed almost entirely of knots, many of which are so small and definite as to differ in no perceptible way from ordinary stars. This progression of types is shown in Figure 308, in which three normal spirals are shown on the left and three barred spirals on the right.

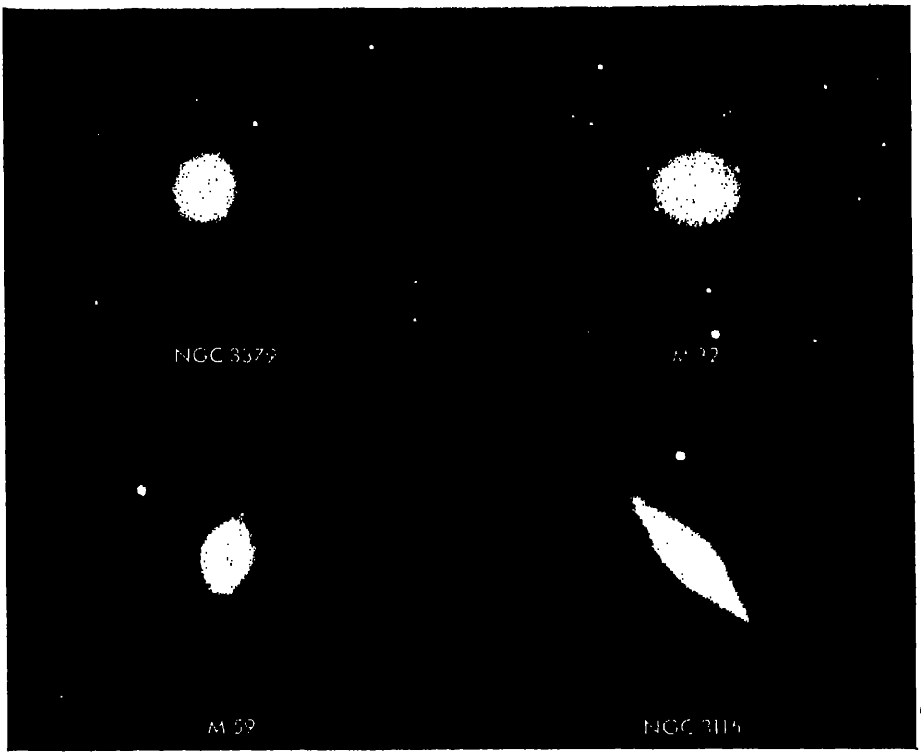


Fig. 309. *Typical Elliptic Galaxies (Hubble).*

The two apparently largest spirals are M 31 Andromedae and M 33 Trianguli. The former is easily traced to a diameter of two degrees and the latter to one degree, corresponding to linear diameters of 28,000 and 13,000 light-years respectively. These dimensions are certainly greatly extended by faint outlying parts of the nebulae (page 464). No other extra-galactic nebula is known to have an apparent diameter as great as half a degree, and most of them appear much smaller.

**Elliptic Nebulae.** Many extra-galactic nebulae show no structural features whatever, when observed under ordinary conditions. In apparent form they are ellipses or lenticular figures in which the ellipticity (ratio

of the difference of longest and shortest axes to the longest) ranges from 0.0 to 0.7. The circular form occurs too frequently to be attributed to accidental tilt of a disk or spheroid; many of them must be actually globular. Figure 309 shows a progression of four elliptic nebulae from the circular form to the lenticular. The elliptic nebulae having the greatest apparent size are the two companions of the Andromeda spiral (Figure 312), with major axes of 8′ (2000 light-years) and 4′ (1000 light-years).

On photographs obtained in 1943 by Baade, using the 100-inch telescope on the best nights with red-sensitive plates and a red filter to exclude the light of the sky, the four nearest elliptic nebulae and the nucleus of the Andromeda spiral are resolved into multitudes of 21st-magnitude stars. The seemingly amorphous galaxies thus appear to be great aggregations of stars, much larger but less dense than the globular star clusters.

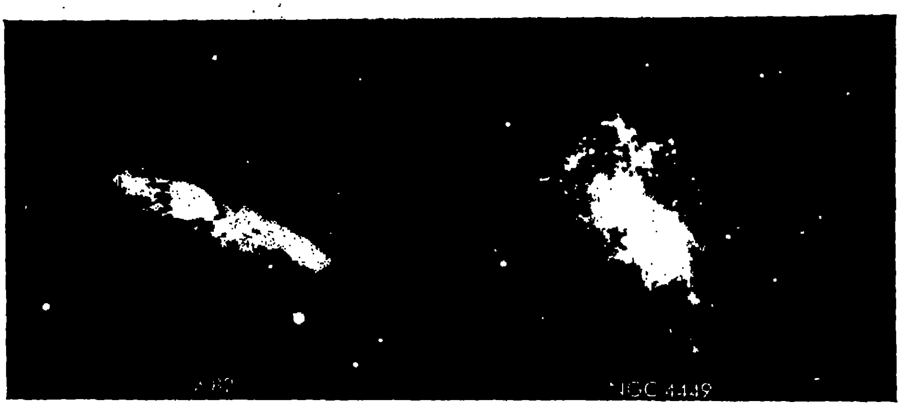


Fig. 310. *Irregular Galaxies (Hubble).*

**Irregular Nebulae.** Irregular galaxies show no evidence of rotational symmetry, and the distribution of their parts is chaotic. More than any other known objects, they resemble the Magellanic clouds. The apparently largest example is NGC 6822 Sagittarii, with extreme dimensions of 10′ × 20′ (2000 × 4000 light-years) and a brighter core about 3′ × 8′ in which stars are densely crowded. This object contains also five diffuse nebulae which resemble those in the Galaxy. Two irregular extra-galactic nebulae are pictured in Figure 310.

**Spectra.** The spectra of extra-galactic nebulae, as far as known, are mainly of stellar types G1 to G5; one exception is of type F, and a few others exhibit, in addition to a continuous or dark-line spectrum, the bright

lines which are characteristic of gaseous nebulae. The bright lines are readily explained by the probable presence of diffuse nebulae within the objects, and the spectra generally are quite in harmony with the view that the objects are galaxies.

The surface faintness of extra-galactic nebulae and the character of their spectra make spectrographic observation extremely difficult. The pioneer work of Fath on the brightest galaxies was followed by that of Slipher, who in the clear air of Flagstaff secured good spectrograms of about fifty. The spectra of fainter nebulae have been observed only at Mount Wilson where Humason, using specially rapid spectrographs with the 100-inch reflector and sometimes giving exposures of many hours, has brought the number up to about 200. Many of Humason's spectrograms, though made with such gigantic equipment, are only about a millimeter long; and yet, under a microscope, they show clearly the H, K, and G lines which identify the solar type of spectrum.

Fig. 311. *The Sombrero Nebula, NGC 4594 Virginis, Photographed by Pease with 60-Inch Mount Wilson Reflector.*

**Rotation and Mass.** As discovered by Slipher and verified at Mount Wilson, the spectral lines of some of the spirals which are seen nearly edgewise are inclined, and, as in the case of a planet (page 261), this inclination denotes rapid[2] axial rotation. It is noticeable in the spectra of the Andromeda nebula, the "Sombrero" nebula NGC 4594 (Figure 311), and a few others; but most extra-galactic nebulae which are

[2] That is, rapid as regards the speed of moving parts. The rotation *periods* are millions of years.

oriented favorably for showing this effect are too small or too faint to permit its detection. The direction of the rotation appears to be that in which the arms of the spiral are trailed, as Slipher inferred from his observations in 1917.

From their rotational speed and size, Hubble estimates that the masses of extra-galactic nebulae are from 600,000,000 to 1,000,000,000 times the mass of the Sun, and that the average density of matter in the observed portion of the universe is $10^{-30}$ grams per cubic centimeter.

**The Andromeda Nebula.** About eight degrees northwest of the bright star $\beta$ Andromedae, and two degrees west of $\nu$, shines the brightest nebula in the sky. It is variously known as Messier 31, NGC 224, and the Great Nebula of Andromeda. To the unaided eye it appears as a small luminous cloud or misty star of the fifth magnitude; in a small telescope it is seen as an elliptical spot about two degrees long in the southwest-northeast direction and half a degree wide, with brightness increasing rapidly toward the center to form a sharp nucleus; and in large telescopes it presents, in addition, longitudinal dark lanes. Only on photographs does its true character appear, and here the dark lanes are seen to be interstices between the whorls of a flat spiral lying in a plane which makes an angle of about 15° with the line of sight (Figures 312–314).

Two elliptical nebulae are associated with the great spiral, and appear with it on Figure 312. The larger and fainter of these, NGC 205 (often called Caroline Herschel's nebula, having been first recorded by that lady), is an elongated ellipse located 35′ northwest of the main nucleus; the other, Messier 32 or NGC 221, is nearly globular and is located 24′ south of the nucleus. The bright condensation near the southwest end of the spiral, which on the photograph is clearly resolved into stars, bears the NGC number 206.

Assiduous study of the Andromeda nebula has yielded many discoveries which, though most of them have been mentioned in preceding pages, may conveniently be summed up here. More than a hundred novae have been discovered in it in twenty years, chiefly in the nuclear region. By far the brightest, and the nearest to the nucleus, was the seventh-magnitude supernova of 1885. The spiral contains many variable stars, of which forty are known to be Cepheids. On and around its face and even beyond the companion nebulae appear 140 faint objects which resemble distant star clusters. (On the reproduction in Figure 312 they are not distinguishable from stars.) The spectrum of the nuclear region and that of M 32 are of the G type; that of NGC 205 is of type F. The spectral lines of the nucleus and also those of the companion nebulae show a radial velocity of −300 km./sec. Those of the nuclear region are inclined and so indicate a rotation, the southwest portion of the nebula approaching more rapidly than the remainder, and the spiral whorls trailing.

The periods and magnitudes of the Cepheids indicate a distance of 805,000 light-years, a figure which is confirmed by the magnitudes of faint novae, other variables, and giant stars. At that distance, a degree is subtended by a length of about 14,000 light-years. Stebbins, by observations with a photoelectric cell, and Shapley, by microphotometer measurements on photographs, have shown that the spiral nebula

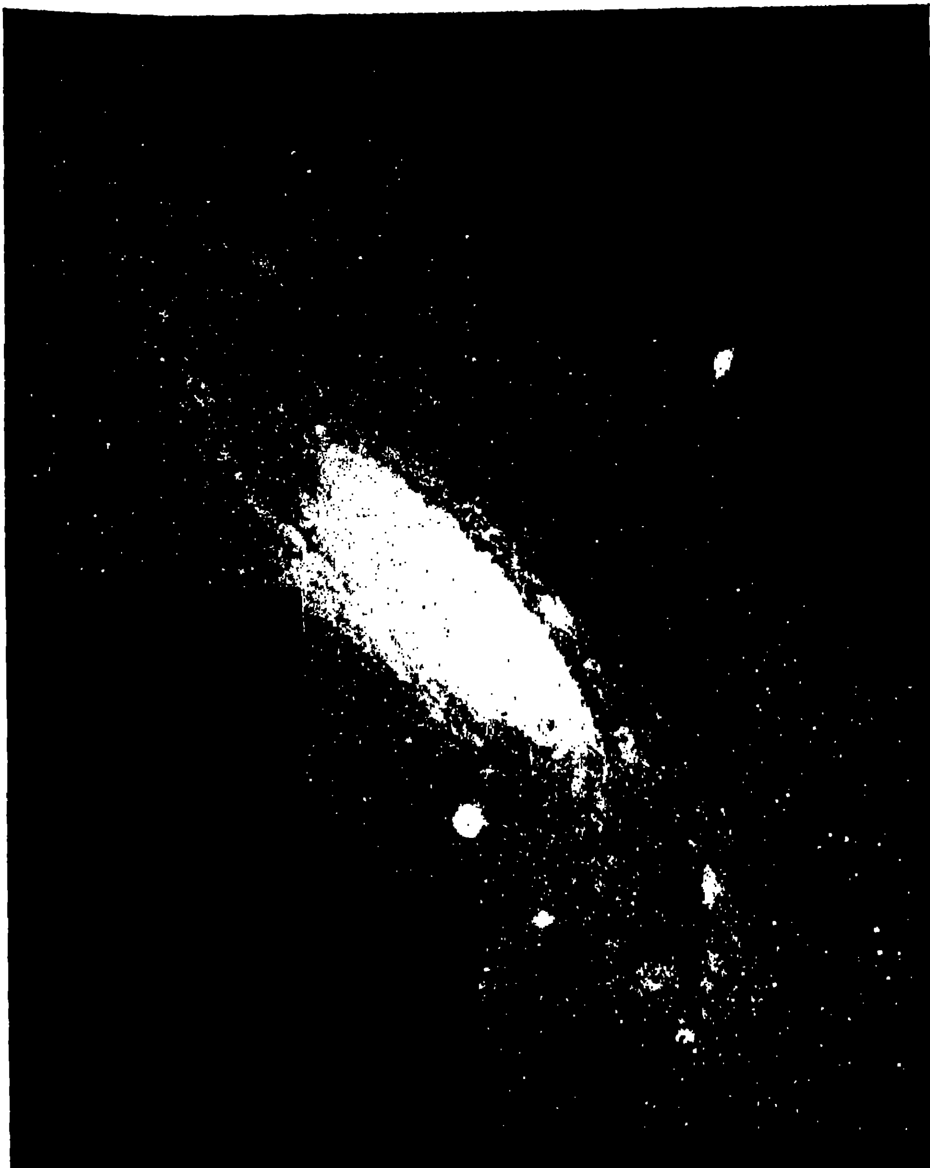


Fig. 312. *The Great Nebula of Andromeda. Photographed by Duncan, 1933 August 19–20, with 100-inch Hooker reflector equipped with a Ross correcting lens. The three photographs, each of 2h. 30m. exposure, were fitted together by E. R. Hoge. The large round object directly below the center of the main nebula is the elliptical nebula M 32. NGC 205 is in the upper right corner of the middle plate. Scale: one inch = 19'.5 = 4500 light-years.*

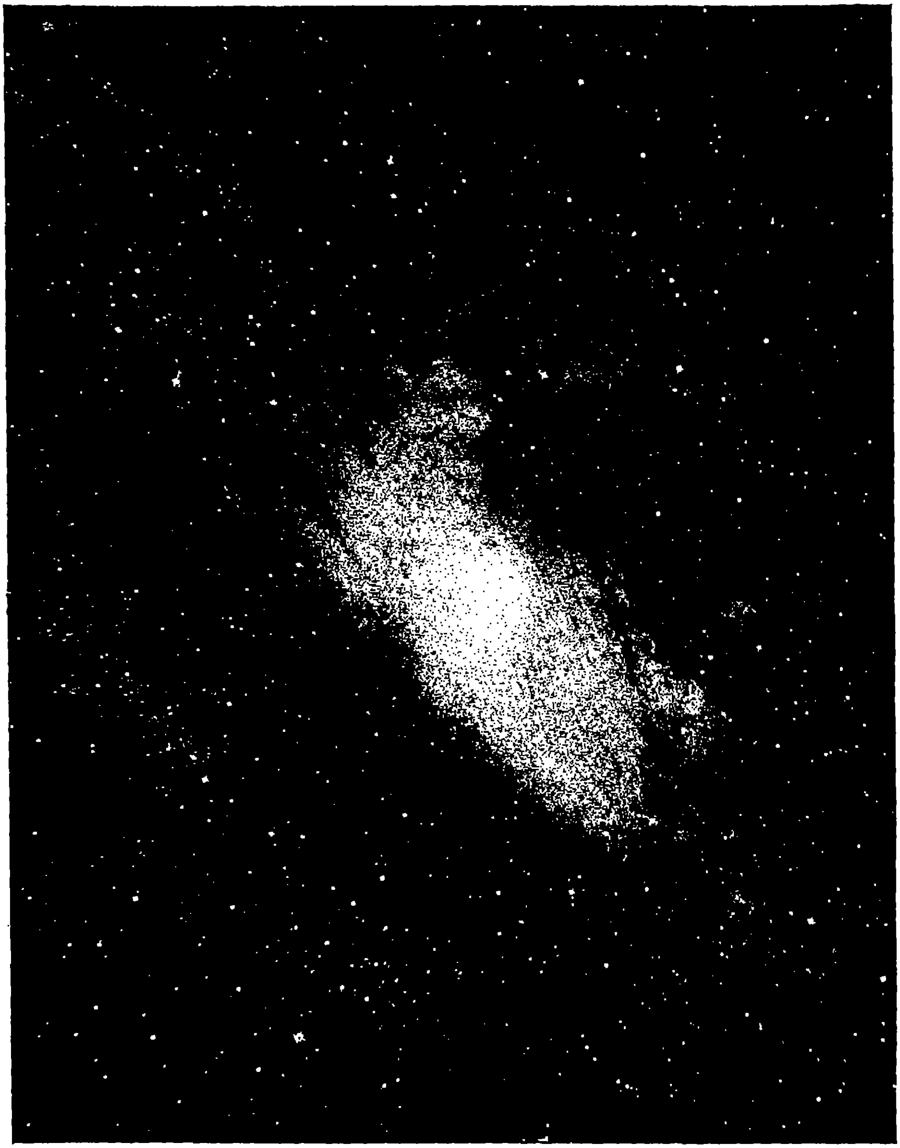


Fig. 313. *Central Part of the Great Nebula of Andromeda, Photographed by Duncan with 100-Inch Telescope, 1920 September 16–17. Exposure 9 hours.*

extends faintly to a diameter of more than four degrees and encloses the two companions; its real diameter is thus of the order of 60,000 light-years.

**The Galactic System Compared with Other Galaxies.** The spiral in Andromeda is the largest extra-galactic nebula whose dimensions have been determined, and yet it appears to be far exceeded, both in size and in mass, by our own Galactic System. Until a few centuries ago, it was fashionable to believe that the Earth, being the abode of man, was the most important place in the universe; but the

fashion has changed and modern astronomers are reluctant to concede such a distinction even to our Galaxy. Shapley would once have solved the difficulty by considering our system to be a small super-galaxy; not a populous cluster of nebulae such as the one in Coma Berenices, but a group such as Stephan's quintet (Figure 315) in Pegasus. The disparity between the Galactic System and other galaxies is not so great as it seemed a few years ago, before the faint outlying portion of the Andromeda nebula was detected or the extent of absorbing material in our own system was recognized; and further study may prove it to be illusory.

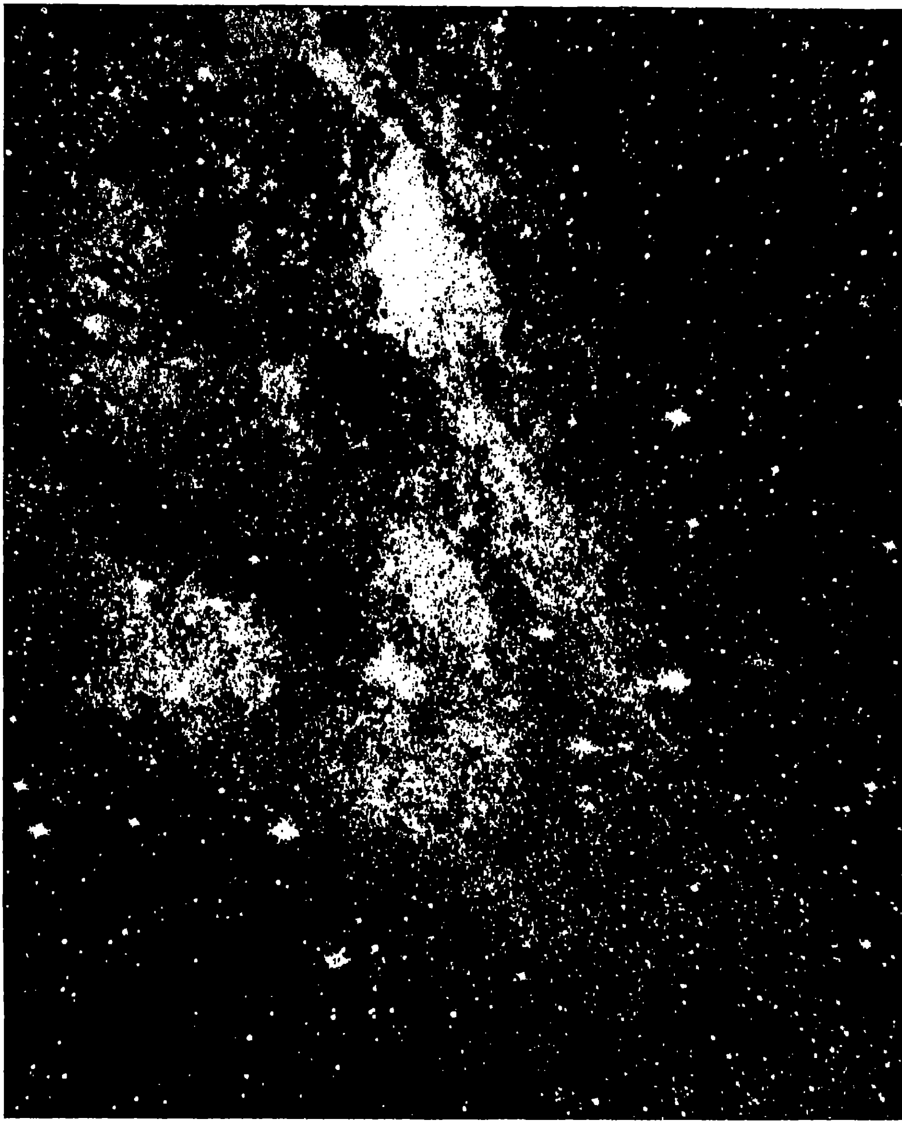


Fig. 314. *South-Preceding End of the Great Nebula of Andromeda, Photographed by Duncan with 100-Inch Telescope, 1925 July 24. Exposure 2 hours.*

Fig. 315. *Stephan's Quintet of Nebulae in Pegasus, Photographed by Ritchey with 60-Inch Reflector, 1916 August 26–27. Exposure 7¾ hours.*

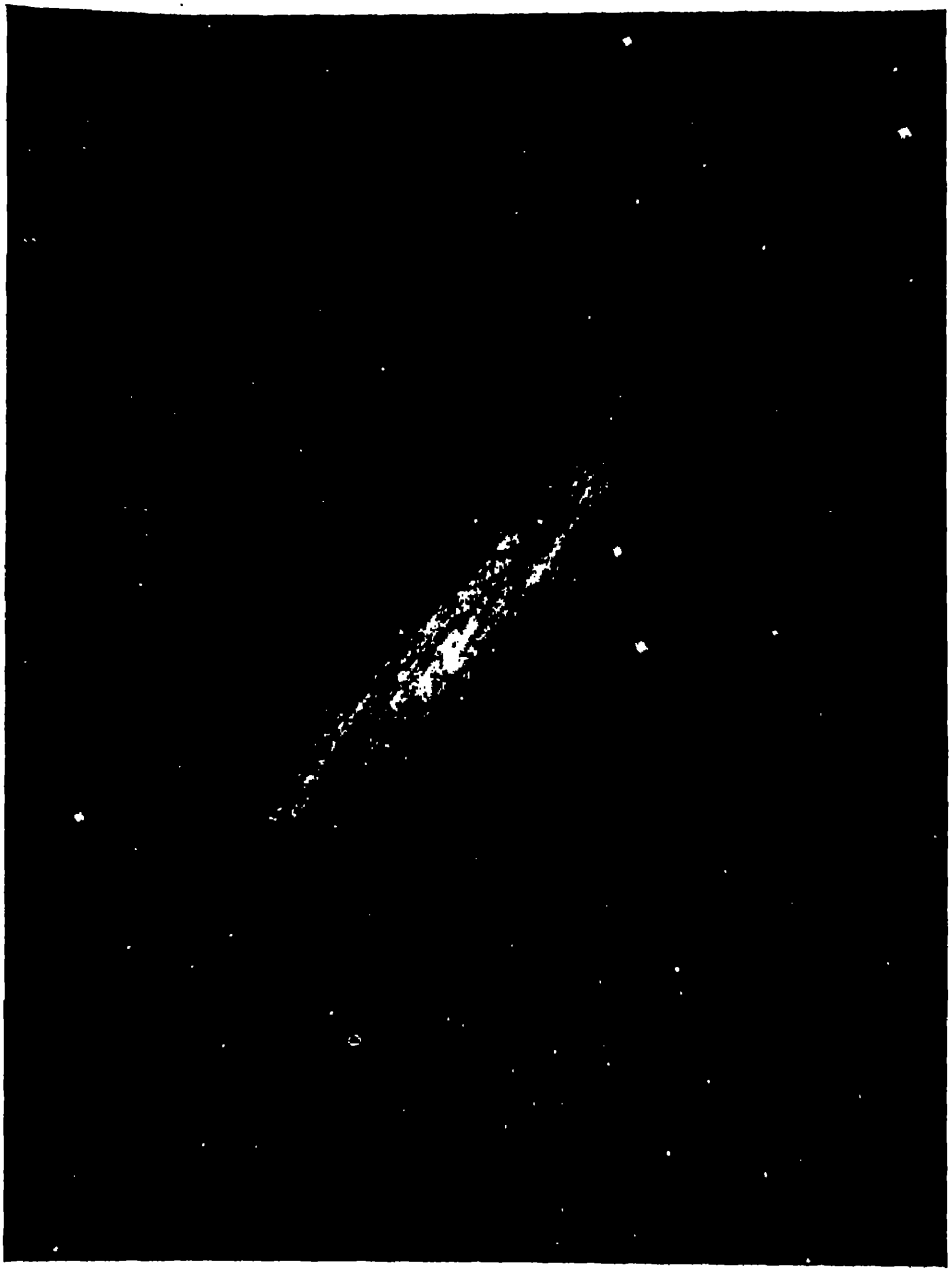


Fig. 316. *The Spiral Nebulae NGC 253 Sculptoris, photographed by Duncan with 100-inch reflector and extra-rapid plate, 1945 September 2. Exposure 50 minutes. Reproduced from a contact print on original scale, 1 mm = 16″. North side is at left.*

**The Radial Velocities of Extra-Galactic Nebulae.** Slipher's studies showed before 1920 that the lines of the spectra of extra-galactic nebulae are greatly displaced, denoting radial velocities much greater than those of stars. Some of the nearer objects, including the Andromeda nebula, show displacements toward the violet, but these nebulae are so situated

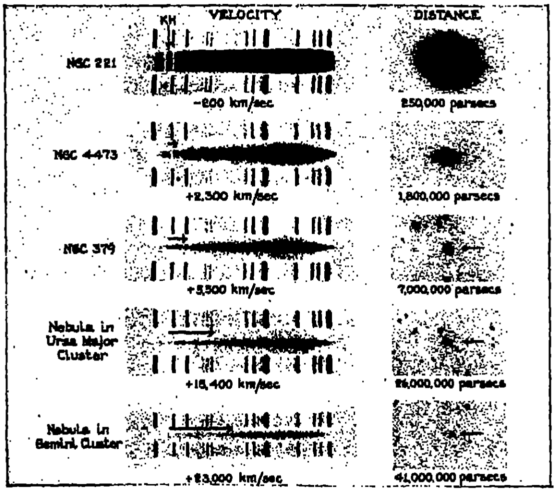



Fig. 317. *Extragalactic Nebulae and Their Spectra, Photographed by Humason.*

*The arrows above the nebular spectra point to the H and K lines of calcium and show the amounts these lines are displaced toward the red end of the spectra. The comparison spectra are of helium.*

*The direct photographs (on the same scale and with approximately the same exposure times) illustrate the decrease in size and brightness with increasing velocity or red-shift.*

*NGC 4473 is a member of the Virgo cluster and NGC 379 is a member of a group of nebulae in Pisces.*

that much of their radial velocity is due to the motion of the Solar System as it is carried toward them by the rotation of our own Galaxy. In the spectra of all others studied, the displacement is toward the red, and in most of them it is so great (corresponding to speeds of many thousands of kilometers per second) that astronomers have hesitated to interpret it as a Doppler-Fizeau effect and have referred to it noncommittally as the **red shift.** However, as no better interpretation has been offered, and as the shift of different lines appears to be proportional to their wave length

(page 168), the corresponding velocities are now pretty generally accepted as real.

By 1928 it was evident that the greater red shifts belonged to the more distant nebulae, and since that date this **velocity-distance relation** has been confirmed and extended by Humason's spectrograms and Hubble's determinations of distance. Hubble and Humason find, in short, that the velocity of recession of distant nebulae is simply proportional to their distance (Figure 317), and is 558 km./sec. per million parsecs, or **170 km./sec. per million light-years.** That is, a nebula 10,000,000 light-years away is receding still farther at a speed of 1700 km./sec., and one 100,000,000 light-years distant is retreating at 17,000 km./sec. The greatest red shift so far observed indicates a speed of recession of 42,000 km./sec., and belongs to a nebula of the eighteenth magnitude in the Boötes cluster, at an estimated distance of 247,000,000 light-years.

**The Expanding Universe.** That the galaxies appear to be receding from our own position in space confers no particular distinction upon this position; their retreat is incidental to a general expansion of the universe. This statement may be made clearer by the analogy of a sheet of rubber on which there are dots an inch apart. If the rubber is stretched to twice its dimensions, neighboring dots will become two inches apart and distances which were two inches will become four, those which were three will become six, and so on; so that, no matter which dot we choose as a point of reference, every other dot will have receded from it by an amount proportional to its original distance.

Various possible pictures of the universe have been developed from the theory of relativity by Einstein, de Sitter, Eddington, Lemaître, and others. Some of these pictures represent the universe in a state of expansion, and harmonize with the relation of distance and velocity which has been found from observations of the nebulae. The observed velocity-distance relation implies that the distances between the galaxies are doubled every 1300 million years. Eddington infers that the galaxies must have begun to separate from one another as recently as $10^{10}$ years ago, a figure which is hard to reconcile with the evidence that the stars are several times $10^{12}$ years old and that even the Earth is at least as old as 1300 million years. It is evident that, as Eddington points out, if the velocity-distance relation persists, the distances between the galaxies will become so great and their mutual recession so rapid that even light cannot pass between them. Each galaxy will then be an island universe indeed, with no connection or communication with any other.

According to the theory of relativity (page 251), the four-dimensional universe of space and time in which we dwell is *finite*, being curved in a fifth dimension so that a train of light waves traveling forever in the same direction must return after long ages to the starting point. It has even been suggested that some of the extra-

galactic nebulae appearing on photographs are identical with others at the opposite points of the celestial sphere, seen the long way around the universe. This relativistic world is analogous to the world of a two-dimensional being who is confined to the surface of a sphere; his two-dimensional space is curved in a third dimension, and if he travels forever in the same direction (i.e., along a great circle), he must return to the starting point. Though unbounded, his world is finite.

**The Question of Man's Place in the Universe.** Not so very long ago, man regarded himself as the center and crowning glory of the universe. "For him did his high Sun flame, and his river billowing ran." The purpose of the Sun was to give him light by day and that of the Moon to light his way by night, and the stars were mentioned as having been "made also." Copernicus inaugurated an important change in this attitude when he showed that the Earth was but one of a number of little balls which revolved around the Sun. Modern astronomy has gone much farther and shown that the Solar System is but a tiny speck in the great Galactic System and that the latter is only one of myriads of galaxies.

Man has lost his exalted position not only in space but also in time. It appears that the stars existed millions of millions of years before the Earth was born; that it was thousands of millions of years later that terrestrial life began at the shore of the primordial sea; that hundreds of millions more elapsed before man appeared; and that during all but a small fraction of his few hundred thousand years of existence he has been only little superior to other animals. The characterization of human history as "a brief and discreditable episode in the life of one of the meaner planets" is not indefensible.

Reacting from the orthodox views of a few centuries ago, many have adopted the doctrine of the plurality of inhabited worlds, imagining that other planets circle other suns and that each is peopled by varied beings. It is now fairly certain that most of the visible universe, so far as it concerns man or any other form of life which we know, is sterile. Mars and Venus seem to be the only planets of the Solar System, except the Earth, which could possibly support life. Among stellar systems, those composed of two or more stars of comparable mass seem to be the rule rather than single stars with retinues of planets. Of stars which possibly have planets, some are variable and subject their surroundings to intolerable extremes of heat and cold. Even of possibly existing planets few are likely to be in the critical condition of temperature, surface gravity, and atmosphere necessary to the delicate thing called life. Yet, if but one star in a

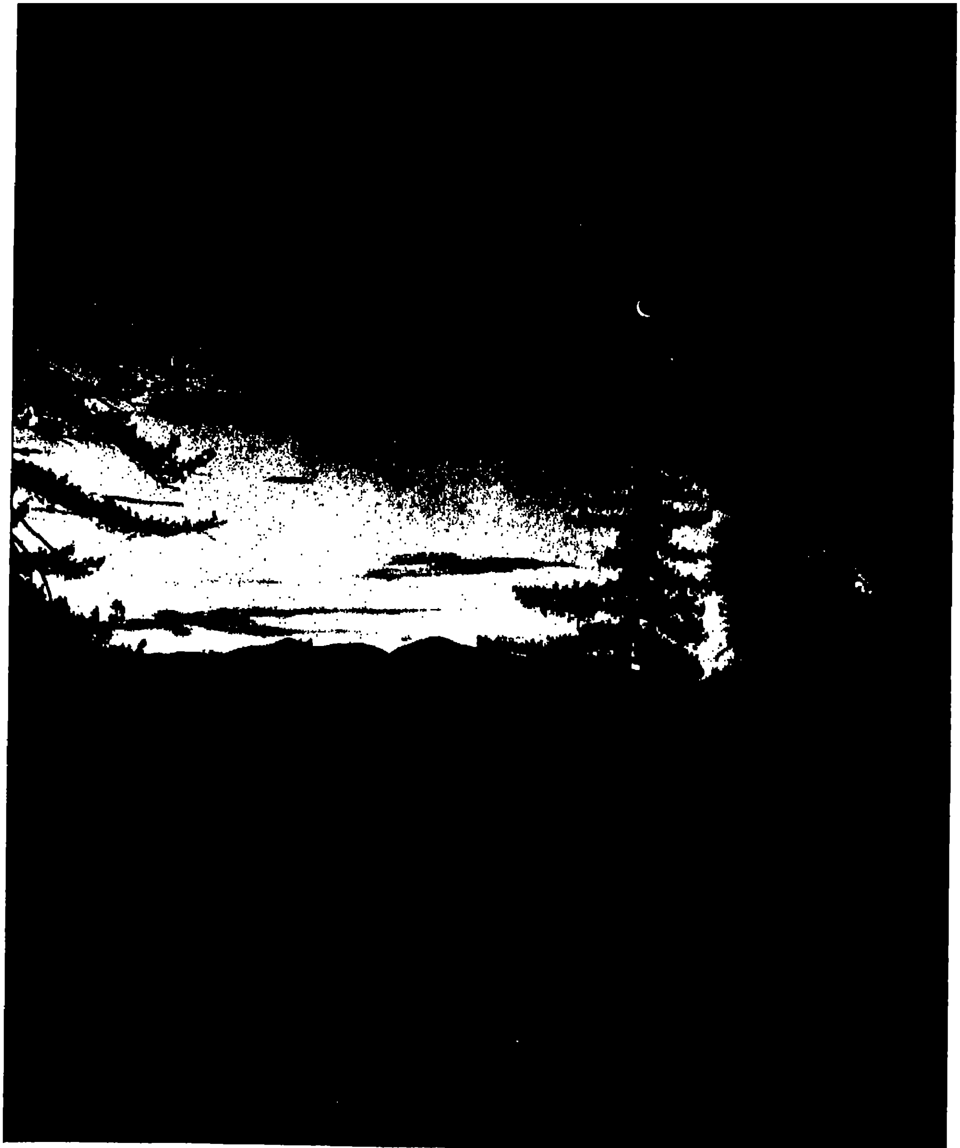


Fig. 318. *The End of the Night. Venus and the waning Moon at dawn, photographed at Mount Wilson by Glenn Moore, 1937.*

million possesses one habitable planet, there are in the Galactic System alone many thousand such abodes, and among them may dwell beings of a far higher order than ourselves.

> Think you this mould of hopes and fears
> Could find no statelier than his peers
> In yonder hundred million spheres?

# TABLES

# TABLES

## THE GREEK ALPHABET

| | | | | | |
|---|---|---|---|---|---|
| Α, α | Alpha | Ι, ι | Iota | Ρ, ρ | Rho |
| Β, β | Beta | Κ, κ | Kappa | Σ, σ, ς | Sigma |
| Γ, γ | Gamma | Λ, λ | Lambda | Τ, τ | Tau |
| Δ, δ | Delta | Μ, μ | Mu | Υ, υ | Upsilon |
| Ε, ε | Epsilon | Ν, ν | Nu | Φ, φ | Phi |
| Ζ, ζ | Zeta | Ξ, ξ | Xi | Χ, χ | Chi |
| Η, η | Eta | Ο, ο | Omicron | Ψ, ψ | Psi |
| Θ, θ | Theta | Π, π, ϖ | Pi | Ω, ω | Omega |

## SYMBOLS

### Symbols of the Sun, Moon, and Planets

| | | | |
|---|---|---|---|
| ☉ | The Sun. | ♃ | Jupiter. |
| ☾ | The Moon. | ♄ | Saturn. |
| ☿ | Mercury. | ⛢ | Uranus. |
| ♀ | Venus. | ♆ | Neptune. |
| ⊕ | The Earth. | ♇ | Pluto. |
| ♂ | Mars. | | |

### Signs of the Zodiac

| | | | | | |
|---|---|---|---|---|---|
| 1. | ♈ | Aries. | 7. | ♎ | Libra. |
| 2. | ♉ | Taurus. | 8. | ♏ | Scorpius. |
| 3. | ♊ | Gemini. | 9. | ♐ | Sagittarius. |
| 4. | ♋ | Cancer. | 10. | ♑ | Capricornus. |
| 5. | ♌ | Leo. | 11. | ♒ | Aquarius. |
| 6. | ♍ | Virgo. | 12. | ♓ | Pisces. |

### Other Symbols

| | |
|---|---|
| ☌ | Conjunction, or having the same longitude or right ascension. |
| ☍ | Opposition, or differing 180° in longitude or right ascension. |
| □ | Quadrature, or having a geocentric angular distance of 90°. |
| ☊ | Ascending node. |
| ☋ | Descending node. |

| | | | |
|---|---|---|---|
| ° | Degrees. | h | Hours. |
| ′ | Minutes of arc. | m | Minutes of time. |
| ″ | Seconds of arc. | s | Seconds of time. |

## THE CONSTELLATIONS

| Name | Genitive | Abbreviation[a] |
|---|---|---|
| Andromeda | Andromedae | And |
| Antlia[b] | Antliae | Ant |
| Apus[b] | Apodis | Aps |
| Aquarius | Aquarii | Aqr |
| Aquila | Aquilae | Aql |
| Ara | Arae | Ara |
| Argo (Navis)[c] | Argus | |
| Aries | Arietis | Ari |
| Auriga | Aurigae | Aur |
| Bootes | Bootis | Boo |
| Caelum[b] | Caeli | Cae |
| Camelopardalis[b] | Camelopardalis | Cam |
| Cancer | Cancri | Cnc |
| Canes Venatici[b] | Canum Venaticorum | CVn |
| Canis Major | Canis Majoris | CMa |
| Canis Minor | Canis Minoris | CMi |
| Capricornus | Capricorni | Cap |
| Cassiopeia | Cassiopeiae | Cas |
| Centaurus | Centauri | Cen |
| Cepheus | Cephei | Cep |
| Cetus | Ceti | Cet |
| Chamaeleon[b] | Chamaeleontis | Cha |
| Circinus[b] | Circini | Cir |
| Columba[b] | Columbae | Col |
| Coma Berenices[b] | Comae Berenices | Com |
| Corona Australis | Coronae Australis | CrA |
| Corona Borealis | Coronae Borealis | CrB |
| Corvus | Corvi | Crv |
| Crater | Crateris | Crt |
| Crux[b] | Crucis | Cru |
| Cygnus | Cygni | Cyg |
| Delphinus | Delphini | Del |
| Dorado[b] | Doradus | Dor |
| Draco | Draconis | Dra |
| Equuleus | Equulei | Equ |
| Eridanus | Eridani | Eri |
| Fornax[b] | Fornacis | For |
| Gemini | Geminorum | Gem |
| Grus[b] | Gruis | Gru |
| Hercules | Herculis | Her |
| Horologium[b] | Horologii | Hor |
| Hydra | Hydrae | Hya |
| Hydrus[b] | Hydri | Hyi |
| Indus[b] | Indi | Ind |
| Lacerta[b] | Lacertae | Lac |
| Leo | Leonis | Leo |
| Leo Minor[b] | Leonis Minoris | LMi |
| Lepus | Leporis | Lep |
| Libra | Librae | Lib |
| Lupus | Lupi | Lup |
| Lynx[b] | Lyncis | Lyn |
| Lyra | Lyrae | Lyr |
| Mensa[b] | Mensae | Men |
| Microscopium[b] | Microscopii | Mic |
| Monoceros[b] | Monocerotis | Mon |
| Musca[b] | Muscae | Mus |
| Norma[b] | Normae | Nor |
| Octans[b] | Octantis | Oct |
| Ophiuchus | Ophiuchi | Oph |
| Orion | Orionis | Ori |
| Pavo[b] | Pavonis | Pav |
| Pegasus | Pegasi | Peg |
| Perseus | Persei | Per |
| Phoenix[b] | Phoenicis | Phe |
| Pictor[b] | Pictoris | Pic |
| Pisces | Piscium | Psc |
| Piscis Australis | Piscis Australis | PsA |
| Reticulum[b] | Reticuli | Ret |
| Sagitta | Sagittae | Sge |
| Sagittarius | Sagittarii | Sgr |
| Scorpius | Scorpii | Sco |
| Sculptor[b] | Sculptoris | Scl |
| Scutum (Sobieskii)[b] | Scuti | Sct |
| Serpens | Serpentis | Ser |
| Sextans[b] | Sextantis | Sex |
| Taurus | Tauri | Tau |
| Telescopium[b] | Telescopii | Tel |
| Triangulum | Trianguli | Tri |
| Triangulum Australe[b] | Trianguli Australis | TrA |
| Tucana[b] | Tucanae | Tuc |
| Ursa Major | Ursae Majoris | UMa |
| Ursa Minor | Ursae Minoris | UMi |
| Virgo | Virginis | Vir |
| (Piscis) Volans[b] | Volantis | Vol |
| Vulpecula[b] | Vulpeculae | Vul |

[a] Adopted in 1922 by the International Astronomical Union.
[b] Of modern origin.
[c] This constellation, which is very large, is often divided into four: Carina (Car), Puppis (Pup), Pyxis (Pyx), and Vela (Vel).

## THE CHEMICAL ELEMENTS

| Atomic Number | Name | Symbol[a] | Atomic Number | Name | Symbol[a] |
|---|---|---|---|---|---|
| 1 | Hydrogen | H | 48 | Cadmium | Cd |
| 2 | Helium | He | 49 | Indium | In |
| 3 | Lithium | Li | 50 | Tin | Sn |
| 4 | Beryllium | Be | 51 | Antimony | Sb |
| 5 | Boron | B | 52 | Tellurium | Te |
| 6 | Carbon | C | 53 | Iodine | I |
| 7 | Nitrogen | N | 54 | Xenon | Xe |
| 8 | Oxygen | O | 55 | Caesium | Cs |
| 9 | Fluorine | F | 56 | Barium | Ba |
| 10 | Neon | Ne | 57 | Lanthanum | La |
| 11 | Sodium | Na | 58 | Cerium | Ce |
| 12 | Magnesium | Mg | 59 | Praseodymium | Pr |
| 13 | Aluminum | Al | 60 | Neodymium | Nd |
| 14 | Silicon | Si | 61 | Illinium | Il |
| 15 | Phosphorus | P | 62 | Samarium | Sm |
| 16 | Sulphur | S | 63 | Europium | Eu |
| 17 | Chlorine | Cl | 64 | Gadolinium | Gd |
| 18 | Argon | A | 65 | Terbium | Tb |
| 19 | Potassium | K | 66 | Dysprosium | Dy |
| 20 | Calcium | Ca | 67 | Holmium | Ho |
| 21 | Scandium | Sc | 68 | Erbium | Er |
| 22 | Titanium | Ti | 69 | Thulium | Tm |
| 23 | Vanadium | V | 70 | Ytterbium | Yb |
| 24 | Chromium | Cr | 71 | Lutecium | Lu |
| 25 | Manganese | Mn | 72 | Hafnium | Hf |
| 26 | Iron | Fe | 73 | Tantalum | Ta |
| 27 | Cobalt | Co | 74 | Tungsten | W |
| 28 | Nickel | Ni | 75 | Rhenium | Re |
| 29 | Copper | Cu | 76 | Osmium | Os |
| 30 | Zinc | Zn | 77 | Iridium | Ir |
| 31 | Gallium | Ga | 78 | Platinum | Pt |
| 32 | Germanium | Ge | 79 | Gold | Au |
| 33 | Arsenic | As | 80 | Mercury | Hg |
| 34 | Selenium | Se | 81 | Thallium | Tl |
| 35 | Bromine | Br | 82 | Lead | Pb |
| 36 | Krypton | Kr | 83 | Bismuth | Bi |
| 37 | Rubidium | Rb | 84 | Polonium | Po |
| 38 | Strontium | Sr | 85 | Virginium[b] | — |
| 39 | Yttrium | Y | 86 | Radon | Rn |
| 40 | Zirconium | Zr | 87 | Alabamine[b] | — |
| 41 | Columbium | Cb | 88 | Radium | Ra |
| 42 | Molybdenum | Mo | 89 | Actinium | Ac |
| 43 | Masurium | Ma | 90 | Thorium | Th |
| 44 | Ruthenium | Ru | 91 | Protoactinium | Pa |
| 45 | Rhodium | Rh | 92 | Uranium | U |
| 46 | Palladium | Pd | 93 | Neptunium[c] | Np |
| 47 | Silver | Ag | 94 | Plutonium[c] | Pu |

[a] Derived from the Latin form of the name.
[b] Existence not universally accepted.
[c] "Transuranian" element; produced artificially.

## THE ORBITS OF THE PLANETS

| Planet | Mean Distance from Sun (a) ⊕ = 1 | Mean Distance from Sun (a) Millions of Miles | Period ($P$) | Eccentricity ($e$) | Inclination ($i$) | Long. of Node (☊) | Long. of Perihelion ($\pi$) |
|---|---|---|---|---|---|---|---|
| | | | | | ° | ° | ° |
| Mercury | .387 | 36.0 | 88.0 days | .206 | 7.0 | 47.6 | 76.5 |
| Venus | .723 | 67.2 | 224.7 | .007 | 3.4 | 76.1 | 130.7 |
| Earth | 1.000 | 92.9 | 365.3 | .017 | ... | .... | 101.9 |
| Mars | 1.524 | 141.5 | 687.0 | .093 | 1.9 | 49.1 | 334.9 |
| Jupiter | 5.203 | 483.3 | 11.86 yrs. | .048 | 1.3 | 99.8 | 13.3 |
| Saturn | 9.54 | 886. | 29.46 | .056 | 2.5 | 113.1 | 91.8 |
| Uranus | 19.19 | 1783. | 84.0 | .047 | 0.8 | 73.7 | 169.7 |
| Neptune | 30.07 | 2793. | 164.8 | .009 | 1.8 | 131.1 | 44.1 |
| Pluto | 39.46 | 3666. | 247.7 | .249 | 17.1 | 109.5 | 223.4 |

## CHARACTERISTICS OF THE SUN, MOON AND PLANETS

| Object | Symbol | Mean Diameter Miles | Mass ⊕ = 1 | Density Water = 1 | Axial Rotation | Mean Surface Gravity ⊕ = 1 | Albedo | Magnitude When Brightest |
|---|---|---|---|---|---|---|---|---|
| Sun | ☉ | 864,000 | 332,000 | 1.4 | $24^d.7$ (equatorial) | 27.9 | | − 26.7 |
| Moon | ☾ | 2,160 | .0123 | 3.3 | $27^d\ 7.7^h$ | .16 | .07 | − 12.6 |
| Mercury | ☿ | 3,010 | .056 | 3.8 | $88^d$ | .27 | .07 | 0 ± |
| Venus | ♀ | 7,580 | .82 | 4.9 | $30^d$ ? | .85 | .59 | − 4 ± |
| Earth | ⊕ | 7,918 | 1.00 | 5.5 | $23^h\ 56^m$ | 1.00 | .29 | |
| Mars | ♂ | 4,220 | .108 | 4.0 | $24^h\ 37^m$ | .38 | .15 | − 2 ± |
| Jupiter | ♃ | 87,000 | 318. | 1.3 | $9^h\ 50^m \pm$ | 2.6 | .56? | − 2 ± |
| Saturn | ♄ | 72,000 | 95. | .7 | $10^h\ 15^m \pm$ | 1.2 | .63? | 0 ± |
| Uranus | ♅ | 31,000 | 14.6 | 1.3 | $10^h.8 \pm$ | .9 | .63? | + 5.7 |
| Neptune | ♆ | 33,000 | 17.2 | 1.3 | $16^h$ ? | 1.0 | .73 | + 7.6 |
| Pluto | ♇ | 4,000? | <.1 | | | | | + 14 |

SATELLITES OF THE PLANETS

| Name | Stellar Mag. | Mean Dist. from Planet $''^{a}$ | Mean Dist. from Planet Miles | Revolution Period d | h | m | Diameter Miles | Discoverer |
|---|---|---|---|---|---|---|---|---|
| SATELLITE OF THE EARTH | | | | | | | | |
| Moon[b] | −12.6 | 530 | 238,857 | 27 | 07 | 43 | 2160 | |
| SATELLITES OF MARS | | | | | | | | |
| Phobos | 12 | 8 | 5,800 | 0 | 07 | 39 | 10? | Hall, 1877 |
| Deimos | 13 | 21 | 14,600 | 1 | 06 | 18 | 5? | Hall, 1877 |
| SATELLITES OF JUPITER | | | | | | | | |
| V | 13 | 48 | 112,600 | 0 | 11 | 57 | 100? | Barnard, 1892 |
| Io | 5 | 112 | 261,800 | 1 | 18 | 28 | 2300 | Galileo, 1610 |
| Europa | 6 | 178 | 416,600 | 3 | 13 | 14 | 2000 | Galileo, 1610 |
| Ganymede | 5 | 284 | 664,200 | 7 | 03 | 43 | 3200 | Galileo, 1610 |
| Callisto | 6 | 499 | 1,169,000 | 16 | 16 | 32 | 3200 | Galileo, 1610 |
| VI[b, d] | 14 | 3037 | 7,114,000 | 250 | 16 | | 100? | Perrine, 1904 |
| VII[b, d] | 16 | 3113 | 7,292,000 | 260 | 01 | | 40? | Perrine, 1905 |
| X[b, d] | 18 | 3116 | 7,300,000 | 260 | | | 15? | Nicholson, 1938 |
| XI[b, c, d] | 18 | 5990 | 14,000,000 | 692 | | | 15? | Nicholson, 1938 |
| VIII[b, c, d] | 16 | 6240 | 14,600,000 | 739 | | | 40? | Melotte, 1908 |
| IX[b, c, d] | 17 | 6360 | 14,900,000 | 758 | | | 20? | Nicholson, 1914 |
| SATELLITES OF SATURN | | | | | | | | |
| Mimas | 12 | 27 | 115,000 | 0 | 22 | 37 | 400? | W. Herschel, 1789 |
| Enceladus | 12 | 34 | 148,000 | 1 | 08 | 53 | 500? | W. Herschel, 1789 |
| Tethys | 11 | 43 | 183,000 | 1 | 21 | 18 | 800? | G. Cassini, 1684 |
| Dione | 11 | 55 | 234,000 | 2 | 17 | 41 | 700? | G. Cassini, 1684 |
| Rhea | 10 | 76 | 327,000 | 4 | 12 | 25 | 1100? | G. Cassini, 1672 |
| Titan | 8 | 177 | 759,000 | 15 | 22 | 41 | 2600? | Huygens, 1655 |
| Hyperion | 13 | 214 | 920,000 | 21 | 06 | 38 | 300? | G. Bond, 1848 |
| Iapetus[b] | 11 | 515 | 2,210,000 | 79 | 07 | 56 | 1000? | G. Cassini, 1671 |
| Phoebe[b, c, d] | 14 | 1870 | 8,034,000 | 550 | | | 200? | W. Pickering, 1898 |
| SATELLITES OF URANUS | | | | | | | | |
| Ariel[c, e] | 16 | 14 | 119,000 | 2 | 12 | 29 | 600? | Lassell, 1851 |
| Umbriel[c, e] | 16 | 19 | 166,000 | 4 | 03 | 28 | 400? | Lassell, 1851 |
| Titania[c, e] | 14 | 32 | 272,000 | 8 | 16 | 56 | 1000? | W. Herschel, 1787 |
| Oberon[c, e] | 14 | 42 | 364,000 | 13 | 11 | 07 | 900? | W. Herschel, 1787 |
| SATELLITE OF NEPTUNE | | | | | | | | |
| Triton[b, c] | 13 | 16 | 220,000 | 5 | 21 | 03 | 3000? | Lassell, 1846 |

[a] Apparent distance, in seconds, as seen from the Sun.
[b] Orbit highly inclined to plane of planet's equator.
[c] Revolution retrograde.
[d] Orbit considerably eccentric.
[e] Orbit inclined 82° to plane of planet's orbit.

## WAVE-LENGTHS OF SPECTRAL LINES

| Hydrogen | Calcium | Sodium | Helium | Mercury |
|---|---|---|---|---|
| $H\alpha$ 6562.79 | 4226.73 | $D_1$ 5895.92 | 10830.3 | 5790.65 |
| $H\beta$ 4861.33 | | $D_2$ 5889.95 | 6678.15 | 5769.59 |
| $H\gamma$ 4340.46 | H 3988.47* | | $D_3$ 5875.62 | 5460.74 |
| $H\delta$ 4101.74 | K 3933.67* | | 5015.68 | 4916.04 |
| $H\epsilon$ 3970.07 | | | 4471.48 | 4358.35 |
| $H\zeta$ 3889.06 | | | 3888.65 | 4077.81 |
| $H\eta$ 3835.40 | | | 4685.75* | 4046.56 |

* Ionized atoms.

## THE RINGS OF SATURN

| | |
|---|---|
| Radius of outer limit of ring system | 86,300 miles |
| Width of outer ring (A) | 11,100 " |
| Width of Cassini's division | 2,200 " |
| Width of Ring B | 18,000 " |
| Width of crepe ring (C) | 11,000 " |
| Distance from inner edge of Ring C to surface of Saturn | 6,000 " |
| Thickness of ring . . . less than | 100 " |

## THE PRINCIPAL STELLAR SPECTRAL TYPES

| Class | Distinguishing Feature | Example | Number of Stars Brighter than Mag. 6.25 | Color Index | Effective Temperature |
|---|---|---|---|---|---|
| O | Ionized helium | $\zeta$ Puppis | 20 | − 0.3 | > 30,000° |
| B | Neutral helium | $\epsilon$ Orionis | 696 | − 0.3 | 20,000 |
| A | Strong hydrogen lines | Sirius | 1885 | 0.0 | 11,000 |
| F | Intermediate class | Procyon | 720 | + 0.3 | 7,500 |
| G | Many metallic lines, strong H and K | The Sun | 609 | + 0.6 | 6,000 |
| K | Intermediate class | Arcturus | 1719 | + 1.0 | 4,200 |
| M | Titanium oxide bands | Antares | 457 | + 1.5 | 3,000 |
| N | Carbon bands | 19 Piscium | 8 | + 2.5 | 3,000 |

## THE BRIGHTEST STARS

### Stars Brighter than Visual Magnitude 1.50

| | Mag. | Sp. | Dist. in l.-y. | | Mag. | Sp. | Dist. in l.-y. |
|---|---|---|---|---|---|---|---|
| Canis Maj. (Sirius) | − 1.58 | A0 | 8.6 | α Orionis (Betelgeuse) | 1.0 to 1.4 | M2 | 300 |
| Argûs (Canopus) | − 0.86 | F0 | 100 | α Crucis | 1.05 | B1 | 220 |
| Centauri | + 0.06 | G0 | 4.3 | α Tauri (Aldebaran) | 1.06 | K5 | 53 |
| Lyrae (Vega) | 0.14 | A0 | 27 | α Virginis (Spica) | 1.21 | B2 | 120 |
| Aurigae (Capella) | 0.21 | G0 | 42 | β Geminorum (Pollux) | 1.21 | K0 | 29 |
| Bootis (Arcturus) | 0.24 | K0 | 33 | α Scorpii (Antares) | 1.22 | Ma | 250 |
| Orionis (Rigel) | 0.34 | B8 | 540 | α Piscis Australis (Fomalhaut) | 1.29 | A3 | 23 |
| Canis Min. (Procyon) | 0.48 | F5 | 11 | α Cygni (Deneb) | 1.33 | A2 | 400 |
| Eridani (Achernar) | 0.60 | B5 | 70 | α Leonis (Regulus) | 1.34 | B8 | 67 |
| Centauri | 0.86 | B1 | 190 | | | | |
| Aquilae (Altair) | 0.89 | A5 | 15.7 | | | | |

### Stars of Visual Magnitude 1.50 to 2.00

| | Mag. | Sp. | Dist. in l.-y. | | Mag. | Sp. | Dist. in l.-y. |
|---|---|---|---|---|---|---|---|
| Crucis | 1.50 | B1 | 272 | β Argûs | 1.80 | A0 | ? |
| Geminorum (Castor) | 1.58 | A0 | 47 | α Trianguli Australis | 1.88 | K2 | 130 |
| Crucis | 1.60 | Mb | 72 | α Persei | 1.90 | F5 | 130 |
| Canis Majoris | 1.63 | B1 | 326 | η Ursae Majoris | 1.91 | B3 | 130 |
| Ursae Majoris | 1.68 | A0 | 48 | ζ Orionis | 1.91 | B0 | 410 |
| Orionis (Bellatrix) | 1.70 | B2 | 250 | γ Geminorum | 1.93 | A0 | 93 |
| Scorpii | 1.71 | B2 | 204 | α Ursae Majoris | 1.95 | K0 | 109 |
| Argûs | 1.74 | K0 | 326 | ε Sagittarii | 1.95 | A0 | 163 |
| Orionis | 1.75 | B0 | 410 | δ Canis Majoris | 1.98 | F8 | 410 |
| Tauri | 1.78 | B8 | 93 | β Canis Majoris | 1.99 | B1 | 360 |

## CONSTANTS

### Units of Length

1 Angstrom unit = $10^{-8}$ cm.
1 micron = $10^{-4}$ cm. = 0.001 mm.
1 centimeter = 0.3937 inch
1 inch = 2.54 centimeters
1 meter = $10^2$ cm. = 3.28084 feet
1 kilometer = $10^5$ cm. = 0.62137 miles
1 mile = $1.60935 \times 10^5$ cm. = 1.60935 km. = 5280 feet
1 astronomical unit = $1.49678 \times 10^{13}$ cm. = 93,005,000 miles
1 light-year = $9.463 \times 10^{17}$ cm. = $5.880 \times 10^{12}$ miles = 63,300 astron. units

### Units of Time

Sidereal day = 23*h* 56*m* 04.09*s* of mean solar time
Mean solar day = 86400 seconds
Synodical month = 29*d* 12*h* 44*m*; sidereal month = 27*d* 07*h* 43*m*
Tropical year (ordinary) = 365*d* 05*h* 48*m* 46*s* = 31,556,926 seconds
Sidereal year = 365*d* 06*h* 09*m* 10*s* = 31,558,150 seconds
Eclipse year = 346*d* 14*h* 53*m*

### The Earth

Equatorial radius, $a$ = 3963.35 miles
Polar radius, $b$ = 3950.01 miles; flattening, $c = (a - b)/a = 1/297.0$
Acceleration due to gravity, $g$ = 32.17 ft/sec². = 981 cm./sec². (in latitude 45°)
Mass of Earth = $6.6 \times 10^{21}$ tons; velocity of escape from Earth = 6.94 miles/sec

### Earth's Orbital Motion

Solar parallax = 8″.79; constant of aberration = 20″.47
Annual general precession = 50″.26; obliquity of ecliptic = 23° 26′ 50″ (1939)
Orbital velocity = 18.5 miles/sec.
= 29.8 km./sec.; parabolic velocity at Earth = 26.2 miles/sec.

### Solar Motion

Solar apex, R. A. 18*h* 04*m*; Dec. +31°
Speed relative to neighboring stars = 12.2 miles/sec. = 20 km./sec.

### The Galactic System

North pole of galactic plane R. A. 12*h* 40*m*, Dec. +28° (1900)
Center, 325° galactic longitude, = R. A. 17*h* 24*m*, Dec. −30°
Distance to center = 30,000 light-years; diameter = 100,000 light-years
Rotational velocity (at Sun) = 262 km./sec.
Rotational period (at Sun) = $2.2 \times 10^8$ years
Mass = $2 \times 10^{11}$ solar masses

### Extragalactic Nebulae

Red shift = +101 miles/sec. per million light-years

### Radiation Constants

Velocity of light = 299,774 km./sec. = 186,271 miles/sec.
Solar constant = 1.93 gram calories/square cm./minute
Light ratio for one magnitude = 2.512; log ratio = 0.4000
Radiation from a star of zero apparent magnitude = $3 \times 10^{-6}$ meter candles
Total energy emitted by a star of zero absolute magnitude = $5 \times 10^{25}$ horsepower

### Miscellaneous

Constant of gravitation, $G = 6.670 \times 10^{-8}$ c.g.s. units
Mass of the electron, $m = 9.035 \times 10^{-28}$ gm.; mass of the proton = $1.662 \times 10^{-24}$ gm.
1 radian = 57°.2958
= 3437′.75
= 206,265″
$\pi$ = 3.141,592,653,6
No. of square degrees in the sky
= 41,253

## TABLE OF JULIAN DAYS

Add together the year number, the month number, and the day of the month.

| | | | | | |
|---|---|---|---|---|---|
| 1940 | 2429629 | 1963 | 2438030 | 1986 | 2446431 |
| 1941 | 29995 | 1964 | 38395 | 1987 | 46796 |
| 1942 | 30360 | 1965 | 38761 | 1988 | 47161 |
| 1943 | 30725 | 1966 | 39126 | 1989 | 47527 |
| 1944 | 31090 | 1967 | 39491 | 1990 | 47892 |
| 1945 | 31456 | 1968 | 39856 | | |
| 1946 | 31821 | 1969 | 40222 | | |
| 1947 | 32186 | 1970 | 40587 | | |
| 1948 | 32551 | 1971 | 40952 | | |
| 1949 | 32917 | 1972 | 41317 | | |
| 1950 | 33282 | 1973 | 41683 | | |
| 1951 | 33647 | 1974 | 42048 | | |
| 1952 | 34012 | 1975 | 42413 | | |
| 1953 | 34378 | 1976 | 42778 | | |
| 1954 | 34743 | 1977 | 43144 | | |
| 1955 | 35108 | 1978 | 43509 | | |
| 1956 | 35473 | 1979 | 43874 | | |
| 1957 | 35839 | 1980 | 44239 | | |
| 1958 | 36204 | 1981 | 44605 | | |
| 1959 | 36569 | 1982 | 44970 | | |
| 1960 | 36934 | 1983 | 45335 | | |
| 1961 | 37300 | 1984 | 45700 | | |
| 1962 | 37665 | 1985 | 46066 | | |

| Month Number | | |
|---|---|---|
| | Common | Leap |
| Jan. | 0 | 0 |
| Feb. | 31 | 31 |
| March | 59 | 60 |
| April | 90 | 91 |
| May | 120 | 121 |
| June | 151 | 152 |
| July | 181 | 182 |
| Aug. | 212 | 213 |
| Sept. | 243 | 244 |
| Oct. | 273 | 274 |
| Nov. | 304 | 305 |
| Dec. | 334 | 335 |

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